JOHN NAPIER OF MERCHISTON

From the portrait painted from life in 1616, now in the gallery of the University of Edinburgh. Reproduced for Durell's Trigonometry by permission of the University authorities. (See page 169.)

PLANE TRIGONOMETRY

BY

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PREFACE

The principal object in writing this book has been the same as that which has governed the author in writing other mathematical text-books; viz., to bring out the fundamental utilities which underlie and grow out of the principles presented. Not only is the fundamental source of new power in Trigonometry frequently emphasized, but each new process is taken up, not arbitrarily, but for the sake of the economy or new power which it gives.

Among other special features of the book, the following may be mentioned:

Under each case in the solution of triangles two groups of examples are given; one with the degree divided sexagesimally, and the other with the degree divided decimally. The inclusion of the examples in terms of the decimally divided degree meets the new requirements of Harvard, Yale, and Princeton.

A chapter is given on logarithms and their properties. Practical examples are included in this chapter which are not only interesting in themselves, but which afford a review of and a correlation with other branches of mathematics.

When use is made of the line equivalents of the trigonometric ratios, it is specially shown that such treatment is merely a convenient substitute for the ratio treatment, and the method of this substitution is shown and its processes carefully safeguarded.

A chapter is given in which the applications of trigonometry are reduced to a system.

The subject-matter of the text-book is enlivened and made more vital and human by a chapter on the history of trigonometry.

Attention is also called to the method in which logarithmic work is arranged. This form of tabulation is used, for instance, in the designing room in the United States Navy Department and by engineers in general. Among the advantages of this method of arranging logarithmic work are the following:

(1) It abbreviates the work by omitting the equality marks.

(2) It includes within itself the actual numbers whose logarithms are being used.

(3) It facilitates the correction of mistakes by including and presenting in order all the steps of a logarithmic reduction.

(4) The arrangement of the work is such that after the pupil has acquired facility in logarithmic computation, some of the steps in the tabulation may be omitted without changing the general form of tabulation.

The author wishes to express his especial indebtedness to Mr. Howard Smith of the Hill School, Pottstown, Pa., to whom most of the examples are due, and who has made important suggestions concerning other parts of the work. The writer is also under obligation to his colleague, Mr. J. H. Keener, to whom the examples in the General Review Exercise are mainly due. Professor William Betz of the East Rochester High School, Rochester, N.Y., Dr. Henry A. Converse of the Polytechnic Institute, Baltimore, Md., and Professor William H. Metzler of Syracuse University have also aided the writer by important corrections and suggestions.

FLETCHER DURELL.

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CHAPTER I

LOGARITHMS

1. The **logarithm** of a number is the exponent of that power of another number, taken as the base, which equals the given number.

Thus, $1000 = 10^3$, hence $\log 1000 = 3$, 10 being taken as the base; again, if 8 be taken as the base, $4 = 8^{\frac{2}{3}}$, hence $\log 4 = \frac{2}{3}$; also, if 5 be taken as the base, $\log 125 = 3$, $\log \frac{1}{25} = -2$, etc.

The base used is sometimes stated in the context as above; but, when desirable, it is indicated by writing it as a small subscript to the word log.

Thus the above expressions might be written,

$$\log_{10} 1000 = 3;\ \log_8 4 = \tfrac{2}{3};\ \log_5 125 = 3;\ \log_5 \tfrac{1}{25} = -2;\ \text{etc.}$$

In general, by the definition of a logarithm,

$$\text{number} = (\text{base})^{\text{logarithm}},$$

or

$$N = B^l;\ \text{hence } \log_B N = l.$$

2. Uses or Utility of Logarithms. One of the principal uses of logarithms is to simplify numerical work. For instance, by logarithms the numerical work of *multiplying* two numbers is converted into the simpler work of *adding* the logarithms of these numbers. To illustrate this principle we may take the simple case of multiplying two numbers which are exact powers of 10, as 1000 and 100. Thus

$$1000 = 10^3$$
$$100 = 10^2$$
hence $$1000 \times 100 = 10^5 = 100{,}000,$$

the multiplication being performed by the addition of exponents.

Similarly, if $$384 = 10^{2.58433+}$$
and $$25 = 10^{1.39794+},$$

384 may be multiplied by 25 by adding the exponents of $10^{2.58433+}$ and $10^{1.39794+}$, thus obtaining $10^{3.98227+}$, and then getting from a table of logarithms the value of $10^{3.98227+}$, viz. 9600.

In like manner, by the use of logarithms, the process of dividing one number by another is converted into the simpler process of subtracting one exponent, or log, from another; the process of involution is converted into the simpler process of multiplication; and the extraction of a root into the simpler process of division.

The saving of labor effected by the use of logarithms can be increased by committing to memory the logs of certain much used numbers as of 2, 3, $\cdots$ 9, π, $\sqrt{\pi}$, $\frac{1}{\pi}$, $\sqrt{2}$, $\sqrt{3}$, etc.

Also by use of the *slide rule*, the practical use of logarithms is reduced to sliding one rod along another and reading off the number corresponding to the terminal position of one end of a rod. If the teacher can find time, it will be a useful exercise to teach the class the use of the slide rule in connection with the study of this chapter.

3. Systems of Logarithms. Any positive number except unity may be made the base of a system of logarithms.

Two principal systems are in use:

1. The **Common** (or **Decimal**) or **Briggsian** System, in which the base is 10. This system is used almost exclusively when logarithms are employed to facilitate numerical computations.

2. The system termed **Natural** or **Napierian**, in which the base is 2.7182818^+. This system is generally used in algebraic processes, as in demonstrating the properties of algebraic expressions, etc.

EXERCISE I

1. Give the value of each of the following: $\log_3 9$, $\log_3 27$, $\log_4 64$, $\log_4 \frac{1}{16}$, $\log_3 \frac{1}{9}$, $\log_3 \frac{1}{81}$, $\log_{10} \frac{1}{10}$, $\log_{10} .01$, $\log_{10} .001$.

2. Also of $\log_2 32$, $\log_2 \frac{1}{32}$, $\log_2 \frac{1}{128}$, $\log_4 8$, $\log_8 16$.

3. Simplify $\log_2 4 + \log_3 9 + \log_{10} .1 - \log_3 \frac{1}{9}$.

4. Write out the value of each power of 2 up to 2^{20} (thus $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, etc.) in the form of a table.

5. By means of this table multiply 32 by 8, converting the multiplication into an addition of exponents.

6. In like manner convert each of the following multiplications into an addition: 32×16; 64×32; 1024×16; 512×64.

7. Also convert each of the following divisions into a subtraction: $1024 \div 16$; $512 \div 64$; $32768 \div 1024$.

8. Also convert each of the following involutions into a multiplication: $(32)^3$; $(64)^2$; $(32)^4$.

9. Also convert each of the following root extractions into a division: $\sqrt[3]{64}$; $\sqrt[5]{1024}$; $\sqrt[4]{4096}$.

10. Let the pupil make up two examples like those in Ex. 6; in Ex. 8; in Ex. 9.

11. Let the pupil construct a table of powers of 3 and make up similar examples concerning it.

COMMON SYSTEM

4. **Characteristic and Mantissa.** If a given number, as 384, be not an exact power of the base, its logarithm, as 2.58433^+, consists of two parts, the whole number called the *characteristic*, and the decimal part called the *mantissa*.

To obtain a rule for determining the characteristic of a given number (the base being 10), we have,

$$\begin{aligned} 10{,}000 &= 10^4, \text{ hence } \log 10{,}000 = 4; \\ 1000 &= 10^3, \text{ hence } \log 1000 = 3; \\ 100 &= 10^2, \text{ hence } \log 100 = 2; \\ 10 &= 10^1, \text{ hence } \log 10 = 1. \end{aligned}$$

Hence any number between 1000 and 10,000 has a logarithm between 3 and 4; that is, the log consists of 3 and a fraction. But every integral number between 1000 and 10,000 contains four digits. Hence every integral number containing *four* figures has 3 for a characteristic.

Similarly every number between 100 and 1000, and therefore containing *three* figures to the left of the decimal point, has 2 for a characteristic; every number between 10 and 100 (that is, every number containing *two* integral figures) has 1 for a characteristic; and every number between 1 and 10 (that is, every number containing *one* integral figure) has 0 for a characteristic.

Hence, *the characteristic of an integral or mixed number is one less than the number of figures to the left of the decimal point.*

5. Characteristic of a Decimal Fraction.

$$1 = 10^0. \quad \therefore \log 1 = 0;$$

$$.1 = \frac{1}{10} = 10^{-1}. \quad \therefore \log .1 = -1;$$

$$.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}. \quad \therefore \log .01 = -2;$$

$$.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}. \quad \therefore \log .001 = -3, \text{ etc.}$$

Hence the logarithm of any number between .1 and 1 (as of .4 for instance) will lie between -1 and 0 and hence will consist of -1 plus a positive fraction; also the logarithm of every number between .01 and .1 (as of .0372 for instance) will be between -2 and -1, and hence will consist of -2 plus a positive fraction; and so on.

Hence, *the characteristic of a decimal fraction is negative, and is numerically one more than the number of zeros between the decimal point and the first significant figure.*

There are two ways in common use for writing the characteristic of a decimal fraction.

Thus, (1) $\log .0384 = \overline{2}.58433$, the minus sign being placed over the characteristic 2, to show that it alone is negative, the mantissa being positive.

Or (2) 10 is added to and subtracted from the log, giving

$$\log .0384 = 8.58433 - 10.$$

In practice the following rule is used for determining the characteristic of the logarithm of a decimal fraction:

Take one more than the number of zeros between the decimal point and the first significant figure, subtract it from 10, *and annex* -10 *after the mantissa.*

EXERCISE 2

Give the characteristic of:

1. 452.	**6.** .08267.	**11.** 7.
2. 16730.	**7.** 1.0042.	**12.** 6267.3.
3. 767.5.	**8.** 7.92631.	**13.** .000227.
4. 64.56.	**9.** .007.	**14.** 100.58.
5. 9.22678.	**10.** .0000625.	**15.** 23.7621.

16. How many figures to the left of the decimal point (or how many zeros immediately to the right) are there in a number the characteristic of whose logarithm is 3? 2? 5? 1? 0? 4? $8-10$? $7-10$? $9-10$?

17. Can you make up a rule for fixing the decimal point in the number which corresponds to a given logarithm?

6. Mantissas of numbers are computed by methods, usually algebraic, which lie outside the scope of this book. After being computed the mantissas are arranged in tables, from which they are taken when needed. In this connection it is important to note that

The position of the decimal point in a number affects only the characteristic, not the mantissa, of the logarithm of the number.

Thus, if $\log 6754 = 3.82956$

$$\log 67.54 = \log \frac{6754}{100} = \log \frac{10^{3.82956}}{10^2} = \log 10^{1.82956} = 1.82956.$$

In general

$$\begin{aligned}
\log 6754 &= 3.82956\\
\log 675.4 &= 2.82956\\
\log 67.54 &= 1.82956\\
\log 6.754 &= 0.82956\\
\log 0.6754 &= 9.82956 - 10\\
\log 0.06754 &= 8.82956 - 10, \text{ etc.}
\end{aligned}$$

7. Direct Use of a Table of Logarithms; that is *given a number, to find its logarithm.* For methods in detail see Introduction to Logarithmic Tables (Arts. 2–5 and 17).

EXERCISE 3

Using five-place tables find the logarithm of each of the following numbers:

1. 7627.	**10.** .00672.	**19.** 17.6287.
2. 6720.	**11.** .000007.	**20.** 42.
3. 82.	**12.** 400000.	**21.** .000001.
4. 7862.	**13.** 14.6235.	**22.** .0186789.
5. 75.	**14.** .00226725.	**23.** 32679.
6. 157.	**15.** 87.	**24.** 3267.9.
7. 36278.	**16.** .76.	**25.** 326.79.
8. 67.222.	**17.** .000125.	**26.** 32.679.
9. 3.3427.	**18.** 100.25.	**27.** 3.2679.

28. Commit to memory the mantissa for each of the following: 2, 3, 5, π. Then write at sight the log of each of the following, 200, 3000, 50, 100 π, 20, .002, 30, .0005, $\frac{\pi}{100}$, .3, .2, 10 π, 20,000.

Using four-place tables, find the logarithm of each of the following:

29. 12.67.	**36.** 24.68.	**43.** .000036775.
30. 762.8.	**37.** .11116.	**44.** .0026382.
31. 42.68.	**38.** 11.685.	**45.** 28966.
32. 1.2267.	**39.** .0012678.	**46.** 19.572.
33. .0263.	**40.** 965.3.	**47.** .8625.
34. .0012678.	**41.** 1.4676.	**48.** .0100267.
35. 1.0026.	**42.** 1.7628.	**49.** 2.225.

50. Work Ex. 28 for four-place tables.

8. Inverse Use of a Table of Logarithms; that is, *given a logarithm, to find the number corresponding to it* (called its *antilogarithm*). See Introduction to the Logarithmic Tables (Arts. 6 and 17).

EXERCISE 4

Using five-place tables, find the antilogarithm of each of the following:

1. 1.41863.
2. 2.19756.
3. 0.98349.
4. 7.68416.
5. 9.22321 − 10.
6. 6.42857 − 10.
7. 6.59068.
8. 5.74706 − 10.
9. 8.00400.

10. Find log of 2.34578.
11. Find antilog of 2.34578.
12. Find log of 1.03678.
13. Find antilog of 1.03678.
14. Find log of 3.21678.
15. Find antilog of 3.21678.
16. Find antilog of 6.00371.
17. Find log of 6.00371.
18. Find antilog of 4.98672.
19. Find log of 4.98672.

Find the number corresponding to each of the following logarithms, using four-place tables.

20. 1.4082.
21. 2.7332.
22. 3.2335.
23. 9.1546 − 10.
24. 2.0326.
25. 1.0135.
26. 8.0283 − 10.
27. 7.1170 − 10.
28. 5.0019 − 10.
29. 2.6575.
30. 4.3490 − 10.
31. 2.8177.

32. Find antilog of 2.3041.
33. Find log of 2.3041.
34. Find log of 0.4975.
35. Find antilog of 0.4975.
36. Find antilog of 1.6924.
37. Find log of 1.6924.

COMPUTATIONS BY USE OF LOGARITHMS

9. Properties of Logarithms used in Numerical Computations. It is shown in algebra that

$$a^x \cdot a^y = a^{x+y}; \text{ and also that } (a^x)^p = a^{px}.$$

Using these properties of exponents, it can be shown that

1. $\log (mn) = \log m + \log n$.
3. $\log m^p = p \log m$.

2. $\log \left(\frac{m}{n}\right) = \log m - \log n$.
4. $\log \sqrt[p]{m} = \frac{\log m}{p}$.

For $m = 10^x$. $\therefore \log m = x$.
$n = 10^y$. $\therefore \log n = y$.

$$\therefore mn = 10^{x+y} \text{ or } \log mn = x + y = \log m + \log n. \tag{1}$$

$$\text{Also } \frac{m}{n} = \frac{10^x}{10^y} = 10^{x-y}, \text{ or } \log \frac{m}{n} = x - y = \log m - \log n. \tag{2}$$

Also $m^p=(10^x)^p=10^{px}$. $\therefore \log m^p=px=p\cdot\log m$, (3)

and $\sqrt[p]{m}=10^{\frac{x}{p}}$. $\therefore \log\sqrt[p]{m}=\frac{x}{p}=\frac{\log m}{p}$. (4)

Hence:

I. **To multiply numbers:**

Add their logarithms and find the antilogarithm of the sum. This will be the product of the numbers.

II. **To divide one number by another:**

Subtract the logarithm of the divisor from the logarithm of the dividend and obtain the antilogarithm of the difference. This will be the quotient.

III. **To raise a number to a required power:**

Multiply the logarithm of the number by the index of the required power and find the antilogarithm of the product.

IV. **To extract the required root of a number:**

Divide the logarithm of the number by the index of the required root and find the antilogarithm of the quotient.

Ex. 1. Multiply 561.75 by .03286 by the use of logarithms.

$$\begin{aligned}\log(561.75\times .03286) &= \log 561.75+\log .03286\\ \log 561.75 &= 2.74954\\ \log .03286 &= \underline{8.51667-10}\\ \text{antilog } 1.26621 &= 18.4591, \quad \textit{Product.}\end{aligned}$$

The following, however, is the arrangement of work used by many practical computers. It has the advantage of showing all the steps in a complex logarithmic computation. (See p. 12, etc.)

$$\begin{array}{rl} 561.75 & \log 2.74954\\ .03286 & \log \underline{8.51667-10}\\ \textit{Answer}=\mathbf{18.4591} & \log 1.26621\end{array}$$

Observe that "561.75 log 2.74954" reads "561.75, its log is 2.74954," etc.

Ex. 2. Compute the amount of \$1 at 5 per cent compound interest for 20 years.

The amount of \$1 at 5% for 20 years $= (1.05)^{20}$.
1.05 log 0.02119; 20 log 0.42380
Amount = **2.65338** log 0.42380.

If the student will compute the value of $(1.05)^{20}$ by continued multiplication, and compare the labor in such a process with that involved in the above process, he will have a good illustration of the usefulness of logarithms.

Ex. 3. Extract approximately the cube root of 532.768.

532.768 log 2.72653 $\frac{1}{3}$ log 0.90884.
Root = **8.1066** log 0.90884.

10. Cologarithm. In operations involving division, instead of subtracting the logarithm of the divisor, it is usual to add its cologarithm. The cologarithm of a number is obtained by subtracting the logarithm of the number from $10 - 10$. Hence *adding the cologarithm of the divisor gives the same result as subtracting its logarithm.* The use of the cologarithm saves figures, and gives a more orderly and compact statement of the work.

The cologarithm of a number is obtained directly from a table of logarithms by the following rule:

Subtract each figure of the logarithm of the given number from 9 *except the last significant figure, which subtract from* 10.

Ex. 1. Find the colog of 37.16.

log 37.16 = 1.57008.
Hence, colog 37.16 = 8.42992 − 10.

Ex. 2. Divide 52678 by 37.16 by the use of the cologarithm of the divisor.

52678 log 4.72163
37.16 log 1.57008 colog 8.42992 − 10.
Quotient = **1417.58** log 3.15155.

11. *In the* **extraction of the root of a decimal number** *it is best to add to and subtract from the logarithm of the decimal*

number such a multiple of 10 *that the last term of the quotient shall be* 10.

Ex. Extract the seventh root of .0854329.

$$\begin{array}{rl} .0854329 & \log 8.93162 - 10 \\ & \underline{60 \qquad\quad - 60} \\ & 7)\overline{68.93162 - 70} \\ \textit{Root} = \mathbf{.703667} & \log 9.84737 - 10 \end{array}$$

12. Computations involving Negative Numbers. In computing, by the use of logarithms, the value of expressions containing one or more negative factors, *first*, determine the *sign* of the result; *second*, determine the *magnitude* of the result by treating all the factors as if they were positive and using logarithms.

Ex. Compute $\dfrac{-876}{795}$.

The result must be negative, since a negative number divided by a positive number gives a negative quotient.

The magnitude of the result is determined by computing the value of $\dfrac{876}{795}$.

EXERCISE 5

Compute by means of five-place logarithms the value of each of the following:

1. 85×627.
2. 26.27×52.67.
3. 8.25×25675.
4. $\dfrac{1768}{211.6}$.
5. $45 \times 27.68 \times .0967 \times 4.2678$.
6. $(2.67)^3$.
7. $\dfrac{27.8675}{18.678}$.
8. $(.5278)^7$.
9. $\sqrt[3]{156.78}$. Also, if you can, extract the cube root of 156.78 without the use of logarithms. About how much more work in this process than in the logarithmic process? Which process is more likely to be accurate, the long or the short one?
10. $\sqrt[4]{.86785}$. Also extract the square root of the square root of

.86785. About how much longer is this process than the logarithmic work?

11. $\sqrt[7]{-76.526}$. **12.** $\sqrt[3]{-.00021}$. **13.** $\sqrt[5]{-.00062367 \times 7.867}$.

Find the compound interest on:

14. \$15375 for 20 years at 6%. Make the computation without the use of logs. What fraction of the work is avoided by the use of logs?

15. \$323.50 for 12 years at 8%.

16. In 1623 the Dutch bought Manhattan Island from the Indians for \$24. What would this sum amount to at the present time, if it had been placed on interest at 6%, the interest to be compounded annually?

17. By aid of the logs committed to memory in Ex. 28, page 12, compute each of the following: $\frac{200}{376}$; $\frac{100\,\pi}{58}$; $\frac{300 \times 500}{\pi}$.

18. Also obtain the colog of 43560 (the number of square feet in an acre) and use it to find the area in acres of a field 200 ft. × 300 ft.; one 300 ft. × 500 ft.; one 1000 ft. × 2000 ft.

Using four-place logarithms, compute the value of the following:

19. 1.2634×26.42.

20. $.001467 \times 96.8 \times 47.37$.

21. $556.85 \times .00016277 \times 4.6$.

22. $(12.67)^3$.

23. $(3.176)^7$.

24. $\sqrt{\frac{22.93}{16.91}}$.

25. $\frac{.0016666}{.00042635}$.

26. $\sqrt[3]{42.67 \times .10126 \times 9.2}$.

27. $\sqrt[5]{.0000073}$.

28. Work Exs. 17 and 18 by the four-place tables.

29. Why are four-place logarithmic tables sufficiently accurate for the work of a carpenter or land surveyor?

Find the compound interest on:

30. \$359.67 for 8 years at 6%.

31. \$100 for 37 years at 4%.

32. \$4962.75 for 16 years at 5%. Try to compute this without the use of logs. About how much longer is the process without logs? Which process is more likely to be accurate?

13. Complex Computations. By the use of the properties of logarithms demonstrated in Art. 9, the value of a complex numerical expression may be computed.

Ex. 1. Compute $\sqrt{\dfrac{215}{67 \times 52}}$ by the use of logarithms.

$$\log \sqrt{\frac{215}{67 \times 52}} = \tfrac{1}{2} \log \frac{215}{67 \times 52} = \tfrac{1}{2}(\log 215 + \operatorname{colog} 67 + \operatorname{colog} 52).$$

Before looking up the logarithm of any number in the table, it is important to make a scheme or outline of the work, leaving blank the places which are to be filled in by logs taken from the table. Thus the preliminary outline for Ex. 1 would be as follows:

		215 log	. . . .
67 log	. . . . . .	colog	. . . .
52 log	. . . . . .	colog	. . . .
		2)	. . . .
Answer =	. . . . . .	log	. . . .

After looking up and inserting the logarithms and completing the computation, the work will appear as follows:

		215 log	2.33244
67 log	1.82607	colog	8.17393 − 10
52 log	1.71600	colog	8.28400 − 10
		2)	18.79037 − 20
Answer =	**.248422**	log	9.39519 − 10

One advantage of the above method of tabulating logarithmic work is that without essential change in the form of the tabulating, the work may be presented in the above complete form, or in a more condensed form (at the option of the teacher), as by omitting the logs of 67 and 52 and giving only their respective cologs in the tabulation.

Ex. 2. Compute $\dfrac{\sqrt{21.8} \cdot \sqrt[3]{.03678}}{.28756}$ by the use of logarithms.

21.8 log	1.33846	$\frac{1}{2}$ log	0.66923
.03678 log	8.56561 − 10	$\frac{1}{3}$ log	9.52187 − 10
.28756 log	9.45873 − 10	colog	0.54127
Answer =	**5.39975**	log	0.73237

14. Exponential Equations. An exponential equation is one in which the unknown quantity occurs in the exponent of some term or factor, as $a^x = b$. An equation of this kind can often be solved by the use of logarithms.

Ex. Find the value of x in the equation $.3^x = 2$.

Taking the logarithm of each member of the equation,

$$x \log .3 = \log 2.$$

Hence* $x = \dfrac{\log 2}{\log .3} = \dfrac{0.30103}{9.47712 - 10} = \dfrac{0.30103}{-0.52288} = -.575^{+}$, *Ans.*

EXERCISE 6

Using five-place tables, compute the value of the following:

(Do not fail to make an outline of the work in each example before looking up any logarithms.)

1. $\dfrac{\sqrt{21.82} \times \sqrt[3]{.0071725}}{.92678}$.

2. $\dfrac{(\sqrt[5]{.26728})^3}{(.06756)^2}$.

3. $\sqrt{\dfrac{.59 \times 2209}{47 \times .3481}}$.

4. $\sqrt{(.19678)^2 - (.072567)^2}$.

5. $\dfrac{(\sqrt{278.2} \times 2.578)^2}{\sqrt[3]{.00231} \times \sqrt{76.19}}$.

6. $\sqrt[5]{\dfrac{267.85 \times 7 \times .000925 \times 468.765}{(21.67)^2 \times .00096725 \times \sqrt{567.256}}}$.

7. Using the logarithms committed to memory in Ex. 28, Exercise 3, compute each of the following:

$$\sqrt{\frac{300 \times 500}{\pi}}; \quad \sqrt[3]{\frac{300\,\pi}{3.1416}}; \quad \sqrt{\frac{200 \times 30}{37\,\pi}}.$$

8. If there are 39.37 inches in a meter, convert the following into feet: 500 meters; 7294 meters; 300 meters (height of Eiffel Tower). What logs used in the first of these computations could be retained and used in the other computations?

Solve for x:

9. $6^x = 67$.

10. $14^{2x+3} = 2167$.

11. $2.8^x = .1967$.

12. $.85^x = .01978$.

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*If the teacher prefers, the remainder of the work for this example may be arranged as follows:

$$\log x + \log(\log .3) = \log(\log 2).$$
$$\therefore \log x = \mathrm{l} \cdot \log 2 - \mathrm{l} \cdot \log .3.$$

2 log 0.30103 l · log 9.47861 − 10.

.3 log 9.47712 − 10 (or − .52288) l · log (−) 9.71840 − 10 colog 0.28160.

$x = -.5757^{+}$ log 9.76021 − 10.

13. Find the side of a square whose area is equal to that of a parallelogram whose base is 22.678 and whose altitude is 17.375.

14. Find the side of a square whose area is equal to that of a circle whose radius is 13.56.

15. Calculate the value of K in the equation,

$$K = \sqrt{s(s-a)(s-b)(s-c)},$$

when $s = \dfrac{a+b+c}{2}$, and $a = 17.6,\quad b = 21.675,\quad c = 26.427.$

16. Calculate the value of b in the equation, $b = \sqrt{a^2 - c^2}$, when $a = .17623$ and $c = .12673$. (Use $b = \sqrt{(a+c)(a-c)}$, etc.)

17. Find the volume of a sphere whose radius is 14.7, if $V = \frac{4}{3}\pi R^3$ and $\pi = 3.1416$.

18. Given $t = 8,\quad a = 32.17,$ find s, if $s = \frac{1}{2}at^2$.

19. Given $s = \dfrac{a+b+c}{2}$ and $a = .1732,\quad b = .14326,\quad c = .2242,$ find h, if $h = \dfrac{2}{c}\sqrt{s(s-a)(s-b)(s-c)}$.

20. Given $R = 14.16$ and $\pi = \frac{22}{7}$, find S, if $S = 4\pi R^2$.

21. Given $\pi = \frac{22}{7}$ and $D = 23.8$, find V, when $V = \frac{1}{6}\pi D^3$.

22. In how many years will \$1 at compound interest at 5% amount to \$25?

Using four-place tables, compute the value of the following:

23. $\sqrt[3]{\dfrac{529}{67 \times 51.8}}$.

24. $\sqrt{\dfrac{.3756 \times .265}{.227 \times .1678}}$.

25. $\dfrac{16.78}{12.97}\sqrt{\dfrac{12.97}{16.78}}$.

26. $\sqrt[3]{(125)^2 - (67)^2}$.

27. $\dfrac{47.326}{.10021}\sqrt{\dfrac{55400 \times 8}{123456 \times .007}}$.

28. $\sqrt[3]{.2167} \times \sqrt[5]{\dfrac{21.67}{32.77}} \times \sqrt{\dfrac{.16765}{1.76364}}$.

29. $\left\{\sqrt{\dfrac{\sqrt{12.673}\,(26.72)^2}{(36.27)^{\frac{1}{2}} \times .01267}}\right\}^3$.

Solve for x:

30. $2^x = 19$.

31. $4^{2x-3} = 11^{x+1}$.

32. $19.38^{3x} = 81672$.

33. $.17^x = .4782$.

34. Find the side of a square whose area is equal to that of a rectangle whose base is 17.628 and whose altitude is 8.263.

35. Find the volume of a sphere whose radius is 1.1124, using $V = \frac{4}{3}\pi R^3$ and $\pi = \frac{22}{7}$.

36. Given $t = 12$ and $g = 32.17$, find s, if $s = \frac{1}{2}gt^2$.

37. Work Exs. 16–19 above by the use of four-place tables.

38. Work Exs. 7 and 8 above by the use of four-place tables.

GENERAL PROPERTIES OF SYSTEMS OF LOGARITHMS

15. *The* **logarithm of unity** *in any system of logarithms is zero.*

For, if a be the base,

$$1 = a^0. \quad \therefore \log_a 1 = 0.$$

16. *The* **logarithm of the base** *in any system of logarithms is unity.*

For $$a = a^1. \quad \therefore \log_a a = 1.$$

17. *The* **logarithm of zero** *in any system whose base is greater than unity is negative infinity;* that is, as the number approaches 0, the logarithm approaches negative infinity.

For, since $a > 1$, $0 = \frac{1}{\infty} = \frac{1}{a^\infty} = a^{-\infty}$. $\therefore \log 0 = -\infty$.

But in any system whose base is less than unity, the logarithm of zero is positive infinity.

For, since $a < 1$, $0 = a^\infty$. $\therefore \log_a 0 = \infty$.

18. Logarithm of a Product, Quotient, Power, and Root *in any system.*

If a be taken as the base, and m and n be any two numbers, it can be shown in a manner similar to that used in Art. 9 that

1. $\log_a mn = \log_a m + \log_a n$.
2. $\log_a \frac{m}{n} = \log_a m - \log_a n$.
3. $\log_a m^p = p \log_a m$.
4. $\log_a \sqrt[p]{m} = \frac{\log_a m}{p}$.

[Let the pupil supply the proof. See Art. 9; use a for 10.]

19. Changing the Base of a System of Logarithms. Given the logarithm of a given number, r, to a base a, to find the logarithm of r to another base k, we use the following formula:

$$\log_k r = \frac{\log_a r}{\log_a k}.$$

For, let $\log_k r = x$.

Then $k^x = r$ . . . . (1)

by definition of a logarithm.

Take the logarithm of each member of (1) to base a, then $x \log_a k = \log_a r$.

Hence,
$$x = \frac{\log_a r}{\log_a k},$$

or
$$\log_k r = \frac{\log_a r}{\log_a k}.$$

It follows as a special case that if $r = a$,

$$\log_k a = \frac{1}{\log_a k}, \text{ or } \log_k a \cdot \log_a k = 1.$$

Ex. Find the logarithm of .7 to the base 5.

By the formula just proved,

$$\log_5 .7 = \frac{\log_{10} .7}{\log_{10} 5} = \frac{9.84510 - 10}{0.69897}$$
$$= \frac{-0.1549}{0.69897} = -0.2216^+, \textit{ Ans.}$$

EXERCISE 7

In working the first twelve examples in the following exercise use four-place tables in solving the even-numbered examples, and five-place tables in solving the odd-numbered examples.

Find the value of:

1. $\log_5 60$.
2. $\log_6 9.3$.
3. $\log_{3.7} 26.2$.
4. $\log_4 .93$.
5. $\log_{\sqrt{3}} \sqrt{5}$.
6. $\log_{80} 18$.
7. $\log_{1.8} .17362$.
8. $\log_{.8} .2631$.
9. $\log_2 .7261$.
10. $\log_{.021} .08275$.
11. $\log_{1.2} .9267$.
12. $\log_7 \sqrt{3.1416}$.

Find without the use of tables:

13. $\log_3 27$.

14. $\log_2 32$.

15. $\log_9 \frac{1}{81}$.

16. $\log_{\frac{1}{16}} 8$.

17. $\log_2 .125$.

18. $\log_2 .0625$.

19. Find the base of the system of logarithms in which the log of $16 = 4$.

20. If the log of $27 = \frac{3}{4}$, find the base.

21. If $\frac{1}{2} =$ the log of 5, find the base.

22. Given the log of $5\frac{1}{16} = -\frac{4}{3}$, find the base.

23. If the log of $64 = 1.2$, find the base.

24. In how many years will a sum of money double itself at 4% compound interest? at 6%?

25. If $1520 amounts to $10,701.46 in 40 years at compound interest, what is the rate per cent?

26. Who invented logarithms, and when (see p. 169)? Find out all you can about this man and the way in which he invented logarithms.

27. What nation first divided the circle into 360 degrees, and one degree into 60 minutes?

CHAPTER II

DEFINITIONS. TRIGONOMETRIC FUNCTIONS

20. Source of New Power. Illustrations. A spring of water is situated at the point A and a house at B. It is desired to find the length of a pipe needed to connect B with A, A and B being separated by a swamp. How can the length of the pipe be determined without going through the swamp?

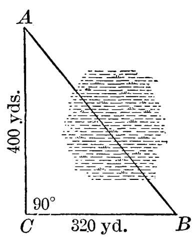


FIG. 1.

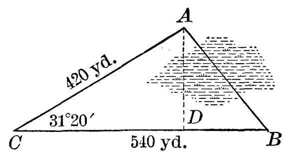


FIG. 2.

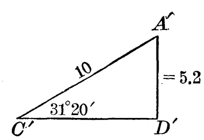


FIG. 3.

If the swamp is situated as in Fig. 1, so that a point C can be taken where CA and CB form a right angle, then CA and CB can be measured and the length of AB computed by the methods of plane geometry. Let the pupil compute AB of Fig. 1.

But if the swamp is situated as in Fig. 2, the above method of computing AB cannot be followed. However, if we take a convenient point C in Fig. 2 and measure the lines AC, CB, and the $\angle C$, the distance AB can be computed *provided we have a table giving the ratios of the sides of all possible right triangles.* Thus from this table we form the triangle given (on enlarged scale) in Fig. 3. Then by the properties of similar triangles we have the proportion $10 : 5.2 = 420 \text{ yd.} : AD$.

From this proportion AD is obtained; afterward AB may be computed from the right triangle ADB by geometry.

Hence the source of new power in trigonometry is a set of tables giving the ratio of each pair of sides in all possible right triangles.

By the aid of such tables it will be found that we are able to find the unknown parts of many triangles which cannot be solved by ordinary geometry. Thus it will be found that if one side AB (Fig. 4) and any two angles (as A and B) of a triangle be known, the other sides (AC and CB) may be computed. By this method, for instance, the distance from the earth to the moon is computed. (For other illustrations of the new power given by trigonometry see Chapter VII.)

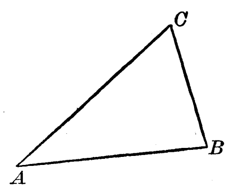


FIG. 4.

21. Trigonometry, as first considered, is that branch of mathematics which determines the remaining parts of a triangle from certain given parts.

Thus it will be found that if any three parts of a triangle are given, provided one of them is a side, the remaining parts may be determined.

Later the word *trigonometry* comes to have a more extended meaning so as to cover the theory of the functions of angles in general wherever these angles may be found. Hence it comes to include much of the theory of wave motion and therefore of particular cases of wave motion, as of sound, light, and electricity. It also becomes largely algebraic in nature.

Plane Trigonometry treats of plane triangles.

See if you can find the derivation of the word *trigonometry.*

22. Trigonometric Functions of an Acute Angle. The fundamental tools or instruments used in trigonometry are the functions of an angle now to be described and defined.

From any point B in one side of an acute angle BAC let fall a perpendicular BC to the other side, forming the right triangle ABC.

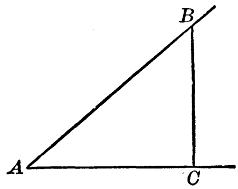


FIG. 5.

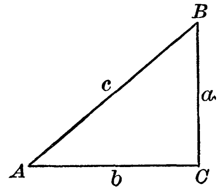


FIG. 6.

Then the ratio $\frac{BC}{AB}$ is termed the *sine* of the angle A.

Similarly,

$$\textit{cosine}\ A = \frac{AC}{AB},\ \textit{cotangent}\ A = \frac{AC}{BC},\ \textit{cosecant}\ A = \frac{AB}{BC},$$

$$\textit{tangent}\ A = \frac{BC}{AC},\ \textit{secant}\ A = \frac{AB}{AC},\ \textit{versed sine}\ A = 1 - \frac{AC}{AB},$$

$$\textit{coversed sine}\ A = 1 - \frac{BC}{AB},$$

or, in general, in a right triangle:

The **sine** *of an acute angle is the ratio of the opposite leg to the hypotenuse.*

The **cosine** *is the ratio of the adjacent leg to the hypotenuse.*

The **tangent** *is the ratio of the opposite leg to the adjacent leg.*

The **cotangent** *is the ratio of the adjacent leg to the opposite leg.*

The **secant** *is the ratio of the hypotenuse to the adjacent leg.*

The **cosecant** *is the ratio of the hypotenuse to the opposite leg.*

The **versed sine** *is* 1 *minus the cosine.*

The **coversed sine** *is* 1 *minus the sine.*

These eight ratios are called the *trigonometric ratios*, or the *trigonometric functions*.

The versed sine and the coversed sine are used so little in

elementary work that we confine our attention mainly to the other six functions. Hence when we speak of the "six functions" we mean the first six trigonometric functions as given above.

The *abbreviations* sin, cos, tan, cot, sec, csc, vers, covers, are ordinarily used for the eight functions.

The cosine, cotangent, cosecant, and coversed sine are termed the **co-functions** of the sine, tangent, secant, and versed sine respectively.

In the above triangle (Fig. 6), denoting the side AB by c, AC by b, and BC by a, we have

$$\sin A = \frac{a}{c} \qquad \sec A = \frac{c}{b}$$

$$\cos A = \frac{b}{c} \qquad \csc A = \frac{c}{a}$$

$$\tan A = \frac{a}{b} \qquad \text{vers } A = 1 - \frac{b}{c}$$

$$\cot A = \frac{b}{a} \qquad \text{covers } A = 1 - \frac{a}{c}$$

Similarly

$$\sin B = \frac{b}{c} \qquad \sec B = \frac{c}{a}$$

$$\cos B = \frac{a}{c} \qquad \csc B = \frac{c}{b}$$

$$\tan B = \frac{b}{a} \qquad \text{vers } B = 1 - \frac{a}{c}$$

$$\cot B = \frac{a}{b} \qquad \text{covers } B = 1 - \frac{b}{c}$$

Or using abbreviations,

$$\text{sin of either acute } \angle = \frac{\perp \text{opp.}}{\text{hyp.}}, \quad \text{cot of either acute } \angle = \frac{\perp \text{adj.}}{\perp \text{opp.}}$$

$$\text{cos of either acute } \angle = \frac{\perp \text{adj.}}{\text{hyp.}}, \quad \text{sec of either acute } \angle = \frac{\text{hyp.}}{\perp \text{adj.}}$$

$$\text{tan of either acute } \angle = \frac{\perp \text{opp.}}{\perp \text{adj.}}, \quad \text{csc of either acute } \angle = \frac{\text{hyp.}}{\perp \text{opp.}}$$

The method of indicating a power of a trigonometric function is shown by the following example: for "the square of the sine of the angle A," that is, for "$(\sin A)^2$," we write "$\sin^2 A$." How then would "the cube of $\cos A$" be written? "The nth power of $\tan A$?"

In this book unless the contrary is stated, in the right triangle ABC, the letter C is supposed to be placed at the vertex of the right angle.

23. Utility of the Trigonometrical Ratios. It will be found that the numerical value of the above trigonometrical ratios for every angle from 0° to 90° may be computed and arranged in tables whence they may be taken and used when needed. These numerical values are used by what is virtually the geometrical principle of similar triangles in solving triangles. Later, however, they become units and elements which can be variously grouped and used in many kinds of algebraic processes.

24. The value of a trigonometric function of an angle *depends only on the size of the angle, not on the length of the lines* chosen to form the ratios.

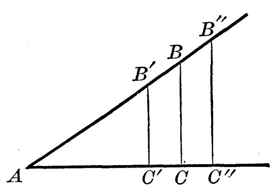


FIG. 7.

Thus, by similar triangles (in Fig. 7),

$$\sin A = \frac{B'C'}{AB'} = \frac{BC}{AB} = \frac{B''C''}{AB''}, \text{ etc.}$$

25. Given two sides of a right triangle, to compute the trigonometric functions for both acute angles of the triangle.

Ex. If in a right triangle $a = 3$, and $b = 4$, find c and the trigonometric ratios of each acute angle.

The hypotenuse $c = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

Hence
$$\sin A = \tfrac{3}{5} \quad \sin B = \tfrac{4}{5}$$
$$\cos A = \tfrac{4}{5} \quad \cos B = \tfrac{3}{5}$$
$$\tan A = \tfrac{3}{4} \quad \tan B = \tfrac{4}{3}$$
etc. etc.

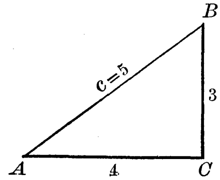


FIG. 8.

In studying trigonometry (and indeed in all mathematical work) the pupil should make the capital letter a in the printed form A and not in the script form $\mathcal{A}$. In other words, he should make the small and capital letters as unlike as possible, and hence make them unlike in shape as well as in size. The reason for this is that the small and capital letters have entirely different meanings; and if as written by the pupil they have the same shape, the pupil is continually mistaking the small letter for the large, and *vice versa*. Similarly the capital letter c should always be written in the form $\mathcal{C}$ and not C.

EXERCISE 8

1. Write the functions of the acute angle B (Fig. 6) in terms of a, b, c. (Let the teacher invert the triangle in various ways.)

2. Construct a right triangle in which $a=8$, $b=6$, $c=10$, and write out the functions of A in this triangle; also of B.

Determine the value of the functions of A in the rt. $\triangle\ ABC$, whose sides are a, b, c, if:

3. $a=6, \quad b=8$.

4. $a=8, \quad b=15$.

5. $a=12, \quad c=13$.

6. $a=39, \quad b=80$.

7. $a=.09, \quad c=.41$.

8. $b=12, \quad c=16.9$.

9. Find the value of the functions of B in Exs. 3–8.

10. In Ex. 2 find the value of

(1) $\sin A \tan A$.

(2) $\sin^2 A + \cos^2 A$.

(3) $\sin A \csc A$.

(4) $1 + \tan^2 A$.

(5) $\sec^2 A - \tan^2 A$.

(6) $\tan A \cot A$.

(7) $\tan A - \dfrac{\sin A}{\cos A}$.

(8) $\cos A \sec A$.

By the use of squared paper construct the angle whose

11. Tangent $= \frac{3}{4}$.

12. Tangent $= \frac{1}{2}$.

13. Tangent $= 1$.

14. Tangent $= 4$.

15. Tangent $= \sqrt{3}$.

16. sine $= \frac{2}{3}$.

17. cosine $= \frac{1}{5}$.

18. secant $= \sqrt{3}$.

19. cosecant $= 5$.

20. Construct with a protractor an angle of 23°. Then construct a right triangle with sides of convenient length having 23° for one of its angles. Measure the sides of this right triangle and hence find sin 23°. Compare this value with the value of sin 23° given in Table V. Determine and test cos 23° and tan 23° in the same way.

21. Treat 37° in the same way; also 52°.

22. On Fig. 2 (p. 24) compute the numerical value of AD; then of CD and DB; then of AB.

23. On Fig. 3, what is the value of $\sin A'$?

24. On Fig. 6, if $AB = 125$, $\angle B = 27°$, and $\sin 27° = .454$, compute AC.

25. Can you suggest some practical problem similar to that given in Art. 20, which could be solved by trigonometry and not by geometry? What is the source of new power in trigonometry which enables us to do this?

26. If by the methods of trigonometry we are able to solve any triangle in which one side and any two angles are given, suggest some practical problem which could be solved by this means (and not by geometry).

In a rt. $\triangle$, given:

27. $a = \sqrt{p^2 + q^2}$, $b = \sqrt{2\,pq}$, find $\sin A$ and $\cos A$.

28. $a = 2\,mn$, $c = m^2 + n^2$, determine $\sin A$, $\sec A$, and $\tan A$.

29. $b = 2\,pq$, $c = p^2 + q^2$, find $\tan A$, $\sin A$, $\csc A$.

30. $a = \sqrt{m^2 + mn}$, $b = \sqrt{mn + n^2}$, find all the functions of B.

31. If $a = 2\sqrt{mn}$ and $c = m + n$, find all the functions of B.

32. If $a = 60$ and $c = 61$, find $\sec A$, $\tan B$, $\cot B$, $\sin A$.

33. If $b = 2.64$ and $c = 2.65$, find the functions of B.

34. If $a = 2\,b$, find the functions of A.

35. If $b = \frac{2}{3}\,c$, find the functions of A.

36. If $a + b = \frac{4}{3}\,c$, find the functions of B.

37. If $a - b = \frac{7}{13}\,c$, find the functions of A.

38. Find the functions of B, if $a = 4\,d$ and $b = 3\,d$.

By use of squared paper construct a rt. $\triangle$, given:

39. $c = 4$ and $\tan A = \frac{3}{2}$.

40. $b = 3$ and $\sin A = \frac{3}{4}$.

41. Find b if $\cos A = .36$ and $c = 4.5$.

42. On Fig. 8, $\sin A =$ what? $\cos B =$ what? Does $\sin A = \cos B$? In like manner, show that $\cos A = \sin B$, $\tan A = \cot B$, $\cot A = \tan B$, $\sec A = \csc B$, $\csc A = \sec B$.

43. Show the same on Fig. 6.

44. In Fig. 6, since c is the hypotenuse, it is evident that it is greater than either leg. Hence $\sin A$, or $\frac{a}{c}$, is always less than 1. What other function of A is always less than 1? Which functions of A are always greater than 1? Which may be either greater or less than 1?

45. Which of the six functions are always proper fractions? improper fractions? may be either proper or improper fractions? Verify this on Fig. 8.

46. If A is any acute angle, is it correct to say that $\sec A$ is always greater than $\sin A$? Why?

47. The values of which of the six functions of A (on Fig. 6) have c for a denominator? a? b?

48. How many of the above examples can you work at sight (*i.e.* for how many can you give results without the use of pencil and paper)?

26. Functions of the Complement of an Angle. From Fig. 6 (page 26).

$$\sin A = \frac{a}{c}; \text{ also } \cos B = \frac{a}{c}.$$

Hence, $\sin A = \cos B,$

or $\boldsymbol{\sin A = \cos (90° - A)}$, since $B = 90° - A$.

Let the pupil show in like manner that

$$\boldsymbol{\cos A} = \sin B = \boldsymbol{\sin (90° - A)},$$
$$\boldsymbol{\tan A} = \cot B = \boldsymbol{\cot (90° - A)},$$

and

$$\boldsymbol{\sec A} = \csc B = \boldsymbol{\csc (90° - A)}.$$

Hence, in general,

Any trigonometric function of an angle is equal to the co-function of the complement of the angle.

By the use of this property,

Any trigonometric function of an angle between 45° *and* 90° *can be reduced to the function of an angle between* 0° *and* 45°.

Thus, $\sin 88° 10' = \cos 1° 50'$.

EXERCISE 9

Express each of the following trigonometric functions as a function of the complementary angle:

1. $\sin 60°$.
2. $\cos 15°$.
3. $\tan 65° 24'$.
4. $\cot 55° 36'$.
5. $\csc 21° 24' 30''$.
6. $\sec 84° 16'$.
7. $\sin 89° 59'$.
8. $\cos 1° 18'$.
9. Given $\tan 60° = \sqrt{3}$, find $\cot 30°$.
10. Given $\sin 30° = \frac{1}{2}$, find $\cos 60°$.
11. Given $\cos A = \frac{x}{y}$, find $\sin (90° - A)$.
12. Given $\sin A = p$, find $\cos (90° - A)$.
13. How many of the examples in this exercise can you work at sight?

RELATIONS OF TRIGONOMETRIC FUNCTIONS OF AN ANGLE

27. Three pairs of reciprocals exist among the trigonometric functions of an acute angle, viz.:

sin and **csc**
cos and **sec**
tan and **cot**

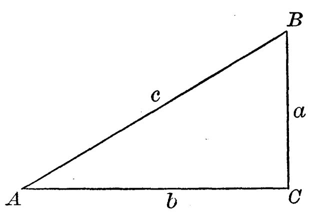


FIG. 9.

For

$$\frac{a}{c} \times \frac{c}{a} = 1. \quad \therefore \sin A \times \csc A = 1.$$

$$\frac{b}{c} \times \frac{c}{b} = 1. \quad \therefore \cos A \times \sec A = 1.$$

$$\frac{a}{b} \times \frac{b}{a} = 1. \quad \therefore \tan A \times \cot A = 1.$$

28. Four equations connect the trigonometric functions of an acute angle in important ways.

For, from Fig. 9,

$$a^2 + b^2 = c^2. \quad \ldots \quad (1)$$

Dividing (1) by c^2,

$$\frac{a^2}{c^2}+\frac{b^2}{c^2}=1, \text{ or } \left(\frac{a}{c}\right)^2+\left(\frac{b}{c}\right)^2=1;$$

that is, $$\sin^2 A+\cos^2 A=1.$$

Dividing (1) by b^2,

$$\frac{a^2}{b^2}+1=\frac{c^2}{b^2}, \text{ or } \left(\frac{a}{b}\right)^2+1=\left(\frac{c}{b}\right)^2;$$

that is, $$\tan^2 A+1=\sec^2 A.$$

Let the student prove in like manner that

$$\cot^2 A+1=\csc^2 A.$$

Also from Fig. 9.

$$\frac{a}{b}=\frac{a}{c}\div\frac{b}{c};$$

that is, $$\tan A=\frac{\sin A}{\cos A}.$$

29. Hence **nine (or more) formulas** give important values for the trigonometric functions. For from the results of Arts. 27 and 28 we readily obtain, for instance,

$$\sin A=\sqrt{1-\cos^2 A}. \qquad \cot A=\frac{\cos A}{\sin A}.$$

$$\cos A=\sqrt{1-\sin^2 A}. \qquad \sec A=\frac{1}{\cos A}.$$

$$\tan A=\frac{\sin A}{\cos A}. \qquad \csc A=\frac{1}{\sin A}.$$

$$\tan A=\frac{1}{\cot A}. \qquad \operatorname{vers} A=1-\cos A.$$

$$\operatorname{covers} A=1-\sin A.$$

30. One trigonometric function of an angle being given, the other functions may be found in either of two ways.

Algebraic Method. By use of the formulas of Art. 29 and equations of Art. 28.

Ex. 1. If $\sin A = \frac{2}{3}$, find the other trigonometric functions of A.

$$\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \tfrac{4}{9}} = \sqrt{\tfrac{5}{9}} = \tfrac{1}{3}\sqrt{5}.$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{2}{3} \div \frac{\sqrt{5}}{3} = \frac{2}{\sqrt{5}} = \frac{2}{5}\sqrt{5}.$$

$$\cot A = \frac{1}{\tan A} = \frac{\sqrt{5}}{2}.$$

$$\sec A = \frac{1}{\cos A} = 1 \div \frac{\sqrt{5}}{3} = \frac{3}{\sqrt{5}} = \frac{3}{5}\sqrt{5}.$$

$$\csc A = \frac{1}{\sin A} = 1 \div \frac{2}{3} = \frac{3}{2}.$$

$$\text{vers } A = 1 - \cos A = 1 - \tfrac{1}{3}\sqrt{5}.$$

$$\text{covers } A = 1 - \sin A = 1 - \tfrac{2}{3} = \tfrac{1}{3}.$$

Ex. 2. If $\tan x = 2$, find the other functions of x.

$$\sec^2 x = 1 + \tan^2 x. \quad \text{(Art. 28.)}$$

$$\therefore \sec^2 x = 1 + 4 = 5.$$

$$\sec x = \sqrt{5}.$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\sqrt{5}} = \frac{1}{5}\sqrt{5}.$$

$$\sin x = \sqrt{1 - \cos^2 x} = \sqrt{1 - \tfrac{1}{5}} = \sqrt{\tfrac{4}{5}} = \tfrac{2}{5}\sqrt{5}, \text{ etc.}$$

Geometric Method. This consists of constructing a right triangle by use of the given function and deriving the required functions from the right triangle.

Ex. 3. Given $\sin A = \frac{2}{3}$, obtain the other trigonometric functions of A by use of the right triangle.

Construct a right triangle whose hypotenuse is 3 and altitude is 2, as ABC.

Then $AC = \sqrt{3^2 - 2^2} = \sqrt{9 - 4} = \sqrt{5}$.

Then from the figure by the definitions of the trigonometric ratios

$$\cos A = \frac{\sqrt{5}}{3}; \quad \tan A = \frac{2}{\sqrt{5}} = \frac{2}{5}\sqrt{5}; \quad \cot A = \frac{\sqrt{5}}{2};$$

$$\sec A = \frac{3}{\sqrt{5}} = \frac{3}{5}\sqrt{5}; \quad \csc A = \frac{3}{2}; \quad \text{vers } A = 1 - \frac{\sqrt{5}}{3};$$

$$\text{covers } A = 1 - \tfrac{2}{3} = \tfrac{1}{3}.$$

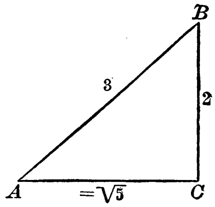


Fig. 10.

As the sides of a right triangle are all positive in sign, in studying the trigonometry of the right triangle we neglect the $\pm$ sign usually placed before a square root radical sign, and take any square root radical as normally plus. When we come to study angles in general, as in Chapters IV and V, it will be necessary carefully to consider whether the sign before a given radical sign is to be taken as $+$ or $-$ (see Art. 61).

EXERCISE 10

Find by means of the formulas the values of the other functions of A, given:

1. $\sin A = \frac{15}{17}$.
2. $\tan A = \frac{12}{5}$.
3. $\sec A = \frac{41}{9}$.
4. $\cos A = \frac{2}{3}$.
5. $\cot A = m$.
6. $\csc A = \sqrt{5}$.
7. $\sin A = 0$.
8. $\cos A = 0$.
9. $\tan A = 0$.
10. $\sin A = 1$.
11. $\sec A = \infty$.
12. $\sin x = 5p$.

Find by geometric methods (squared paper may be used to advantage in constructing diagrams) the other functions of A (or x), given:

13. $\tan A = \frac{3}{4}$.
14. $\cos A = \frac{5}{13}$.
15. $\csc A = \frac{17}{15}$.
16. $\cot A = \frac{3}{2}$.
17. $\sin A = \frac{1}{2}$.
18. $\sec A = 4$.
19. $\tan A = m$.
20. $\sin A = \frac{1}{2}\sqrt{2}$.
21. $\cos x = 1$.

Find by both methods the other functions of the angle named when:

22. $\csc A = \frac{41}{40}$.
23. $\tan A = \dfrac{2mn}{m^2 - n^2}$.
24. $\cot A = \sqrt{2} + 1$.
25. $\sin A = 1$.
26. $\tan 22\frac{1}{2}° = \sqrt{2} - 1$.
27. $\cos A = \frac{5}{8}$.
28. $\sec A = \dfrac{4}{\sqrt{6} - \sqrt{2}}$.
29. $\cos A = K$.
30. $\cot 15° = 2 + \sqrt{3}$.

Express each of the other trigonometric functions of A in terms of:

31. $\sin A$.
32. $\cos A$.
33. $\tan A$.
34. $\cot A$.
35. $\sec A$.
36. $\csc A$.
37. $\text{vers}\ A$.
38. Given $\sin A = \frac{3}{4}$, find $\cot A$.
39. Given $\cos A = \frac{39}{80}$, find $\csc A$.
40. Given $\tan A = \sqrt{3}$, find $\sin A$
41. Given $\csc A = \frac{8}{5}$, find $\sec A$.
42. Given $\sec A = \frac{25}{7}$, find $\cot A$.
43. Given $\cot A = \sqrt{2} - 1$, find $\cos A$.
44. Given $\tan A = \sqrt{6}$, find $\csc A$.

45. Transform the expression $\sin^2 A + \cos A$ so that the only trigonometric function contained in it shall be $\cos A$.

46. Transform $(1 + \tan^2 A) \sec A$ so that it shall contain only $\cos A$.

47. Transform $(\tan A + \cot A) \sec A \cos A$ so that it shall contain only $\sin A$ and $\cos A$.

48. Transform the equation $\cos^2 x - \sin^2 x = \sin x$ so that it shall contain only $\sin x$.

49. Transform $\tan x = 2 + \cot x$ so that it shall contain only $\tan x$.

50. Which of the six functions are always less than 1? Which are always greater than 1? Which may be either greater or less than 1? How can you use this principle in testing the accuracy of examples like Exs. 1–30 of this Exercise?

51. How many of the above examples can you work at sight?

31. Trigonometric Identities.

As stated in algebra, an *identity* is an equality which is true for all values of the unknown quantity (or quantities) contained in it.

Thus $(x + 2)(x - 2) = x^2 - 4$ is an identity, since it is true for all values of x, as for $x = 0, 1, 2, 3, \cdots$, or $-1, -2$, etc.

An *equation* proper (or a conditional equation) is an equality which is true only for a certain special value (or values) of the unknown quantity (or quantities).

Thus $x^2 - x = 2x - 2$ is true only when $x = 1$ or 2, and hence is an equation proper, or conditional equation.

The equality mark used in equations is $=$, and that used in identities is $\equiv$. However, in elementary mathematics it is customary to use the mark $=$ for both equations and identities and let the context decide whether we are dealing with an identity or an equation.

Similarly in geometry the word "circle" is sometimes used to denote an area and sometimes a line (the circumference), the context deciding in each case what is meant. So $8''$ may mean either 8 inches or 8 seconds of angle, etc.

Relations of identity among trigonometrical functions may be proved in either of two ways.

First Method. By use of the formulas for the functions given in Arts. 28 and 29 (and particularly those which reduce the function to sine and cosine) an expression may

be proved identical with another, by reducing one of the given expressions directly to the form of the other.

Ex. 1. Prove $\cot^2 A \cos^2 A = \cot^2 A - \cos^2 A$.

$$\begin{aligned}
\cot^2 A \cos^2 A &= \frac{\cos^2 A}{\sin^2 A} \cos^2 A \\
&= \frac{(1 - \sin^2 A) \cos^2 A}{\sin^2 A} \\
&= \frac{\cos^2 A}{\sin^2 A} - \frac{\sin^2 A \cos^2 A}{\sin^2 A} \\
&= \cot^2 A - \cos^2 A.
\end{aligned}$$

Instead of proving an identity by reducing one member of the identity to the form of the other, it is sometimes more advantageous to reduce both expressions to a common third form, and hence infer their identity by Ax. 1.

Thus we may start with $\cot^2 A \cos^2 A = \cot^2 A - \cos^2 A$ and transform it as follows:

$$\frac{\cos^2 A}{\sin^2 A} \cos^2 A = \frac{\cos^2 A}{\sin^2 A} - \cos^2 A,$$

or

$$\frac{\cos^4 A}{\sin^2 A} = \frac{\cos^2 A - \cos^2 A \sin^2 A}{\sin^2 A}.$$

$$\frac{\cos^4 A}{\sin^2 A} = \frac{\cos^2 A (1 - \sin^2 A)}{\sin^2 A}.$$

$$\frac{\cos^4 A}{\sin^2 A} = \frac{\cos^4 A}{\sin^2 A}.$$

Since the last is plainly an identity, we infer that

$$\cot^2 A \cos^2 A = \cot^2 A - \cos^2 A$$

is also an identity.

Second Method. By use of the values of the functions obtained by applying the definitions of the functions to the right triangle (Art. 22, Fig. 6).

Ex. 2. Prove $\dfrac{\sin A}{\cos A \tan^2 A} = \cot A.$

Substitute $\frac{a}{c}$ for $\sin A$; $\frac{b}{c}$ for $\cos A$; $\frac{a}{b}$ for $\tan A$; $\frac{b}{a}$ for $\cot A$. Then

$$\frac{\sin A}{\cos A \tan^2 A} = \frac{\frac{a}{c}}{\frac{b}{c}\cdot\frac{a^2}{b^2}} = \frac{b}{a} = \cot A.$$

EXERCISE 11

Prove each of the following identities:

(In the solution of identities, the first of the two methods given above is to be preferred, since its use helps fix in mind the fundamental equations and formulas given in Arts. 28 and 29.)

1. $\cos A \tan A = \sin A.$

2. $\sin A \sec A = \tan A.$

3. $\cos A \csc A = \cot A.$

4. $\cos A = \sin A \cot A.$

5. $\sin A = \cos A \tan A.$

6. $\dfrac{1+\cos A}{\sin A} = \dfrac{\sin A}{1-\cos A}.$

7. $\dfrac{1+\sin A}{\cos A} = \dfrac{\cos A}{1-\sin A}.$

8. $\sin^2 A - \cos^2 A = 2\sin^2 A - 1.$

9. $(1-\sin^2 A)\tan^2 A = \sin^2 A.$

10. $(\tan A + \cot A)\sin A \cos A = 1.$

11. $(1-\sin^2 A)\csc^2 A = \cot^2 A.$

12. $(\sin A + \cos A)^2 = 1 + 2\sin A \cos A.$

13. $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2.$

14. $(\csc^2 A - 1)\sin^2 A = \cos^2 A.$

15. $\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A} = \sec A \csc A.$

16. $\dfrac{\cot^2 A}{1+\cot^2 A} = \cos^2 A.$

17. $\tan A + \cot A = \sec A \csc A.$

18. $\tan A + \cot A = \dfrac{\sec^2 A + \csc^2 A}{\sec A \times \csc A}.$

19. $\sin^4 A - \cos^4 A = \sin^2 A - \cos^2 A.$

20. $\dfrac{\sin A}{1-\cot A} + \dfrac{\cos A}{1-\tan A} = \sin A + \cos A.$

21. $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \csc A - \cot A.$

22. $\dfrac{1+\tan A}{1+\cot A}=\dfrac{1-\tan A}{\cot A-1}$.

23. $\cot A+\tan A=\dfrac{1}{\sin A\cos A}$.

24. $\tan^2 A-\sin^2 A=\tan^2 A\sin^2 A$.

25. $\csc^4 A-2\csc^2 A=\cot^4 A-1$.

26. $\sec^4 A(1-\sin^4 A)=2\tan^2 A+1$.

27. $\dfrac{\csc A}{\tan A+\cot A}=\cos A$.

28. $\dfrac{1-\cot^2 A}{1+\cot^2 A}=\sin^2 A-\cos^2 A$.

29. $\dfrac{\cot A-\cos A}{\cot A\cos A}=\dfrac{\cot A\cos A}{\cot A+\cos A}$.

30. $1-\cot^4 A=2\csc^2 A-\csc^4 A$.

31. $\sqrt{1-\sin^2 A}\tan A=\sin A$.

32. $\sin^6 A+\cos^6 A=1-3\sin^2 A\cos^2 A$.

33. $\cos^3 A-\sin^3 A=(\cos A-\sin A)(1+\sin A\cos A)$.

34. Reduce $\tan^6 x\sec^4 x$ to the form $(\tan^8 x+\tan^6 x)\sec^2 x$.

Transform:

35. $\tan^8 x$ into $(\tan^6 x-\tan^4 x+\tan^2 x-1)\sec^2 x+1$.

36. $\sec^{10} y$ into $\sec^2 y\,(1+4\tan^2 y+6\tan^4 y+4\tan^6 y+\tan^8 y)$.

37. $\sqrt{1+\sin x}$ into $\dfrac{\cos x}{\sqrt{1-\sin x}}$.

38. $\dfrac{1}{1+\sin x}$ into $\sec^2 x-\sec x\tan x$.

39. $\dfrac{1+\sin x}{\cos^2 x}$ into $\sec^2 x+\sec x\tan x$.

40. See if you can make up or discover any other trigonometrical identities for yourself.

41. How many of the above examples can you work at sight?

TRIGONOMETRIC FUNCTIONS OF PARTICULAR ANGLES

32. Functions of 45°. The trigonometric functions of 30°, 45°, and 60° are used so frequently that it is of service to determine their values and commit these values to

memory. It is helpful to notice that we determine these values in each case by the use of a right angle, the hypotenuse of which is taken as 1.

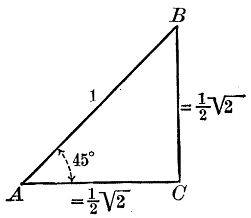


FIG. 11.

Let ABC (Fig. 11) be an isosceles right triangle, the hypotenuse of which, AB, is 1. Then, by geometry, each leg is $\frac{1}{2}\sqrt{2}$ (for $\angle B = 45°$, $\therefore AC = BC$; but $\overline{AC}^2 + \overline{BC}^2 = 1^2$, $\therefore 2\,\overline{BC}^2 = 1^2$, etc.).

By the definitions of the trigonometric functions,

$$\sin 45° = (\tfrac{1}{2}\sqrt{2}) \div 1 = \tfrac{1}{2}\sqrt{2}.$$

$$\cos 45° = (\tfrac{1}{2}\sqrt{2}) \div 1 = \tfrac{1}{2}\sqrt{2}.$$

$$\tan 45° = \frac{\frac{1}{2}\sqrt{2}}{\frac{1}{2}\sqrt{2}} = 1.$$

$$\cot 45° = \frac{\frac{1}{2}\sqrt{2}}{\frac{1}{2}\sqrt{2}} = 1.$$

$$\sec 45° = 1 \div \frac{\sqrt{2}}{2} = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

$$\csc 45° = 1 \div \frac{\sqrt{2}}{2} = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

33. Functions of 30° and 60°. Let ABD (Fig. 12) be an equilateral triangle in which the length of one side is 1. Let AC be $\perp BD$.

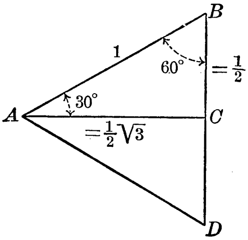


FIG. 12.

Then, by geometry

$$\angle BAD = 60°,$$

and $$\angle BAC = 30°.$$

Also AC bisects BD, hence $BC = \frac{1}{2}$.

$AC = \sqrt{\overline{AB}^2 - \overline{BC}^2} = \sqrt{1 - \frac{1}{4}} = \frac{1}{2}\sqrt{3}.$

Then in the right triangle ABC,

$$\sin 30° = \tfrac{1}{2}.$$

$$\cos 30° = \tfrac{1}{2}\sqrt{3}.$$

$$\tan 30^\circ = \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{3}} = \frac{1}{\sqrt{3}} = \tfrac{1}{3}\sqrt{3}.$$

$$\cot 30^\circ = \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}} = \sqrt{3}.$$

$$\sec 30^\circ = \frac{1}{\frac{1}{2}\sqrt{3}} = \frac{2}{\sqrt{3}} = \tfrac{2}{3}\sqrt{3}.$$

$$\csc 30^\circ = \frac{1}{\frac{1}{2}} = 2.$$

Let the pupil write out in like manner the functions of 60° (that is, of $\angle ABC$ in the $\triangle ABC$).

Of the results obtained in Arts. 32 and 33 those which are most used may be conveniently arranged in a table thus:

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$
cos	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$
tan	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$

34. Functions of 0°. Let ABC (Fig. 13) be a right triangle in which the hypotenuse $AB = 1$ and the angle BAC is small and is diminished and made to approach 0° as a limit. Then if AB remains fixed in length, BC approaches zero and AC approaches 1.

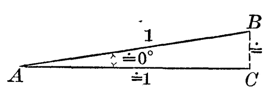


Fig. 13.

At the limit,

$$\sin 0^\circ = \frac{0}{1} = 0. \qquad \sec 0^\circ = \frac{1}{1} = 1.$$

$$\cos 0^\circ = \frac{1}{1} = 1. \qquad \csc 0^\circ = \frac{1}{0} = \infty.$$

$$\tan 0^\circ = \frac{0}{1} = 0. \qquad \text{vers } 0^\circ = 1 - 1 = 0.$$

$$\cot 0^\circ = \frac{1}{0} = \infty. \qquad \text{covers } 0^\circ = 1 - 0 = 1.$$

35. Functions of 90°. Let ABC (Fig. 14) be a right triangle in which BAC is nearly a right angle and approaches 90° as a limit. AB remains fixed in length; hence BC approaches 1 as a limit and AC approaches 0.

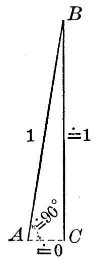


Fig. 14.

At the limit,

$$\sin 90° = \frac{1}{1} = 1. \qquad \sec 90° = \frac{1}{0} = \infty.$$

$$\cos 90° = \frac{0}{1} = 0. \qquad \csc 90° = \frac{1}{1} = 1.$$

$$\tan 90° = \frac{1}{0} = \infty. \qquad \text{vers } 90° = 1 - 0 = 1.$$

$$\cot 90° = \frac{0}{1} = 0. \qquad \text{covers } 90° = 1 - 1 = 0.$$

The results obtained in Arts. 34 and 35 may be conveniently arranged in a table thus:

	0°	90°
sin	0	1
cos	1	0
tan	0	∞
cot	∞	0
sec	1	∞
csc	∞	1

36. Representation of the Trigonometric Functions of an Acute Angle by Lines. If a quadrant of a circle OAB be drawn with center O and radius OB equal to 1, the sine of any angle AOP' is $\frac{M'P'}{OP'} = \frac{M'P'}{1} = M'P'$.

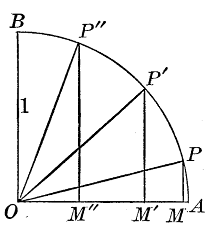


Fig. 15.

Similarly the sine of $\angle AOP = MP$, and sine of $\angle AOP'' = M''P''$.

In other words the sine of any angle AOP in a quadrant whose radius is 1 is represented by the perpendicular let fall from P upon the radius OA.

Hence it is easy to see that, since MP is the sine of $\angle AOP$, if AOP becomes very small and $\doteq 0$, $MP \doteq 0$, and at the limit $\sin 0° = 0$. Also if $\angle AOP''$ increases and $\doteq 90°$, $\sin \angle AOP''$ or $M''P'' \doteq OB$ or 1. Hence at the limit $\sin 90° = 1$.

Similarly $\cos \angle AOP' = \frac{OM'}{OP'} = \frac{OM'}{1} = OM'$. Hence also $\cos \angle AOP = OM$, $\cos \angle AOP'' = OM''$. In other words the cosine of any angle AOP in a quadrant whose radius is 1 is represented by the part of OA intercepted between O and the foot of the line representing the sine.

Hence $\cos 0° = OA$ or 1, and as $\angle AOP$ changes from 0° to 90°, the cosine changes from 1 to 0.

Similarly, (Fig. 16),

$$\tan \angle AOT = \frac{AT}{OA} = \frac{AT}{1} = AT.$$

$$\sec \angle AOT = \frac{OT}{OA} = \frac{OT}{1} = OT.$$

$$\cot \angle AOT = \tan \angle BOR$$

$$= \frac{BR}{OB} = \frac{BR}{1} = BR.$$

$$\csc \angle AOT = \sec \angle BOR$$

$$= \frac{OR}{OB} = \frac{OR}{1} = OR.$$

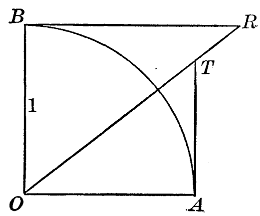


FIG. 16.

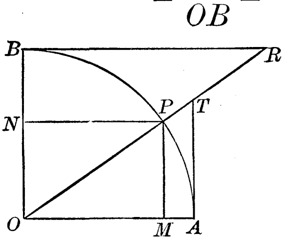


FIG. 17.

The various lines which represent the trigonometric functions of an acute angle AOP may be combined in a single figure (Fig. 17). Let the pupil find the lines on the figure which represent vers $\angle AOP$ and covers $\angle AOP$.

37. Tables of Trigonometric Functions of Angles from 0° to 90° called Natural Functions. By methods which will be explained later (see Art. 116) the values of the trigonometric

functions for angles of every degree and minute from 0° to 90° may be calculated. These values are arranged in tables called Tables of Natural Trigonometric Functions.

EXERCISE 12

By the use of squared paper, construct the following angles, making use of their natural functions:

1. 30°. (Use $\sin 30^\circ = \frac{1}{2}$.) **2.** 45°. **3.** 60°.

4. If $\tan 61^\circ 37' = 1.85$, construct the angle $61^\circ 37'$ on squared paper.

By use of the table of natural tangents, construct:

5. $42^\circ 30'$. **6.** $56^\circ 37'$. **7.** 47.24°. **8.** 72.37°.

By use of the table of natural sines, construct:

9. $61^\circ 23'$. **10.** $47^\circ 15'$. **11.** 52.35°. **12.** 63.84°.

Find the numerical value of:

13. $2 \sin 30^\circ + \cos 60^\circ + \sin 90^\circ$.

14. $b \tan 30^\circ + c \cot 60^\circ + a \tan 0^\circ$.

15. $4 \tan 0^\circ + 4 \sin^2 45^\circ + 2 \cos 45^\circ$.

16. $\tan 30^\circ \cos 90^\circ - 4 \sin 60^\circ + \cos^2 0^\circ$.

17. $\tan 30^\circ \cot 30^\circ - 2 \sin 45^\circ \tan 45^\circ - 6 \cos 60^\circ \cot 45^\circ + \sin 90^\circ$.

18. $\sec 60^\circ \cos 60^\circ - \tan 30^\circ \cot 60^\circ + \tan 60^\circ \cot 30^\circ - 20 \sin 30^\circ$.

19. Show that $(\sin 60^\circ - \sin 45^\circ)(\cos 30^\circ + \cos 45^\circ) = \frac{1}{4}$.

If $P = 0^\circ$, $Q = 30^\circ$, $R = 45^\circ$, $S = 60^\circ$, $T = 90^\circ$, find the value of each of the following expressions:

20. $\sin Q + \cos R - 1$.

21. $\tan^2 P + \tan^2 Q + \tan^2 R$.

22. $\cos P \cos Q \cos R + \sin R \sin S \sin T$.

23. $\sec P + 2 \sin Q + 2 \cos^2 R + \frac{1}{3} \tan^2 S + \operatorname{cosec} T$.

24. Does twice the tangent of $45^\circ =$ the tan of 90°? Why?

25. Does $\sin 30^\circ + \sin 45^\circ = \sin 75^\circ$?

26. Does $\cot 30^\circ + \cot 45^\circ = \cot 75^\circ$?

27. Draw a diagram showing the trigonometric functions as lines when $\angle AOP$ is less than 45°.

28. Also when $\angle AOP$ is greater than 45°.

29. Also when $\angle AOP$ equals 45°.

30. Given that x is greater than $45°$ and less than $90°$, show on a diagram similar to Fig. 17 that $\tan x$ is greater than $\cot x$.

31. Given that x is less than $45°$, show that $\sec x$ is less than $\csc x$.

32. Show that $\cos x$ is always less than $\cot x$.

33. Show that $\sin x < \tan x < \sec x$.

34. Show that $\cot x < \csc x$.

35. If a flagstaff is at a distance of 150 ft. and the angle of elevation (see Art. 88) of the top of the flagstaff is $30°$, find the height of the flagstaff.

36. Find its height if the angle of elevation of the top (at the same distance) is $45°$. Is $60°$.

37. Make up two examples similar to Ex. 35.

38. The Washington Monument is 555 ft. high. At a certain place the angle of elevation of its top is $30°$. Find the distance of the monument from this place.

39. At a certain spot 165 ft. from the top of a particular part of Niagara Falls the angle of depression (see Art. 88) of the bottom of the falls is $45°$. What is the perpendicular extent of the falls?

40. How many of the examples in this exercise can you work at sight?

38. Many **trigonometric equations** involving only acute angles may now be solved.

Ex. 1. Find the value of x which satisfies the equation $\sin x = \frac{1}{2}$.

Since $\sin 30° = \frac{1}{2}$, in the given equation $x = 30°$, *Ans.*

Ex. 2. Solve $\sin x = \cos x$.

Dividing each member by $\cos x$, $\tan x = 1$.

$$\therefore x = 45°, \textit{ Ans.}$$

Ex. 3. Solve $\tan x - 1 = 2 \sin x - 2 \cos x$.

Substituting for $\tan x$, $\quad \dfrac{\sin x}{\cos x} - 1 = 2 \sin x - 2 \cos x.$

Hence, $\quad \sin x - \cos x = 2 \sin x \cos x - 2 \cos^2 x.$

Factoring, $(\sin x - \cos x)(1 - 2 \cos x) = 0$.

Hence, $\sin x - \cos x = 0. \quad \therefore \tan x = 1, \quad x = 45°.$

Also $1 - 2 \cos x = 0. \quad \therefore \cos x = \frac{1}{2}, x = 60°.$

Hence, $\quad x = 45°, 60°, \textit{ Ans.}$

Ex. 4. Given $\sin x = \cos 4\,x$, find x.

By Art. 26 we may substitute for $\sin x$ its equal, $\cos(90° - x)$.

Then
$$\cos(90° - x) = \cos 4\,x.$$
$$\therefore 90° - x = 4\,x.$$
$$5x = 90°.$$
$$x = 18°, \textit{ Ans.}$$

EXERCISE 13

Solve each of the following equations:

1. $\tan^2 x = 3.$
2. $\sin^2 x = \frac{3}{4}.$
3. $\cot x = 3 \tan x.$
4. $\cot^2 x = \frac{1}{3}.$
5. $\sqrt{1 - \sin^2 x} = 1 + \sin x.$
6. $\sec^2 x = 2.$
7. $\tan x + \cot x = 2.$
8. $\sec x = \sqrt{2} \tan x.$
9. $\cos^2 x - \sin^2 x = \sin x.$
10. $\tan^2 x + 2 \sec^2 x = 11.$
11. $3 \cot^2 x + \cot x = 4.$
12. $2 \sin y + \csc y = 3.$
13. $2 \sin x \sqrt{3} + 4 \cos x = 5.$
14. $\sec x = 2 \tan x.$
15. $4 \sin^2 x - \tan^2 x = \cot^2 x.$
16. $\cot x + 2 \tan x = \dfrac{5 \sec x}{2}.$
17. $3 \cos x + \tan x = 1 + 3 \sin x.$
18. $\tan x = 2 \cot x - 1.$
19. $\csc y = 2 \cot y.$
20. $2 \sin x + \cos x = 2.$
21. $2 \sec x - \cos x = 1.$
22. $\sin^2 x + \sin x = \frac{2}{3}.$

Solve:

23. $\sin x = \cos 5\,x.$
24. $\tan y = \cot 8\,y.$
25. $\cos \frac{1}{2} x = \sin x.$
26. $\sec(45° + x) = \csc x.$
27. $\sin y = \cos ny.$
28. $\sin 3\,x = \cos 2\,x.$

29. If a church steeple is at a distance of 80 ft., and the steeple is 80 ft. high, find the angle of elevation of the top of the steeple.

30. If the height of the steeple is 80.5 ft. and the distance of the base is 100 ft., see if you can find the angle of elevation of the top of the steeple by use of the table of natural tangents (pp. 91–96 of the tables).

31. Make up an example similar to Ex. 29.

32. Make up an example similar to Ex. 30.

33. In a right triangle given $c = 62$, $a = 31$, find A.

34. Given $c = 150$, $a = 75$, find B.

35. Given $c = 120$, $b = 60\sqrt{3}$, find A.

36. How many of the examples in this exercise can you work at sight?

39. Tables of Logarithms of the Trigonometric Functions from 0° to 90°. In performing numerical work involving trigonometric functions, it is usually more expeditious to proceed by the use of logarithms. Hence the logarithms of the natural trigonometric functions have been obtained once for all and arranged in tables called Tables of Logarithmic Trigonometric Functions. The use of these tables is explained in the Introduction to the Tables (Arts. 7–11).

EXERCISE 14

By the use of five-place tables, find:

1. log sin 26° 18′.

2. log cos 12° 16′.

3. log tan 36° 18′.

4. log cot 76° 18′.

5. log tan 55° 16′.

6. log tan 15° 18′.

7. log cos 86° 52′.

8. log tan 36°.

9. log sin 4° 6′ 55″.

10. log cos 17° 17′ 30″.

11. log cot 37° 28′ 50″.

12. log sin 78° 59′ 30″.

13. log tan 86° 46′ 5″.

14. log tan 4° 44′ 50″.

15. log cos 45° 48′ 48″.

16. log cot 60° 52′ 6″.

17. We have proved (see Art. 33) that sin 30° = .5. Obtain log .5 and thus show that the value of log sin 30° as given in the table is correct.

18. Similarly verify the value of log sin 45°, and of log tan 60°, as given in the table.

19. In the rt. $\triangle ABC$, $a = b \tan A$. (Why?) If $A = 18° 16'$ and $b = 18.63$, find a.

20. In the rt. $\triangle ABC$, $b = c \cos A$. (Why?) Find b if $c = 18.675$ and $A = 36° 36' 36''$.

By the use of four-place* tables, find:

21. log sin 15.3°.

22. log cos 47.5°.

23. log cot 33.7°.

24. log tan 78.8°.

25. log sin 27.35°.

26. log cos 26.36°.

---

* When the term "four-place tables" is used in connection with angles, the four-place logarithmic tables for the decimally divided degree are meant. See Arts. 18–19 of the tables.

27. log tan 63.78°.

28. log cot 12.65°.

29. log cos 40.16°.

30. log cot 29.23°.

31. In the rt. $\triangle BAC$, $b = a \cot A$. (Why?) If $A = 18.67°$ and $a = .2167$ feet, find b.

32. In the rt. $\triangle ABC$, $a = c \sin A$. (Why?) If $c = 17.65$ and $A = 59.72°$, find a. Also find b, if $b = c \cos A$.

EXERCISE 15

Using five-place tables, find A, given:

1. $\log \sin A = 9.59632 - 10$.
2. $\log \tan A = 9.73777 - 10$.
3. $\log \cos A = 9.90951 - 10$.
4. $\log \cot A = 10.07029 - 10$.
5. $\log \sin A = 9.96159 - 10$.
6. $\log \tan A = 0.44540$.
7. $\log \cos A = 9.53390 - 10$.
8. $\log \tan A = 1.06575$.
9. $\log \sin A = 9.95788 - 10$.
10. $\log \cot A = 1.02921$.
11. $\log \sin A = 8.84501 - 10$.
12. $\log \cos A = 8.84501 - 10$.

By use of four-place tables, find A, given:

13. $\log \sin A = 9.6495 - 10$.
14. $\log \cos A = 9.8063 - 10$.
15. $\log \tan A = 9.7384 - 10$.
16. $\log \cot A = 0.4755$.
17. $\log \cot A = 9.8248 - 10$.
18. $\log \tan A = 0.4422$.
19. $\log \cos A = 9.6351 - 10$.
20. $\log \cos A = 9.8409 - 10$.
21. $\log \tan A = 0.2575$.
22. $\log \cot A = 2.0248$.
23. $\log \tan A = 1.5718$.
24. $\log \sin A = 9.9596 - 10$.
25. $\log \cos A = 9.3129 - 10$.
26. $\log \cot A = 0.5881$.

EXERCISE 16

By use of five-place tables find:

1. $\log \sin 0° 56' 18''$.
2. $\log \tan 1° 16' 37''$.
3. $\log \cos 88° 13' 26''$.
4. $\log \tan 88° 54' 50''$.
5. $\log \cot 1° 18' 36''$.
6. $\log \cos 89° 7' 19''$.
7. $\log \sin 1° 6' 12''$.
8. $\log \cot 88° 16' 32''$.

Find the angle A if:

9. $\log \tan A = 7.88154 - 10$.
10. $\log \cos A = 8.28910 - 10$.
11. $\log \sin A = 8.09600 - 10$.
12. $\log \cot A = 7.90390 - 10$.
13. $\log \tan A = 3.05992$.
14. $\log \cot A = 2.88206$.
15. $\log \sin A = 6.88800 - 10$.
16. $\log \cos A = 7.63702 - 10$.

For "angle whose log sin is" we may write "$\angle$ log sin," or "antilog sin," hence find:

17. $\angle \log \sin 9.82627-10$.

18. $\angle \log \tan 10.90261-10$.

19. $\angle \log \cos 9.06000-10$.

20. $\angle \log \cot 8.09599-10$.

21. $\angle \log \cos 8.09599-10$.

22. $\angle \log \tan = 2.77651$.

23. In the $\triangle ABC$, $a = c \sin A$. Find a if $c = 18.6$ and $A = 26° 18' 48.''$

Find the value of the following:

24. $\dfrac{528.7 \times \cos 83° 16' 24'' \times \tan^2 75° 18' 24''}{672 \cot^2 18° 32' 54'' \times \sin 69° \div \cos^2 15° 16' 34''}$.

25. $\dfrac{265 \times \tan 65° 18' \times \cos^2 14° 28' 12''}{19 \cot^2 11° 16' 24'' \times \sin 75° 15' 45'' \times .7}$.

By use of four-place tables, find:

26. $\log \cos 88.76°$.

27. $\log \sin 0.762°$.

28. $\log \cot 89.267°$.

29. $\log \tan 1.067°$.

30. $\log \tan 88.763°$.

31. $\log \cot 0.765°$.

32. $\log \sin 1.267°$.

33. $\log \cos 89.467°$.

Find angle A if:

34. $\log \cot A = 8.1067-10$.

35. $\log \tan A = 8.2574-10$.

36. $\log \cos A = 8.1360-10$.

37. $\log \sin A = 8.0440-10$.

38. $\log \tan A = 2.1080$.

39. $\log \cot A = 2.0532$.

40. $\log \sin A = 7.9100-10$.

41. $\log \cos A = 7.9932-10$.

Find:

42. $\log \cot 88.676°$.

43. $\log \tan 88.676°$.

44. $\angle \log \cot 8.1078-10$.

45. $\angle \log \tan 8.0295-10$.

46. $\angle \log \cos 8.0959-10$.

47. $\angle \log \sin 8.0371-10$.

48. $\log \tan 88.68°$.

49. In the rt. $\triangle ABC$, $a = c \sin A$. (Why?) Find a if $c = 126.27$, and $A = 1.267°$.

50. In the rt. $\triangle ABC$, $b = a \cot A$. (Why?) Find b if $a = 0.4267$, and $A = 2.166°$.

51. Find the value of $\dfrac{632.7 \times \cos 78.16° \times \tan^2 71.62°}{426.8 \times \sin 13.25° \times \cot^2 12.47° \times .8}$.

52. Find the value of $\dfrac{326 \times \tan 38.25 \times \cos^2 88.627}{43 \times \cot 0.826° \times \sin^2 2.467°}$.

EXERCISE 17. REVIEW

1. In the right $\triangle ABC$, given $\tan A = \frac{8}{15}$ and $a = 16$, find b, c, and the other functions of A.

2. If $\cos A = \dfrac{8}{17}$, find the value of $\dfrac{\sin A + \tan A}{\cos A - \cot A}$.

3. Show that $\cos 60° \cos 30° + \sin 60° \sin 30° = \cos 30°$.

4. Show that $\dfrac{\cot 45° + \cot 90°}{1 - \cot 45° \cot 90°} = 1$.

(Work Exs. 5–12 without the use of tables.)

5. Which is greater, $\sin 49°$ or $\cos 49°$?

6. If $\sin A = \frac{3}{5}$, is A greater or less than $45°$?

7. If $\tan A = 2$, is A greater or less than $60°$?

8. Which is the greater, $\tan 37°$ or $\cot 37°$?

9. If $A = 60°$, show that $\sin \frac{1}{2} A = \sqrt{\dfrac{1 - \cos A}{2}}$.

10. If $A = 60°$, show that $\cot \frac{1}{2} A = \sqrt{\dfrac{1 + \cos A}{1 - \cos A}}$.

11. Which is greater, $\sin 45°$ or $\frac{1}{2} \sin 90°$? $\sin 60°$ or $2 \sin 30°$? $\tan 30°$ or $\frac{1}{2} \tan 60°$?

12. If $x = 30°$ and $y = 60°$, show that $\sin x \cos y + \cos x \sin y = \sin (x + y)$.

13. Prove $\dfrac{1 + \cot A}{1 - \cot A} = \dfrac{\sec A + \csc A}{\sec A - \csc A}$.

14. Prove $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \dfrac{\sin^2 A}{\cos^2 A}$.

15. Prove $\dfrac{1 + \cos A}{1 - \cos A} = (\csc A + \cot A)^2$.

16. If $x = 30°$, show that $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$.

17. If $x = 30°$, show that $\sin 3x = 3 \sin x - 4 \sin^3 x$.

18. If $x = 30°$, show that $\cos 3x = 4 \cos^3 x - 3 \cos x$.

Solve the following trigonometric equations: —

19. $\tan x + 3 \cot x = 4$.

20. $2 \sec^2 x - \tan^2 x = 5$.

21. $3 \csc^2 x - 2 \cot x = 4$.

If $P=0°$, $Q=30°$, $R=45°$, $S=60°$, $T=90°$, find the value of:

22. $\cos^2 Q+\cos^2 S+\cos^2 T+2\cos Q\cos S\cos T$.

23. $\sec Q(1+\tan R)-\sin^3 T(\cos R+\sin S\cos Q)$.

24. $\dfrac{1+\tan^2 S}{2-\tan^2 R}+3(\cos P\sin^2 R-\sin S)$.

25. If $25\sin A=7$, find $\cot A$ and $\csc A$.

26. If $p\cot\theta=\sqrt{r^2-p^2}$, find $\sin\theta$.

27. If i denotes the angle of incidence of a ray of light falling on a piece of glass, and r the angle of refraction, then $\sin i=\frac{3}{2}\sin r$. Find r when $i=27°\ 17'$.

28. If at a distance of 300 ft. the angle of elevation of the top of one of the big trees of California is 45°, how tall is the tree?

29. If at a distance of 300 ft. the angle of elevation of the top of a tree were 42°, see if you can find out how tall the tree would be. (Why are we able to determine this height by trigonometry and not by geometry ?)

30. Who first, and at what date, defined the sine of an angle as the ratio between two lines (see p. 165)? Give the different substitutes for this idea of the sine that had been used before this time. Why is the ratio definition of the sine superior to each of these?

31. Explain the origin and literal meaning of the word sine (see p. 166).

32. Who first invented each of the other trigonometric ratios, and at what time (see pp. 162, 164)?

33. Give some of the various names used for these ratios, with the names of the inventors of these names.

34. What nation first used the trigonometrical identity

$$\sin^2 A+\cos^2 A=1 \text{ (see p. 172)?} \quad \tan x=\frac{\sin x}{\cos x}?$$

35. Give an account of the computation of trigonometric tables (see pp. 168–170).

CHAPTER III

RIGHT TRIANGLES

40. Two Cases arise in the trigonometrical solution of right triangles.

Case I. *Given* **one side** *and* **an acute angle.**

Case II. *Given* **two sides.**

In each of these cases it will be observed that three parts are really given, since the right angle is known.

Case I

41. The solution of Case I is effected as follows:

Subtract the given angle from 90°. *This will give the unknown angle.*

The unknown sides may then be found by means of the following:

1. *Either leg* = (*sine of* ∠ *opposite*) × *hypotenuse.*
2. *Either leg* = (*cosine of* ∠ *adjacent*) × *hypotenuse.*
3. *Either leg* = (*tangent of* ∠ *opposite*) × *other leg.*
4. *Hypotenuse* = (*secant of either acute* ∠) × (*leg adjacent to that* ∠).

Also (either leg) = (cot of ∠ adjacent) × (other leg);
hyp. = (csc of either acute ∠) × (leg opposite that ∠).

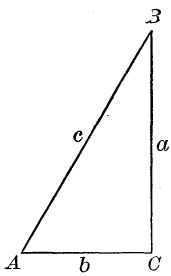


Fig. 18.

Proof

By def., $\sin A = \frac{a}{c}$. $\therefore a = c \sin A$.

Also $\cos B = \frac{a}{c}$. $\therefore a = c \cos B$.

$\tan A = \frac{a}{b}$. $\therefore a = b \tan A$.

Also $\sec B = \frac{c}{a}$. $\therefore c = a \sec B$.

Similarly it may be proved that:

$$b = c \sin B, \quad b = c \cos A, \quad b = a \tan B, \text{ and } c = b \sec A.$$

Ex. 1. Given $A = 55° \, 43' \, 29''$, $c = 415.18$, find the remaining parts of the right triangle.

We first draw a diagram (Fig. 19) of the triangle to be solved, and on this diagram write the known magnitudes (415.18 for c, and $55° \, 43' \, 29''$ for A). We also indicate the parts to be computed (a, b, B) by annexing the = mark to each of these. During the numerical computation, as soon as the result for any part is ascertained, this result should be entered on the diagram after the proper = mark.

$\angle B = 90° - 55° \, 43' \, 29'' = \mathbf{34° \, 16' \, 31''}.$

$a = 415.18 \sin 55° \, 43' \, 29''.$ (Art. 41, 1)

$\therefore \log a = \log 415.18 + \log \sin 55° \, 43' \, 29''.$

415.18	log	2.61824
55° 43′ 29″	log sin	9.91716 − 10
a = **343.085**	log	2.53540

Also $b = 415.18 \cos 55° \, 43' \, 29''.$ (Art. 41, 2)

$\therefore \log b = \log 415.18 + \log \cos 55° \, 43' \, 29''.$

415.18	log	2.61824
55° 43′ 29″	log cos	9.75064 − 10
b = **233.821**	log	2.36888

(As a check use $a = b \tan A$.)

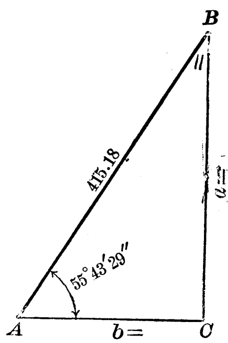


FIG. 19.

Ex. 2. Given $a = .0723$, $B = 31° \, 47' \, 7''$, find the remaining parts of the right triangle.

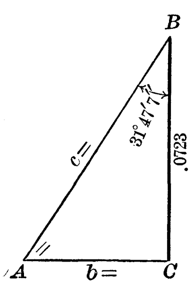


FIG. 20.

$\angle A = 90° - 31° \, 47' \, 7'' = \mathbf{58° \, 12' \, 53''}.$

$b = .0723 \tan 31° \, 47' \, 7''$

.0723	log	8.85914 − 10
31° 47′ 7″	log tan	9.79216 − 10
b = **.448022**	log	8.65130 − 10

$c = .0723 \sec 31° \, 47' \, 7''$

$$= \frac{.0723}{\cos 31° \, 47'' \, 7'}$$

		.9723 log	8.85914 − 10
31° 47′ 7″ log cos	9.92943 − 10	colog cos	0.07057
c =	**.0850567**	log	8.92971 − 10

(As a check use $b = c \cos A$.)

Ex. 3. By use of four-place tables solve the right triangle in which $b = 21.635$, $A = 47.23°$.

$$\angle B = 90° - 47.23° = \mathbf{42.77°}.$$

Also $\quad a = 21.635 \tan 47.23°. \qquad$ (Art. 41, 3)

$$\therefore \log a = \log 21.635 + \log \tan 47.23°.$$

21.635	log	1.3352
47.23°	log tan	0.0339
$a = \mathbf{23.394}$	log	1.3691

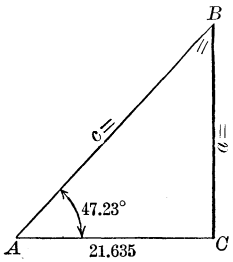


FIG. 21.

By Art. 41, 4, $c = 21.635 \sec 47.23° = \dfrac{21.635}{\cos 47.23°}$

$$\therefore \log c = \log 21.635 + \text{colog} \cos 47.23°$$

21.635	log	1.3352
47.23°	colog cos	0.1681
$c = \mathbf{31.864}$	log	1.5033

(As a check use $a = c \cos B$.)

42. First Estimates. Graphical Solutions. In the solutions of triangles fully one half the mistakes commonly made, and those the most important ones, are eliminated by making a rough mental forecast of the results before proceeding with the exact numerical work.

Thus in solving Ex. 1 of Art. 41, the pupil should first of all observe that, the hypotenuse being 415.18, each of the legs will be less than 415.18; and also that, since angle B is less than angle A, side b must be less than side a. If then as a result of his exact numerical calculation, the pupil finds a leg greater than 415.18, or a less than b, he knows at once that a mistake has been made.

Similarly it is useful, by means of the rule and protractor, to make a drawing according to scale of the triangle to be solved, and from the figure to determine as accurately as possible the dimensions of the unknown parts by measuring them according to scale. Such results should be accurate enough to aid in eliminating any large errors in the numerical work. (Indeed, if the work be neatly done, the results obtained from the diagram will be accurate enough for many practical purposes.)

43. Exact checks of the numerical accuracy of the work of solving right triangles are obtained by calculating some side or angle of the triangle by a formula different from those already used in the computation, and observing whether the results thus obtained accord with those obtained in the first solution.

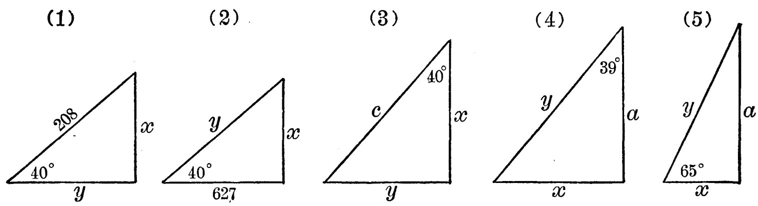
Thus, to check the accuracy of the solution given for Ex. 1, Art. 41, determine whether $\tan A = \frac{a}{b}$; that is, compute the value of the fraction $\frac{343.085}{233.821}$ and also obtain from the table the value of $\tan 55° 43' 29''$ and observe whether these two values accord.

EXERCISE 18

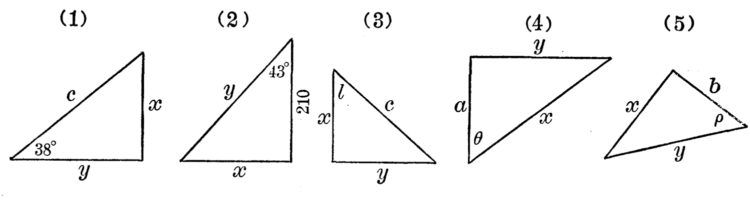
State at sight the formula value of x (or of x and y) in each of the following triangles:

Thus in Ex. 1, (1), $x = 208 \sin 40°$.

1.



2.



3. Make up an example similar to Ex. 2.

By use of five-place tables solve each of the following triangles, given:

(In working each example outline all the work carefully before looking up any logs — see Ex. 1, p. 18.)

4. $A = 28°$, $b = 12$.

5. $A = 78°$, $c = 26.735$.

6. $A = 46° 18'$, $b = 48.527$.

7. $A = 28° 17'$, $c = 24.16$.

8. $B = 54° \ 43'$ $c = 1123$.

9. $B = 37° \ 19'$, $b = 293.8$.

10. $A = 38° \ 16' \ 24''$, $c = 3.6289$.

11. $B = 72° \ 16' \ 42''$, $a = 22.684$.

12. Given $c = .52684$, $B = 63° \ 18' \ 48''$; find a.

13. Given $A = 37° \ 25' \ 20''$, $c = .356$; find b.

Find the remaining parts in each of the following right triangles, given:

14. $A = 63° \ 28' \ 40''$, $a = 256.43$.

15. $c = 13.867$, $A = 87° \ 16' \ 30''$.

16. $A = 51° \ 9' \ 6''$, $c = .19678$.

17. $a = 126.78$, $A = 26° \ 18' \ 36''$.

18. Given $A = 5° \ 16' \ 32''$, $b = .96156$; find c.

19. Given $A = 37° \ 14' \ 15''$, $b = 217$; find a.

20. If the top of the Statue of Liberty in New York harbor is 301 ft. above the water surface, and a boat in the harbor finds the angle of elevation of the top of the statue to be 12°, how far is the boat from the statue?

21. If a certain point on the brink of the Grand Cañon of the Colorado is known to be a horizontal distance of 3 miles from the Colorado River and the angle of depression of the river is 17°, how deep is the cañon at that place and how far from the observer is the river in a straight line?

22. Which of the examples in Exercise 22 are you able to solve by Case I? Solve one of these.

23. Make up a similar practical problem for yourself and solve it, as for instance one concerning the Bunker Hill monument (221 ft. high).

Solve the following right triangles, by use of four-place tables, having given:

24. $A = 32.6°$, $b = 18$.

25. $A = 56°$, $c = 2.678$.

26. $B = 38.2°$, $c = .7685$.

27. $B = 82.5°$, $a = 12.56$.

28. $A = 37.67°$, $c = 126.7$.

29. $B = 76.25°$, $a = .926$.

30. $A = 21.32°$, $a = 16.256$.

31. $B = 66.27°$, $b = .0087$.

32. Given $c = .6243$, $B = 51.25°$; find a.

33. Given $A = 77.26°$, $c = .5163$; find b.

34. Given $B = 39.29°$, $b = 41.67$; find a.

Find the remaining parts in each of the following right triangles, given:

35. $c = 13.13$, $A = 88.17°$.

36. $B = 42.16°$, $a = .5252$.

37. Given $A = 5.26°$, $b = 128.6$; find c.

38. Given $B = 87.267°$, $c = 22.67$; find a.

39. Given $A = 4.276°$, $a = 26.32$; find b.

40. Work Exs. 20–23 by four-place tables.

Solve without the use of tables, having given:

41. $A = 30°$, $b = 7$.

42. $A = 45°$, $c = 12$.

43. $B = 60°$, $b = 25$.

44. $B = 30°$, $a = 1000$.

45. $A = 60°$, $a = 2000$.

46. $B = 30°$, $c = 1200$.

47. $A = 45°$, $b = 200$.

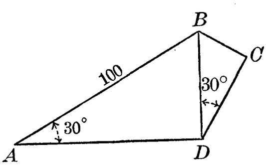
48. $A = 30°$, $c = 20\,d$.

49. Solve Exs. 6 and 7 of this exercise without the use of logarithms (*i.e.* by the use of the Tables of Natural Sines, etc., pp. 91–96).

50. How many of Exs. 41–48 can you solve at sight without drawing a figure?

51. On the figure if $\triangle ADB$ and DCB are right $\triangle$s, find BD, BC, and DC at sight.

52. On Fig. 52, p. 93, if $OP = 1$, what is the value of OQ? of PQ? of QN? of ON?



Case II

TWO SIDES GIVEN

44. The Solution of Case II is effected as follows:

Find one of the angles of the given triangle by using that one of the following trigonometric ratios which contains the two given sides:

1. *sine of either acute* $\angle = \dfrac{\perp\, opp.}{hyp.}$.

2. *cosine of either acute* $\angle = \dfrac{\perp\, adj.}{hyp.}$.

3. *tangent of either acute* $\angle = \dfrac{\perp\, opp.}{\perp\, adj.}$.

Find the remaining parts of the triangle by Case I (but if the hypotenuse and a leg are given, the other leg may be found by one of the formulas, $a=\sqrt{(c+b)(c-b)}$, $b=\sqrt{(c+a)(c-a)}$).

Ex. 1. Given $a = 317$, $c = 438$, find the remaining parts of the right triangle ABC.

$$\sin A = \frac{317}{438}. \qquad \text{(Art. 44, 1)}$$

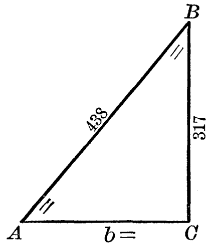


FIG. 22.

Hence $\log \sin A = \log 317 + \operatorname{colog} 438$

	317	log 2.50106
	438	log 2.64147 colog 7.35853 − 10
$A =$	**46° 21′ 55″**	log sin 9.85959 − 10

$B = 90° - 46° 21' 55'' =$ **43° 38′ 5″**.

$b = 438 \cos 46° 21' 55''.$ (Art. 41, 2)

	438	log 2.64147
	46° 21′ 55″	log cos 9.83888 − 10
$b =$	**302.24**	log 2.48035

$\left(\text{As a check use } \tan A = \frac{a}{b}.\right)$

Ex. 2. By use of four-place tables, solve the right triangle in which $a = 3.104$, $b = 2.965$.

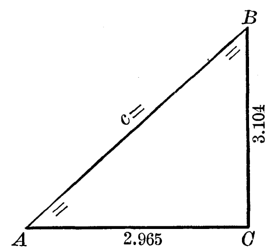


FIG. 23.

$$\tan A = \frac{3.104}{2.965}. \qquad \text{(Art. 44)}$$

$$B = 90° - A.$$

$$c = \frac{3.104}{\cos B}. \qquad \text{(Art. 41)}$$

	3.104 log	0.4920
	2.965 colog	9.5279 − 10
$A =$	**46.31°** log tan	0.0199

$B = 90° - 46.31° =$ **43.69°**.

	3.104 log	0.4920
	43.69° colog cos	0.1408
$c =$	**4.293** log	0.6328

45. Sources of Power in Trigonometrical Solution of Triangles. There is danger that the pupil form mechanical habits of solving triangles without realizing the nature or

meaning of what he is doing. He should constantly realize that he is able to do what he is doing because some one before him has computed the legs of every possible right triangle whose hypotenuse is 1, and the other parts when each leg is 1, and arranged the results in tables (natural sines, etc.,) and that he uses these results (and therefore uses the work done in computing them) by the geometrical principle of similar triangles. Also that some one else has made the pupil's work easier by looking up the logarithms of all the numbers in the natural tables and arranging them in other tables, and that the pupil is using this work also.

46. Special Case. Given the hypotenuse and a leg nearly equal, the angle between them will be very small. If this angle be found directly from the parts given, it will be found in terms of the cosine. Since the cosine of a small angle changes slowly as the angle varies, such a solution will not be accurate in the last figures. A more accurate solution is obtained by first calculating the third side by the use of the formula $a = \sqrt{(c+b)(c-b)}$ and finding the angle mentioned in terms of the sine.

Ex. Given $c = 412$, $b = 410$, solve the triangle.

By the formula, $a = \sqrt{(412+410)(412-410)}$

$= \sqrt{822 \times 2}$.

$\therefore \log a = \frac{1}{2}(\log 822 + \log 2)$.

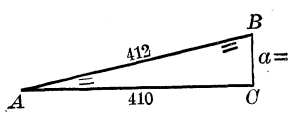


FIG. 24.

822 log 2.91487

2 log 0.30103

2)3.21590

$a =$ **40.546** log 1.60795

Also $\sin A = \dfrac{40.546}{412}$

40.546 log 1.60795

412 colog 7.38510 − 10

$A =$ **5° 38′ 52″** log sin 8.99305 − 10

$B = 90° - 5° 38' 52'' =$ **84° 21′ 8″**.

EXERCISE 19

Using five-place tables, solve in full the following right triangles, given:

(In working each example outline all the work carefully before looking up any logs — see Ex. 1, p. 18.)

1. $c = 18.4,\ \ a = 10.7.$

2. $c = 37.266,\ \ a = 20.46.$

3. $a = 26.725,\ \ c = 39.626.$

4. $a = 5,\ \ b = 6.$

5. $c = .89672,\ \ a = .68425.$

6. $b = 14.222,\ \ c = 21.678.$

7. $a = .0628,\ \ b = .0487.$

8. $a = .1777,\ \ c = .25643.$

9. Given $a = 4$ yd., $b = 9$ ft., find A.

10. Given $a = 8.701$ yd., $b = 21.645$ yd., find $\angle A$.

11. Given $b = .26725,\ \ c = .39626$, find $\angle B$.

12. Solve in full if $a = 6,\ \ b = 6.$

13. Find A if $a = .02678,\ \ b = .05537.$

14. Solve in full if $c = 117.32,\ \ a = 112.67.$

Suggestion. First use $b = \sqrt{c^2 - a^2} = \sqrt{(c + a)(c - a)}.$

15. Solve in full if $b = 358,\ \ c = 362.$

16. Solve in full if $a = 26.63,\ \ c = 27.99.$

17. If the Mt. Washington railway at a certain place rises 3596 ft. for 3 mi. of the length of the track, what angle on the average does the track make with the horizon?

18. The carpenter's rule for constructing $\frac{3}{4}$ of a right angle is to construct a right triangle whose legs are 5 and 12 inches and take the greater acute angle in the triangle. How far is this from being correct?

19. Which of the examples in Exercise 22 are you able to solve by the methods of Case II? Solve two of these.

20. Make up a similar practical problem for yourself and solve it.

Solve by use of four-place tables, having given:

21. $c = 23.7,\ \ a = 15.7.$

22. $c = .562,\ \ b = .3962.$

23. $a = 33.29,\ \ b = 27.28.$

24. $a = 5,\ \ b = 8.$

25. $b = 6.7,\ \ c = 9.7.$

26. $b = .12675,\ \ a = .14296.$

27. $c = 132.96,\ \ b = 100.82.$

28. $a = .07282,\ \ c = .11111.$

29. $a = 2367,\ \ b = 1827.6.$

30. Given $a = 11$, $c = 16$, find A.

31. Given $a = 27.82$, $b = 33.67$, find B.

32. Given $c = 156.7$, $b = 148.2$, solve in full.

First use $a = \sqrt{c^2 - b^2} = \sqrt{c + b)(c - b)}$.

33. Given $c = 862$, $a = 854$, solve in full.

34. Given $a = 98.6$, $b = 63.4$, find A.

35. Given $c = .4367$, $b = .1967$, find B.

36. Work Exs. 17–20 by the four-place tables.

Without the use of tables solve in full each of the following right triangles, given:

37. $a = 13$, $b = 13$.

38. $c = 18$, $a = 9$.

39. $c = 200$, $b = 100$.

40. $a = \sqrt{3}$, $b = 1$.

41. $c = 6$, $a = 3\sqrt{3}$.

42. $c = \sqrt{2}$, $b = 1$.

43. $c = 100$, $a = 50\sqrt{3}$.

44. $a + c = 18$, $b = 6\sqrt{3}$.

45. Solve Exs. 3 and 4 of this Exercise without the use of logarithms.

46. How many of Exs. 37–43 are you able to solve at sight without drawing a figure?

47. Isosceles Triangles. If certain parts of an isosceles triangle be given, the unknown parts may often be determined by *dividing the isosceles triangle into two equal right triangles by means of a perpendicular drawn from the vertex to the base, and by solving one of the right triangles thus formed.*

Ex. 1. If the vertex angle of an isosceles triangle is 42° 30′ and a leg is 47.6, find the base.

Draw the altitude OD. Then $\angle AOD = 21° 15'$. Hence, in the right $\triangle$ AOD, we have a side and an acute angle given, to find the base AD (Case I). Hence

$$AD = 47.6 \sin 21° 15'.$$

$$\begin{array}{rl} 47.6 & \log 1.67761 \\ 21° 15' & \log \sin 9.55923 - 10 \\ \hline AD = 17.252 & \log 1.23684 \end{array}$$

$$AB = 2\,AD = \mathbf{34.504}.$$

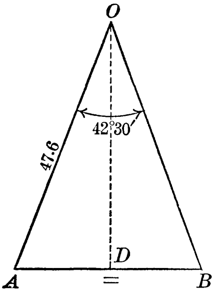


FIG. 25.

Ex. 2. By use of four-place tables, solve the isosceles triangle whose base is 12.25 and vertex angle 28.22°.

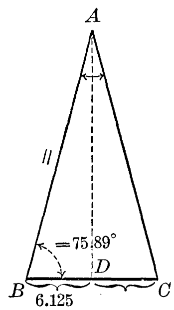


FIG. 26.

Draw the altitude AD.

Then $\angle BAD = \frac{1}{2}(28.22°) = 14.11°$.

$\angle B = 90° - 14.11° = \mathbf{75.89°}$.

$$AB = 6.125 \sec 75.89° = \frac{6.125}{\cos 75.89°}$$

6.125	log	0.7872
75.89°	colog cos	0.6130
$AB =$ **25.129**	log	1.4002

48. A regular polygon may be *divided into equal right triangles by lines drawn from the center to the vertices and by the apothems to the sides*. Hence if certain parts of a regular polygon are given, the remaining parts may often be determined by dividing the polygon into right triangles and solving one of these triangles.

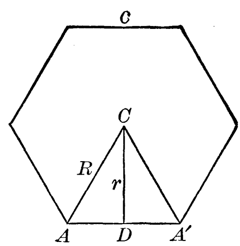


FIG. 27.

It is to be observed that one of the right triangles, as ACD of Fig. 27, has the radius of the circle circumscribed about the polygon for its hypotenuse AC, and the radius of the inscribed circle, CD, for a leg. Hence, $\angle ACA' = \frac{360°}{n}$, where n denotes the number of sides of the polygon, and $\angle ACD$ of the right triangle $= \frac{180°}{n}$.

EXERCISE 20

Using five-place tables, solve each of the following isosceles triangles, given:

1. Base $= 120$, base $\angle = 60°$.
2. Leg $= 216$, vertex $\angle = 110°$.
3. Base $\angle = 56° 18'$, leg $= 8.7265$.
4. Base $\angle = 38° 17' 50''$, altitude $= 31.42$.

5. Base $\angle = 55° \, 18' \, 24''$, altitude $= 762.89$.

6. Base $= 8.2364$, altitude $= 7{\cdot}8$.

7. Vertex $\angle = 113° \, 17'$, base $= .12692$.

8. Altitude $= 4835$, base $= 9248$.

9. One side of a regular pentagon is 12. Find the apothem, radius, perimeter, and area of the pentagon.

10. One side of a regular decagon is 1. Find the apothem, radius, perimeter, and area of the decagon.

11. The radius of a circle is 16 feet. Find the side, apothem, and area of a regular inscribed dodecagon.

12. Find the same magnitudes for a regular dodecagon which is circumscribed about a circle whose radius is 17.

13. The diagonal of a regular pentagon is 14; find the side, apothem, perimeter, and area of the pentagon.

14. The apothem of a regular heptagon is 0.69786; find the perimeter and area of the heptagon.

If m denotes the base, h the altitude, l the leg, C the vertex angle, and D the base angle of an isosceles triangle, find:

15. h, m, and C, in terms of D and l.

16. D, l, and C, in terms of m and h.

17. D, C, and m, in terms of h and l.

18. C, h, and l, in terms of D and m.

19. D, h, and l, in terms of C and m.

20. Solve the isosceles triangle in which a leg $= 2.62731$ and the altitude $= 1.76683$.

21. If a chord 22.67 ft. in length subtends an arc $127° \, 23'$, what is the radius of the circle?

22. If the radius of a circle is 105.27 ft., what is the length of a chord which subtends an arc of $54° \, 13'$?

23. The side of a regular polygon of fourteen sides inscribed in a circle is 21.6 ft.; find the side of a regular twenty-sided polygon inscribed in the same circle.

24. The radius of a circle is R; show that each side of a regular inscribed polygon of n sides is $2\,R \sin\left(\frac{180°}{n}\right)$, and that each side of a regular circumscribed polygon is $2\,R \tan\left(\frac{180°}{n}\right)$.

25. Each side of a regular polygon of n sides is m; show that the radius of the circumscribed circle is equal to $\frac{m}{2}\csc\left(\frac{180°}{n}\right)$, and the radius of the inscribed circle is equal to $\frac{m}{2}\cot\left(\frac{180°}{n}\right)$.

26. If the chord of an arc of 36° is 24, find the chord of an arc of 12° in the same circle.

27. If the chord of an arc of 48° is 36, find the chord of an arc of 66° in the same circle.

Using four-place tables, solve the isosceles triangle in which:

28. Leg $= 36.72$, base $\angle = 32.6$.

29. Base $= 1600$, base $\angle = 67.4°$.

30. Vertex $\angle = 117.72°$, altitude $= 17.83$.

31. Base $= .7368$, altitude $= .4864$.

32. Altitude $= 112.67$, leg $= 128.7$.

33. Leg $= 67.87$, base $\angle = 32.73°$.

34. Altitude $= .11683$, base $\angle = 76.18°$.

35. Base $= 31.26$, altitude $= 21.73$.

36. Vertex $\angle = 151.7°$, leg $= .4363$.

37. One side of a regular octagon is 14. Find the apothem and area of the octagon.

38. The apothem of a regular pentagon is 19.7. Find the perimeter of the pentagon.

39. A regular decagon is inscribed in a circle whose radius is 1.76. Find the side and apothem of the decagon.

40. Find the magnitude of the various parts of a regular heptagon circumscribed about a circle whose radius is 21.

41. The diagonal connecting two alternate vertices of a regular dodecagon is 18. Find the side, apothem, and area of the dodecagon.

42. If a chord of 37.82 ft. subtends an arc of 118.3°, find the radius of the circle.

43. If the radius of a circle is 100, what is the length of a chord which subtends an arc of 67.7°?

Without the use of the tables, solve the following :

44. The base of an isosceles triangle is 50, and the vertex angle is 120°. Find the base angle and altitude.

45. The leg of an isosceles triangle is 100, and the altitude is 50. Find the base angle and base.

46. The altitude of an isosceles triangle is 10, and the base angle is 60°. Find a leg and the base.

47. The leg of an isosceles triangle is $6\sqrt{2}$, and the base is 12. Find the base angle, vertex angle, and altitude.

48. The radius of a circle is 2. Find the number of degrees in an arc which subtends a chord whose length is $2\sqrt{3}$.

49. The diagonal of a square is 10. Find the side of the square.

50. How many of Exs. 44–49 can you work at sight?

AREAS

49. General Method of computing Area of a Right Triangle. If b denote the base, a the altitude, and K the area of a right triangle, by geometry $K = \frac{1}{2}ab$.

$$\therefore \log \boldsymbol{K} = \log \boldsymbol{a} + \log \boldsymbol{b} + \text{colog } 2.$$

Ex. 1. Given $A = 37° \, 19'$, $b = 308$, find the area of the right triangle.

To find log a and then the area we proceed as follows:

$a = 308 \tan 37° \, 19'$. (Art. 41)

$$\begin{array}{rl} 308 \log & 2.48855 \\ 37° \, 19' \log \tan & \underline{9.88210 - 10} \\ a \log & 2.37065 \\ 308 \log & 2.48855 \\ 2 \text{ colog} & \underline{9.69897 - 10} \\ K = \mathbf{36155} \log & 4.55817 \end{array}$$

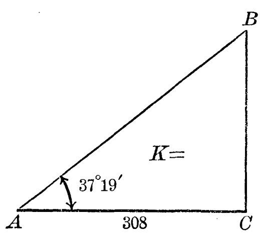


FIG. 28.

Ex. 2. Find the area of a right triangle in which the hypotenuse is 417 and the base 356.

$$\begin{aligned} a &= \sqrt{c^2 - b^2} = \sqrt{(417)^2 - (356)^2} \\ &= \sqrt{(417 + 356)(417 - 356)} = \sqrt{773 \times 61}. \end{aligned}$$

$$\therefore \log a = \tfrac{1}{2}(\log 773 + \log 61).$$

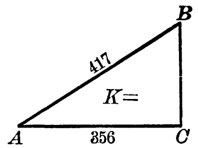


Fig. 29.

$K = \frac{1}{2}ab$. $\therefore \log K = \log a + \log b + \text{colog } 2$.

$$\begin{array}{rlrl} 773 & \log 2.88818 & \frac{1}{2}\log & 1.44409 \\ 61 & \log 1.78533 & \frac{1}{2}\log & 0.89267 \\ & & 356 \log & 2.55145 \\ & & 2 \text{ colog} & \underline{9.69897 - 10} \\ & & K = \mathbf{38652.7} \log & 4.58718 \end{array}$$

Ex. 3. By use of four-place tables find the area of the right triangle in which $A = 37.32°$ and $b = 308$ (see Fig. 28).

$$\log K = \log a + \log 308 + \text{colog } 2.$$

To find $\log a$, $\quad a = 308 \tan 37.32°$.

$$\begin{array}{rl} 308 \log & 2.4886 \\ 37.32° \log \tan & \underline{9.8821} \\ a \log & 2.3707 \\ 308 \log & 2.4886 \\ 2 \text{ colog} & \underline{9.6990 - 10} \\ K = \mathbf{36167} \log & 4.5583 \end{array}$$

50. Formulas for Area of a Right Triangle. The area of a right triangle may often be obtained more readily by the use of a formula involving only the particular parts of the triangle given. Denoting the area of a right triangle by K, let the pupil show that

When the *two legs* are given, $\boldsymbol{K = \frac{1}{2} ab}$.

When *an acute angle and the hypotenuse* are given,

$$\boldsymbol{K = \frac{1}{2} c^2 \sin A \cos A} \quad (\text{or } = \tfrac{1}{2} c^2 \sin B \cos B).$$

When *the hypotenuse and a leg* are given,

$$\boldsymbol{K = \frac{1}{2} a\sqrt{(c+a)(c-a)}} \quad (\text{or } = \tfrac{1}{2} b\sqrt{(c+b)(c-b)}).$$

When *an acute angle and a leg* are given,

$$\boldsymbol{K = \frac{1}{2} a^2 \tan B} \quad (\text{or} = \tfrac{1}{2} b^2 \tan A),$$

or

$$\boldsymbol{K = \frac{1}{2} a^2 \cot A} \quad (\text{or} = \tfrac{1}{2} b^2 \cot B).$$

By geometry, what is the method or formula for computing the *area of an isosceles triangle?* of a *regular polygon?* The formulas given above for computing the area of a right triangle are sometimes useful in computing the area of an isosceles triangle, or of a regular polygon.

EXERCISE 21

Using five-place tables, compute the area of the right triangle in which:

1. $A = 28° 18'$, $b = 216$.
2. $B = 72°$, $a = 196$.
3. $A = 21° 16' 30''$, $c = 31.967$.
4. $c = 46.72$, $b = 32.54$.
5. $B = 63° 18'$, $c = 124.72$.
6. $a = 192.7$, $b = 212.97$.
7. $a = 0.73216$, $c = .9125$.
8. $c = 927.8$ ft., $b = 759.8$ ft.
9. Given $a = 2.5$ and $K = 4.27$, find b, c, and A.
10. Given $K = 7.256$ and $A = 26° 18'$, find a, b, and c.
11. Given $K = 55.686$ and $c = 15.67$, find a, b, and A.

Compute the area of the isosceles triangle in which:

12. Base $= 12.67$, leg $= 9.267$.
13. Base $= .67892$, altitude $= .26217$.
14. Base angle $= 68° 18'$, leg $= .2892$.
15. Vertex angle $= 105° 17'$, altitude $= 13.67$.
16. Vertex angle $= 113° 18'$, leg 25.6.
17. Given area $= 16.72$ and base $= 6.37$, find altitude, leg, and base angle.
18. Given area $= .9273$ and base angle $= 27° 18'$, find leg, base, and altitude.
19. Given area $= 22.76$ and vertex angle $= 117° 55'$, find leg, base, and altitude.
20. Find the area of the regular pentagon whose perimeter is 3.35.
21. Find the area of the regular dodecagon whose apothem is 1.7267.
22. Find the area of a regular heptagon inscribed in a circle whose radius is 0.7516.
23. Given a regular octagon whose apothem is 2.27; find the difference between its area and that of the inscribed circle.
24. Given $n = 9$ and $K = 30$, find r, c, and R.
25. Given $n = 11$ and $K = 35$, find the perimeter.
26. Given $n = 5$ and $K = 37$, find p and R.
27. If n denotes the number of sides, R the radius, and C the central angle of any regular polygon, prove that $K = nR^2 \sin \frac{1}{2} C \cos \frac{1}{2} C$.

Using four-place tables, find the area of each of the following right triangles, given:

28. $A = 34.6°$, $a = 67.8$.

29. $B = 84°$, $a = 100$.

30. $A = 18.62°$, $b = 72.36$.

31. $a = .16376$, $b = .19762$.

32. $b = 8.42$, $c = 11.26$.

33. $B = 39.24°$, $c = 23.68$.

34. $c = 5000$, $a = 3000$.

35. $A = 47°$, $a = .0087$.

Solve the following right triangles, given:

36. $b = 6.37$, $K = 26.38$.

37. $K = 1200$, $A = 63.18°$.

38. $K = .4962$, $c = .1635$.

Find the area of each of the following isosceles triangles, given:

39. Base $= .7262$, leg $= .5263$.

40. Altitude $= 12.36$, leg $= 17.27$.

41. Altitude $= 86.27$, base $= 111.63$.

42. Base angle $= 42.67°$, leg $= 17.43$.

43. Vertex angle $= 100.24°$, altitude $= 8.217$.

44. Vertex angle $= 78.32°$, leg $= .6526$.

In an isosceles triangle:

45. Given area $= 192.67$ and base $= 43.64$, find altitude, leg, and base angle.

46. Given area $= 0.7362$ and base angle $= 37.43°$, find leg, base, and altitude.

47. Given area $= 1367.8$ and vertex angle $= 113.28°$, find base, leg, and altitude.

48. Given area $= .1025$, and leg $= .4916$, find the base, altitude, and angle at the base.

49. Find the area of a regular decagon whose perimeter is 27.63.

50. Find the area of a regular pentagon whose apothem is .4782.

51. Find the area of a regular heptagon inscribed in a circle whose radius is 116.2.

52. Given the side of a regular octagon as 5.33, find the difference between the area of the octagon and that of the circumscribed circle.

In a regular polygon:

53. Given $n = 7$ and $K = 14$, find c, r, and R.

54. Given $n = 11$ and $K = 1000$, find r, c, and R.

55. Given $n = 9$ and $K = 47$, find r, c, and R.

56. Given $n = 14$ and $K = 800$, find the perimeter.

Without the use of the tables, find the area of each of the following right triangles, given:

57. $a = 100$ and $A = 60°$.

58. $b = 600$ and $c = 1200$.

59. $a = 26.3$ and $b = 21.2$.

60. $B = 60°$ and $a = 90$.

61. $a = 80$ and $c = 160$.

62. $b = 40$ and $c = 40\sqrt{2}$.

63. $c = 4000$ and $A = 30°$.

64. $A = 45°$, $b = 120$.

Also of each of the following isosceles triangles, given:

65. Vertex $\angle = 120°$, leg $= 100$.

66. Base $\angle = 30°$, base $= 200$.

67. Leg $= 40$, altitude $= 20$.

68. Vertex $\angle = 90°$, leg $= 400$.

EXERCISE 22. APPLICATIONS

Solve, using either set of tables:

1. The angle of elevation (see Art. 88) of the top of a cliff, measured from a point 225 ft. from the base, is 60°. How high is the cliff?

2. At a point 170 ft. from a tower, and on a level with its base, the angle of elevation of the top of the tower is found to be 70° 18′ [70.3°]. What is the height of the tower?

3. The angle of elevation of the sun is 65° 30′ [65.5°] and the length of a tree's shadow, on a level plane, is 52 ft. Find the height of the tree.

4. If the Eiffel Tower is 984 ft. high, what will be the angle of elevation of its top, when viewed at a distance of a mile?

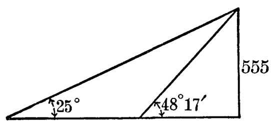
5. The length of a kite string is 700 ft., and the angle of elevation of the kite is 44° 36′ [44.6°]. Find the height of the kite supposing the kite string to be straight.

6. One of the equal sides of an isosceles triangle is 62.8 ft., and one of the equal angles is 52° 18′ 36″ [52.31°]. Find the base, altitude, and area of the triangle.

7. What is the elevation of the sun, if a tree 82.6 ft. high casts a shadow 105.8 ft. long on a horizontal plane?

8. A ladder, 25 ft. long, leans against a house and reaches to a point 21.6 ft. from the ground. Find the angle between the ladder and the house, and the distance the foot of the ladder is from the house.

Why are we able to solve an example like this by trigonometry when we are not able to do so by geometry?



9. The Washington Monument is 555 ft. high. How far apart are two observers who from points due west of the monument observe its angles of elevation to be 25° and 48° 17′ [48.28°] respectively?

10. If the Grand Cañon of the Colorado is 5000 ft. deep, what will be the angle of depression of the river flowing through it when viewed from the brink of the cañon at a horizontal distance of 3 mi.?

11. If a hillside has a slope of 7°, a dam 10 ft. high will force the water how far back up the hillside?

12. A tower 125 ft. high stands on the bank of a river. The angle subtended by the tower at the edge of the opposite bank is 23° 31′ [23.52°]. Find the width of the river.

13. What is the height of a hill if its angle of elevation taken at the foot of the hill is 40° 18′ [40.3°] and if this angle taken 150 yd. from the foot of the hill and on a level with the foot is 28° 42′ [28.7°]?

14. From the summit of a hill, there are observed two consecutive milestones on a straight horizontal road running from the base of the hill. The angles of depression (see Art. 88) are found to be 12° and 7° respectively. Find the height of the hill.

15. A valley is crossed by a horizontal bridge, whose length is l. The sides of the valley make angles m and n with the plane of the horizon. Show that the height of the bridge above the bottom of the valley is $\dfrac{l}{\cot m + \cot n}$.

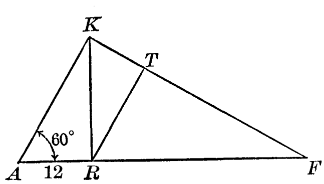
16. Upon a hill overlooking the sea stands a tower 70 ft. high. From a ship the angle of elevation of the base and top of the tower are respectively 15° 4′ [15.07°] and 15°40′ [15.67°]. What is the height of the hill and the horizontal distance of the ship from the tower?

17. Given:

$\angle AKF = \angle ARK = \angle RTF = 90°.$

$\angle KAR = 60°$ and $AR = 12$.

Without the use of the tables find the length of all the other lines in the figure.

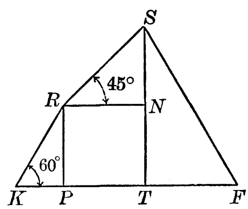


18. A boy standing m feet behind and opposite the middle of a football goal, sees that the angle of elevation of the nearer crossbar is A, and the angle of elevation of the crossbar at the other end of the field is C. Prove that the length of the field is $m(\tan A \cot C - 1)$.

19. A railroad embankment is 7 ft. high. If the top of the embankment is 8 ft. wide and the sides slope at an angle of 43°, what will be the width of the base?

20. If the Metropolitan Life Insurance building of New York City is 700 ft. high, how far from the building is an observer when the angle of elevation of the top of the building is 7° 36′ [7.6°]?

21. A man standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank is 60°; when he retires 50 m. from the edge of the river, the angle of elevation is 30°. Without the use of the tables find the height of the tree and the width of the river.



22. Given: $KP = 6$ m.;

$\angle K = \angle F = 60°$; $\angle SRN = 45°$;

and $RNTP$ a square.

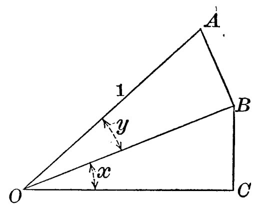
Without the use of the tables find the lengths of KR, PR, RS, ST, SF, and TF.

23. A tower and a monument stand on the same horizontal plane. The height of the tower is 35.6 m. and the angles of depression of the top and base of the monument, as observed from the top of the tower, are respectively 5° 16′ 48″ [5.28°] and 8° 18′ 30″ [8.3°]. How high is the monument?

24. A flagstaff stands on the roof of a building. From a point A on the ground the angles of elevation of the foot and the top of the flagstaff are 37° and 46°, respectively. From a point B, 250 ft. farther off and in line with A and the base of the building immediately below the flagstaff, the angle of elevation of the top of the flagstaff is 27° 30′ [27.5°]. Find the length of the flagstaff.

25. From the top of a lighthouse, 150 ft. above the sea level, the angle of depression of a buoy situated between the lighthouse and the shore was 62° 14′ [62.23°] and that of a point on the shore in a straight line with the buoy was 12° 10′ [12.17°]. Find the distance, in feet, of the buoy from the shore.

26. The base of a rectangle is 50.62 and its diagonal is 71.6. Find the altitude of the rectangle and the angle which the diagonal makes with the base.



27. Given:

$$OA = 1,$$
$$\angle ABO = \angle BCO = 90°.$$

Express AB, OB, BC, OC in terms of trigonometric functions of x and y.

28. The Singer building of New York City is 612 ft. high. Make up some problem concerning this which can be solved by trigonometry.

29. The diagonals of a rhombus are 42.28 and 30.58. Find the sides and angles.

30. Make up (or collect) as many different examples as you can showing the practical uses of the solution of right triangles by trigonometry, each example being distinct from the rest either in principle or in the field of its application.

31. Who first, and at what date, taught the trigonometric solution of triangles in the same general way as is done at present?

CHAPTER IV

GONIOMETRY

TRIGONOMETRIC FUNCTIONS OF ANGLES IN GENERAL

51. Angles greater than 90°. In solving oblique triangles, angles greater than 90° may occur. Hence it is important to learn what the trigonometric functions of an obtuse angle are. Similarly the radius of a rotating wheel, as in a dynamo, generates angles greater than 360° and by successive rotations generates angles unlimited in size.

In astronomy, the heavenly bodies, by successive rotations about an axis, and by revolutions in an orbit, also generate angles unlimited in size.

Hence a general method is needed of determining the trigonometric functions of angles unlimited in size.

52. The Four Quadrants. Definitions. Let AC (Fig. 30) be the horizontal diameter of a circle $ABCD$, and BD the diameter perpendicular to AC.

Then AOB, BOC, COD, and DOA are termed the *first*, *second*, *third*, and *fourth* quadrants of the circle.

On Fig. 31 the four parts into which a plane is divided by the lines XX' and YY' are also termed quadrants and are numbered in the same order as the quadrants of a circle.

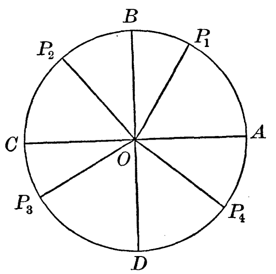


FIG. 30.

In treating of the properties of angles in general, it is convenient, wherever possible, to let the angles start at the same place, as OA (that is, to have the vertex and a side in common).

Let the rotating radius start in the position OA and rotate toward the position OB (in the direction contrary to that in which the hands of a clock move, or counter-clockwise).

The ∡ AOP_1, AOP_2, AOP_3, AOP_4 are called angles in the first, second, third, and fourth quadrants respectively.

The **initial line** of an angle is the rotating radius, which generates the angle, in its first position, as AO.

The **terminal line** of an angle is the rotating radius in its final position, as OP_2 for $\angle AOP_2$.

By continuing the rotation of OA, angles greater than 360° will be generated. If two angles differ by 360°, or by any exact multiple of 360°, they will have the same terminal line.

Coterminal angles are angles which have the same terminal line, as 37°, 397°, and 757°.

In general an angle is said to be of or in that quadrant in which its terminal line lies.

53. Negative Angles. In algebra it is shown that negative quantity is quantity exactly opposite in some respect, as, for instance, in direction, from other quantity taken as positive. Hence if the rotating radius move from the position OA (Fig. 30) toward the position OD (that is, in the same direction with the hands of a clock, or clockwise), a negative angle, as the acute $\angle AOP_4$, will be generated. If the radius continue to rotate in this direction, a whole series of negative angles will be formed similarly.

54. Rectangular Coördinates. In order to define the trigonometric functions of angles greater than 90°, and of negative angles, two straight lines, XX' and YY' (Fig. 31), intersecting at the point O and perpendicular to each other, are taken and called **axes.** The signs of other lines used are determined by their position with

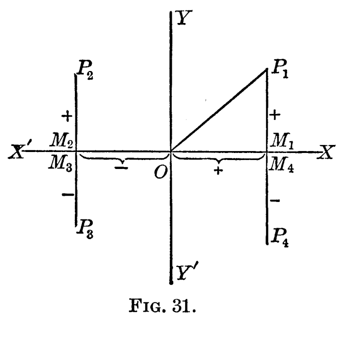


FIG. 31.

reference to these axes Lines drawn from YY' to the right (and $\parallel XX'$) are taken as $+$; lines drawn from YY' to the left (and $\parallel XX'$) are taken as $-$. Lines drawn from XX' above (and $\parallel YY'$) are taken as $+$; lines drawn from XX' below (and $\parallel YY'$) are taken as $-$.

The **origin** is the point in which the axes intersect, as the point O on Fig. 31.

The **ordinate** of a point is the distance of the point above or below the axis XX'. The **abscissa** of a point is the distance of the point to the right or left of the YY' axis. Thus, the ordinate of P_1 is M_1P_1; the abscissa of P_1 is OM_1.

Coördinates is the general term for abscissa and ordinate of a point. The coördinates of a point may be written together in parenthesis with abscissa first and a comma between. Thus if $OM_1 = a$, and $M_1P_1 = b$, the coördinates of P_1 are (a, b).

The **distance of a point** is the line drawn from the origin to the point, thus on Fig. 31 the distance of P_1 is OP_1. The distance of a point is independent of sign.

55. Definitions of Trigonometric Functions of Any Angle.

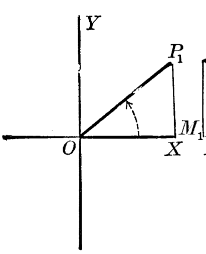


FIG. 32.

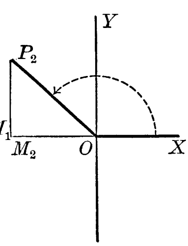


FIG. 33.

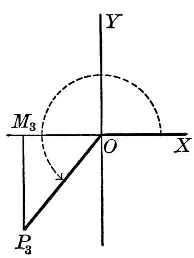


FIG. 34.

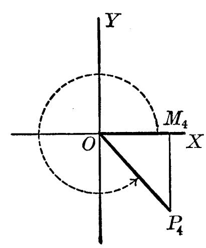


FIG. 35.

If we regard an angle as formed by an initial line and a line drawn from the origin to a point whose abscissa and ordinate are considered, then

sine of an angle = ratio of ordinate to distance;

cosine of an angle = ratio of abscissa to distance;

tangent of an angle = *ratio of ordinate to abscissa;*

cotangent of an angle = *ratio of abscissa to ordinate;*

secant of an angle = *ratio of distance to abscissa;*

cosecant of an angle = *ratio of distance to ordinate.*

Thus in Figs. 32, 33, 34, 35, $\sin \angle XOP_1 = \dfrac{M_1P_1}{OP_1}$,

$\sin \angle XOP_2 = \dfrac{M_2P_2}{OP_2}$, $\sin \angle XOP_3 = \dfrac{M_3P_3}{OP_3}$, $\sin \angle XOP_4 = \dfrac{M_4P_4}{OP_4}$.

Let the pupil point out in like manner the other trigonometric functions of the angles XOP_1, XOP_2, XOP_3, XOP_4.

56. Trigonometric Functions represented by Lines.

If a circle (Fig. 36) be drawn with O as a center and a radius OA, equal to 1, and with M_1P_1, M_2P_2, M_3P_3, M_4P_4, perpendicular to XX',

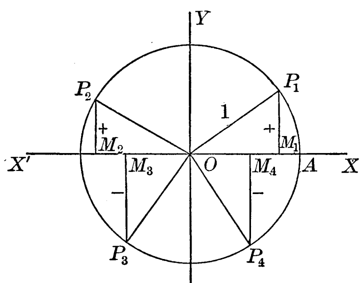


FIG. 36.

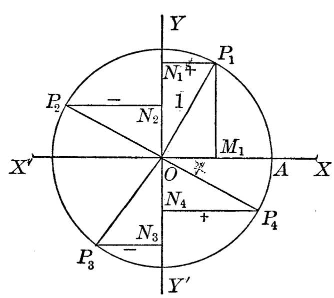


FIG. 37.

$$\sin \angle AOP_1 = \frac{M_1P_1}{OP_1} = \frac{M_1P_1}{1} = M_1P_1.$$

Similarly, $\sin \angle AOP_2 = M_2P_2$; $\sin \angle AOP_3 = M_3P_3$; and $\sin \angle AOP_4 = M_4P_4$. Or, in the circle as described, *the sine of an angle is represented by a line drawn from the terminal end of the arc intercepted by the angle, and perpendicular to the horizontal diameter.*

Similarly if (in Fig. 37) N_1P_1, N_2P_2, N_3P_3, N_4P_4 are perpendicular to YY',

$$\cos \angle AOP_1 = \frac{N_1P_1}{OP_1} = \frac{N_1P_1}{1} = N_1P_1;$$

$\cos \angle AOP_2 = N_2P_2$; $\cos \angle AOP_3 = N_3P_3$; $\cos \angle AOP_4 = N_4P_4$.

Or, in the circle as described, *the cosine of an angle is represented by a line drawn from the terminal end of the arc intercepted by the angle, and perpendicular to the vertical diameter.*

Similarly (in Fig. 38), if TT' is tangent to the circle at A,

$$\tan \angle AOP_1 = \frac{AT_1}{OA} = \frac{AT_1}{1} = AT_1;$$

$\tan \angle AOP_2 = AT_2$; $\tan \angle AOP_3 = AT_3$; $\tan \angle AOP_4 = AT_4$.

Or in the circle as described, *the tangent of an angle is represented by a line drawn touching the initial end of the arc intercepted by the angle, and terminated by the radius to the other end of the arc, produced.*

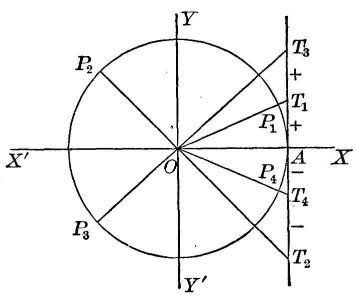


FIG. 38.

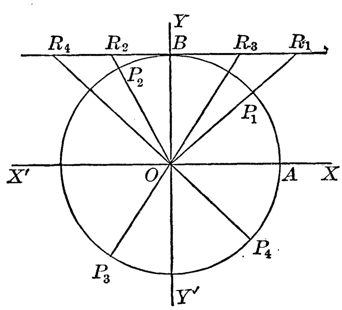


FIG. 39.

Similarly (in Fig. 39), if R_1R_4 is tangent to the circle at the point B,

$$\cot \angle AOP_1 = \tan \angle BOR_1 = \frac{BR_1}{OB} = \frac{BR_1}{1} = BR_1;$$

$\cot \angle AOP_2 = BR_2$; $\cot \angle AOP_3 = BR_3$; $\cot \angle AOP_4 = BR_4$; or in the circle as described *the cotangent of an angle is repre-*

sented by a line which is the tangent of the complement of the given angle.

On Fig. 38 the secants of the four angles used are readily shown to be represented by OT_1, OT_2, OT_3, OT_4; or, in general, *the secant of an angle is represented by a line drawn from the center through the terminal end of the arc intercepted by the angle, and terminated by the tangent.*

Similarly on Fig. 39 the cosecants of the four angles used are represented by OR_1, OR_2, OR_3, OR_4; or, in general, the *cosecant of an angle is represented by a line which is the secant of the complement of the angle.*

It will be convenient to draw a figure for an angle in each quadrant showing the lines which represent the functions of that angle.

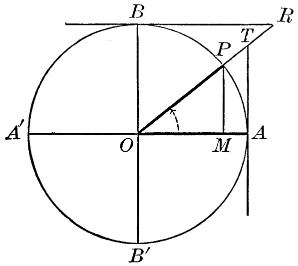


Fig. 40.

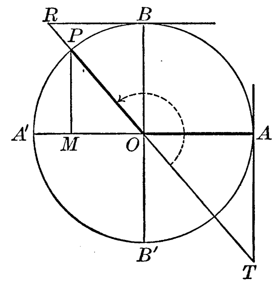


Fig. 41.

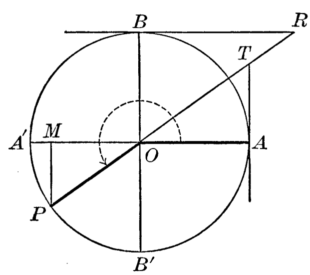


Fig. 42.

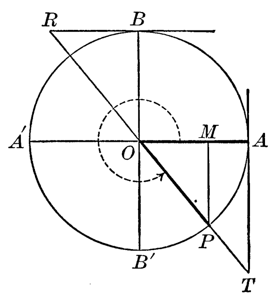


Fig. 43.

The lines which represent the various trigonometric functions of an angle are not the same as the trigonometric functions which they represent, but they have many of the same properties as the functions or ratios. It is often

easier to perceive these properties by the use of the lines, than by the use of the ratios which the lines represent.

In deriving the properties of the trigonometric functions of angles greater than 90° we shall derive them from the lines representing the functions; but in such cases we give some specimen proofs showing how these properties may be derived from the ratio definitions (of Art. 55), and in other cases leave it as an exercise for the pupil to derive the proofs from the ratios if the teacher considers it desirable.

57. Signs of the Trigonometric Functions in the Different Quadrants. Of the lines representing the sines of angles in the different quadrants, viz. M_1P_1, M_2P_2, M_3P_3, M_4P_4 (Fig. 36), the first two are above the horizontal axis, and are therefore plus in sign; the last two are below, and therefore minus. Hence the signs of the sines of angles in the four quadrants are respectively $+, +, -, -$.

The students may obtain the same results from Figs. 32–35 by using the general definitions of trigonometric functions given in Art. 55.

Similarly in Fig. 37 the cosine lines N_1P_1, N_2P_2, N_3P_3, N_4P_4 are $+, -, -, +$, respectively; and in Fig. 38 the tangent lines AT_1, AT_2, AT_3, AT_4 are $+, -, +, -$, respectively.

Since the sine of a quantity and of its reciprocal must be the same, the sign of the cotangent in the various quadrants must be the same as that of the tangent; that of the secant, the same as the cosine; that of the cosecant, the same as the sine.

Or, proceeding geometrically, on Fig. 39, the cotangent lines BR_1, BR_2, BR_3, BR_4 are $+, -, +, -$.

The secant is considered as plus when it is drawn in the same direction from the center as the terminal radius (thus OT_2, Fig. 38, is opposite in direction from OP_2 and is therefore negative). Hence the secant lines OT_1, OT_2, OT_3 OT_4 have the signs $+, -, -, +$, respec-

tively. Similarly the cosecant lines (Fig. 39) OR_1, OR_2, OR_3, OR_4 have the signs $+$, $+$, $-$, $-$.

The results thus obtained may be arranged in a table as follows:

	I	II	III	IV
sine and cosecant	+	+	−	−
cosine and secant	+	−	−	+
tangent and cotangent	+	−	+	−

EXERCISE 23

In which quadrant is each of the following angles?

1. 123°.
2. 155°.
3. 215°.
4. 285°.
5. 338°.
6. 415°.
7. $-18°$.
8. $-125°$.
9. 612°.
10. $-500°$.
11. 1111°.
12. $-222°$.
13. $-1826°$.
14. 2625°.
15. $-1500°$.

16. Find the signs of the functions of the angles in Exs. 1, 3, and 5.

Give two positive and two negative angles each of which is coterminal with:

17. 25°. 18. $-30°$. 19. 100°. 20. $-100°$.

Find the smallest possible angle coterminal with:

21. 425°.
22. 780°.
23. $-300°$.
24. 875°.
25. $-1760°$.
26. 1493°.

In which quadrant does an angle lie:

27. If its sin is positive and cos negative?
28. If its tan is positive and sin negative?
29. If its cot is negative and cos negative?
30. If its csc is negative and cot positive?
31. If its cos is positive and tan negative?
32. If its sec is negative and tan negative?

33. A railroad embankment is 9 ft. high and 43 ft. wide at the base. If each of its sides makes an angle of 27° 15′ [27.25°] with the horizontal, how wide is the top of the embankment?

34. If a railroad embankment is 7 ft. high and 28 ft. 9 in. wide at the top, and one side has a slope of 23° 30′ [23.5°] and the other a slope of 32° 45′ [32.75°], how wide is the base?

35. Make up a similar example for yourself.

58. Functions of 0°, 90°, 180°, 270°, 360°. In Arts. 34 and 35 it is shown that $\sin 0° = 0$ and $\sin 90° = 1$. Similar results are readily perceived for other quadrants by the use of a figure showing the sines as lines in the different quadrants.

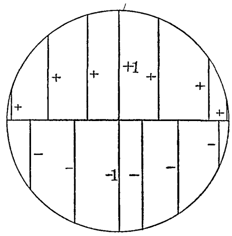


FIG. 44.

Thus in Fig. 44 in the first quadrant the sine increases from 0 to 1; in the second quadrant it decreases from 1 to 0; in the third it decreases from 0 to -1; in the fourth quadrant it increases from -1 to 0. Hence the sines of 0°, 90°, 180°, 270°, 360°, in order, are 0, 1, 0, -1, 0. Similarly in the first quadrant (Fig. 45) the cosine decreases from 1 to 0; in the second quadrant it decreases from 0 to -1; in the third quadrant it increases from -1 to 0; in the fourth quadrant it increases from 0 to 1. Hence the cosines of 0°, 90°, 180°, 270°, 360°, in order, are 1, 0, -1, 0, 1.

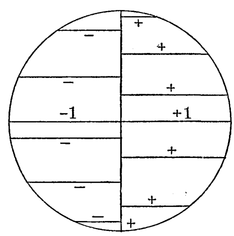


FIG. 45.

Similarly from Fig. 38, or from the formula $\tan x = \dfrac{\sin x}{\cos x}$, it is clear that the tangent in the different quadrants changes from 0 to ∞; from $-\infty$ to 0; from 0 to ∞; from $-\infty$ to 0. Hence the tangents of 0°, 90°, 180°, 270°, 360°, in order, are 0, $\pm\infty$, 0, $\pm\infty$, 0.

The changes in the value of the cotangent, the secant, and the cosecant, and the values of these functions for the above-mentioned angles may be obtained from geometrical figures in like manner, but these values are obtained more readily from the reciprocal formulas

$$\cot = \frac{1}{\tan}; \quad \sec = \frac{1}{\cos}; \quad \csc = \frac{1}{\sin}.$$

Thus, $$\sec 180° = \frac{1}{\cos 180°} = \frac{1}{-1} = -1.$$

Obtaining the values of the required functions thus and arranging all the results obtained in a table, we have

	0°	90°	180°	270°	360°
sin	0	1	0	–1	0
cos	1	0	–1	0	1
tan	0	∞	0	∞	0
cot	∞	0	∞	0	∞
sec	1	∞	–1	∞	1
csc	∞	1	∞	–1	∞

In the above table ∞ is to be taken as $+$ or $-$ according to the side from which it is approached (see Art. 57).

EXERCISE 24

Find the numerical value of:

1. $5 \sin 90° + 7 \cos 180° + 8 \sin 30°$.
2. $m \sin 0° + p \cos 90° + c \cot 360°$.
3. $b \cos 90° - c \tan 180° + b \cot 270°$.
4. $(a^2 - c^2) \cos 180° + 4\, ac \sin 90°$.
5. $2 \tan 0° \sin 90° - 4 \sec 0° \sin 270° + 5 \csc 90° \cos 0° \cot 270°$.
6. $a \cos 180° \sec 360° - b \tan 180° \sin 270° - a \sin 90° \sec 0° + b \sin 90° \cos 270°$.
7. $m \sin 270° \csc 90° + n \cos 180° \csc 270° \cot 270° - m \sec 180°$.
8. $6\, m \csc 90° \cos^2 0° - 17\, n \sec^2 0° \cot^2 270° + 3\, m \sin 270° \sec 360°$.
9. Show that

$$4 \cos^2 45° \sec 0° + 6 \tan^2 30° \sin 270° + 12 \cot^2 45° \cos 180° - 4 \tan^2 45° \csc 270° = -8.$$

59. Trigonometric Functions of Angles greater than 360°. It is evident that the trigonometric functions of angles from 360° to 720° are the same in order as those from 0° to 360°. Similarly for every succeeding 360°, the functions repeat themselves.

Hence to find the functions of an angle greater than 360°,

Divide the angle by 360° *and find the required trigonometric function of the remainder.*

Ex. Sin $766° = \sin(2 \times 360° + 46°) = \sin 46°$.

60. Formulas for the Acute Angle extended to any Angle. The equations and formulas proved in Arts. 27–29 concerning the function of an acute angle are true for the functions of any angle.

Thus, on each of the Figs. 40–43, $\overline{MP}^2 + \overline{OM}^2 = \overline{OP}^2$.

That is, $$\sin^2 x + \cos^2 x = 1.$$

Also in each quadrant the $\triangle$ OMP, OAT, OBR are similar.

$$\therefore AT : OA = MP : OM, \text{ or } \tan x : 1 = \sin x : \cos x,$$

$$\text{or} \qquad \tan x = \frac{\sin x}{\cos x}.$$

Let the pupil prove in like manner,

$$\sin x = \frac{1}{\csc x}, \ \cos x = \frac{1}{\sec x}.$$

Or these results may be proved directly from the ratio definitions of the trigonometric functions of any angle.

For if angle XOP of Figs. 32–35 be denoted by x, in any quadrant

$$\overline{\text{abs. } P}^2 + \overline{\text{ord. } P}^2 = \overline{\text{dist. } P}^2,$$

$$\therefore \left(\frac{\text{abs. } P}{\text{dist. } P}\right)^2 + \left(\frac{\text{ord. } P}{\text{dist. } P}\right)^2 = 1.$$

Hence, $$\sin^2 x + \cos^2 x = 1.$$

Let the pupil prove in a similar manner that

$$\tan^2 x + 1 = \sec^2 x, \text{ and } \cot^2 x + 1 = \csc^2 x.$$

Also $$\tan x = \frac{\text{ord. } P}{\text{abs. } P} = \frac{\frac{\text{ord. } P}{\text{dist. } P}}{\frac{\text{abs. } P}{\text{dist. } P}} = \frac{\sin x}{\cos x}, \text{ or } \tan x = \frac{\sin x}{\cos x}.$$

Also, $$\frac{\text{ord. } P}{\text{dist. } P} \times \frac{\text{dist. } P}{\text{ord. } P} = 1, \ \frac{\text{abs. } P}{\text{dist. } P} \times \frac{\text{dist. } P}{\text{abs. } P} = 1, \ \frac{\text{ord. } P}{\text{abs. } P} \times \frac{\text{abs. } P}{\text{ord. } P} = 1;$$

or $$\sin x \times \csc x = 1, \ \cos x \times \sec x = 1, \ \tan x \times \cot x = 1.$$

61. One function of an angle being given, the other functions may be found in a manner similar to that used in Art. 30. Owing to the fact that for angles less than 360°, two angles correspond to any given function, two sets of answers are found in each example.

Ex. 1. Given $\cos x = -\frac{4}{5}$, find the other functions of x.

By the table of signs (Art. 57) a negative cosine occurs in both the second and third quadrants.

2d quadrant. $\sin x = \sqrt{1 - (\frac{4}{5})^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$,

$$\tan x = \frac{\sin x}{\cos x} = -\frac{3}{4}, \text{ etc.}$$

3d quadrant. $\sin x = \sqrt{1 - (\frac{4}{5})^2} = \sqrt{\frac{9}{25}} = -\frac{3}{5}$,

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4}, \text{ etc.}$$

Ex. 2. Given $\tan x = 2$, find the remaining functions of x.

The positive tangent occurs (see Art. 57) in both the first and third quadrants.

1st quadrant. $\sec^2 x = 1 + \tan^2 x = 1 + 4 = 5$, $\sec x = \sqrt{5}$,

$$\cos x = \frac{1}{\sec x} = \frac{1}{\sqrt{5}} = \frac{1}{5}\sqrt{5}, \text{ etc.}$$

3d quadrant. $\sec^2 x = 1 + 4$, $\sec x = -\sqrt{5}$,

$$\cos x = \frac{1}{-\sqrt{5}} = -\frac{1}{5}\sqrt{5}, \text{ etc.}$$

In case solutions are sought by the geometrical method, the following figures may be used in Exs. 1 and 2 respectively.

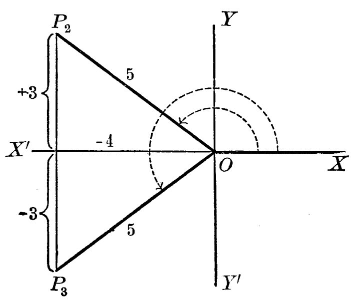


FIG. 46.

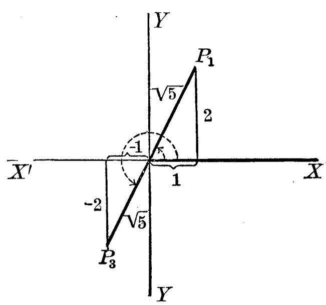


FIG. 47.

EXERCISE 25

1. Find the numerical value of sin 390°; also of cos 390°, tan 390°, and sec 390°.

2. Find the numerical value of cos 780°; also of tan 780°, sin 780°, and cot 780°.

3. Find the values of sin, cos, tan, and cot of the following angles:

4. 1860°. **6.** $-675°$. **8.** $-1740°$.

5. $-330°$. **7.** 750°. **9.** 2205°.

10. Given $\cos x = -\frac{3}{5}$, find the other functions of x.

11. Given $\tan x = -\frac{15}{8}$, find the other functions of x.

12. Given $\sin x = -\frac{5}{13}$, find the other functions of x.

13. Given $\cot x = 2$ and $\sin x$ negative, find the other functions of x.

14. Given $\sec x = -m$ and $\tan x$ negative, find the other functions of x.

15. Given $\tan x = -3$, find the other functions of x when x is an angle in the fourth quadrant.

16. Given $\sec x = -6$, find the other functions of x if $\tan x$ is positive.

17. Verify geometrically the results obtained in Exs. 10–16.

18. Given $\cot y = \frac{2}{5}\sqrt{5}$ and $\cos y$ negative, find $\sin y$ and $\csc y$.

19. Given $\tan x = -\frac{1}{3}\sqrt{3}$ and $\cos x$ positive, find the other functions of x.

20. If θ is in the second quadrant and if $\operatorname{cosec} \theta = \frac{13}{5}$, find the value of $\dfrac{\cot \theta + \sec \theta}{\tan \theta + \cos \theta}$.

21. Find the value of $\dfrac{\cos \theta + \cot \theta}{\csc \theta + \sec \theta}$, if θ is in the fourth quadrant and $\tan \theta = -\frac{12}{5}$.

62. Trigonometric Functions of $90° + x$ in terms of functions of x. The trigonometric functions of $90° + x$ may be reduced to functions of x by use of the following formulas:

$$\sin (90° + x) = \cos x. \qquad \cot (90° + x) = -\tan x.$$

$$\cos (90° + x) = -\sin x. \qquad \sec (90° + x) = -\csc x.$$

$$\tan (90° + x) = -\cot x. \qquad \csc (90° + x) = \sec x.$$

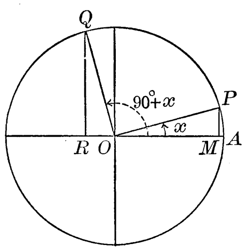


Fig. 48 *a*.

For, let $\angle AOP$ (Fig 48 *a*) be any angle x in the first quadrant. Let POQ be a right angle. Let $OP = OQ = 1$.

Then $\angle RQO = \angle MOP$. (*sides* $\perp$)

$\therefore \triangle RQO = \triangle MOP$. (*hyp. and acute* $\angle \doteq$)

$\therefore \sin(90° + x) = RQ = OM = \cos x$.

$\cos(90° + x) = OR = -PM = -\sin x$.

$$\tan(90° + x) = \frac{\sin(90° + x)}{\cos(90° + x)} = \frac{\cos x}{-\sin x} = -\cot x.$$

Let the pupil supply the proofs for $\cot(90° + x)$, $\sec(90° + x)$, and $\csc(90° + x)$.

The same results may readily be obtained for angles ending in the second, third, and fourth quadrants by use of the following diagrams.

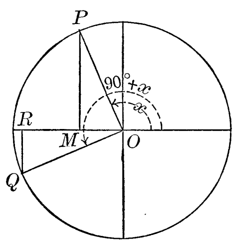


Fig. 48 *b*.

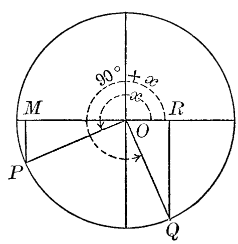


Fig. 48 *c*.

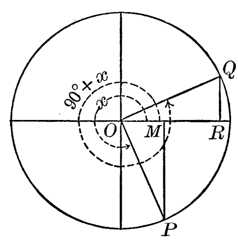


Fig. 48 *d*.

Ex. 1. Find the value of sin 300°.

$$\sin 300° = \sin(90° + 210°) = \cos 210°$$
$$= -\sin 120° = -\cos 30° = -\tfrac{1}{2}\sqrt{3}.$$

Ex. 2. Reduce tan 923° to a function of an angle less than 90°.

$$\tan 923° = \tan(720° + 203°) = \tan 203° \qquad \text{(Art. 59)}$$
$$= -\cot 113° = \tan 23°.$$

Ex. 3. Simplify $\cos(630° + A)$.

$$\cos(630° + A) = \cos(270° + A)$$
$$= -\sin(180° + A)$$
$$= -\cos(90° + A) = \sin A.$$

EXERCISE 26

Find the numerical value of:

1. $\sin 210°$.
2. $\cos 300°$.
3. $\tan 120°$.
4. $\cot 150°$.
5. $\sec 1215°$.
6. $\sec 900°$.
7. $\tan 210°$.
8. $\sin 330°$.
9. $\cos 240°$.
10. $\cos 225° + 3 \sin 330° - \tan 225°$.
11. $\cot 840° - 3 \tan 420° + 2 \sec 480°$.

Express each of the following trigonometric ratios in terms of a ratio of some positive angle not greater than 45°:

12. $\sin 142°$.
13. $\tan 163°$.
14. $\cos 310°$.
15. $\sec 185°$.
16. $\cot 265°$.
17. $\tan 315°$.
18. $\cos 110°$.
19. $\sin 567°$:
20. $\cot 1415°$.
21. $\csc 1200°$.
22. $\cos 117°$.
23. $\tan 428°$.
24. $\sin (280° \ 16')$.
25. $\cot (2100° \ 17')$.
26. $\csc 1325°$.
27. $\cos 82°$.
28. $\tan 1060°$.
29. $\tan 840°$.
30. Prove $\sin 330° \cos 390° = \cos 570° \sin 510°$.
31. Prove $\tan 45° \sec 1080° \cos 570° \sin 510°$
$- \sin 330° \tan 225° \cos 390° = 0$.
32. Find the value of $6 \sec^2 1080° \tan^2 135° \sin 1890°$
$+ 8 \cot 45° \cos 1140° + \csc 630° \tan 225° \cos 720° \sin 1830°$.

Simplify the following expressions:

33. $5 \sin (90° + x) - 6 \cos (180° + x)$.
34. $a \sin (90° + x) + b \cos (270° + x) - c \tan (180° + x)$.
35. $p \sin (180° + x) \cos (180° + x)$.
36. $(a + b) \sin (270° + x) - (a - b) \cos (270° + x)$.

63. Trigonometric Functions of a Negative Angle. The trigonometric functions of a negative angle may be converted into functions of a positive angle by use of the following formulas:

$$\sin(-x) = -\sin x. \qquad \cot(-x) = -\cot x.$$
$$\cos(-x) = \cos x. \qquad \sec(-x) = \sec x.$$
$$\tan(-x) = -\tan x. \qquad \csc(-x) = -\csc x.$$

For let $\angle AOP$ (Fig. 49) be a positive angle, x, and $\angle AOQ$ an equal negative angle. Let $OP = OQ = 1$.

Then the right triangles OMP and OMQ are equal.

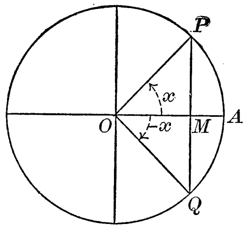


FIG. 49.

Hence,

$$\sin(-x) = MQ = -MP = -\sin x$$

$$\cos(-x) = OM = \cos x$$

$$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x}$$

$$= -\tan x.$$

Let the pupil supply the proofs for $\cot(-x)$, $\sec(-x)$, and $\csc(-x)$.

The same results are readily obtained for angles in the other quadrants by the use of appropriate diagrams.

Ex. 1. Find the numerical value of $\cos(-225°)$.

$$\cos(-225°) = \cos 225°,$$

$$= -\sin 135° \qquad \text{(Art. 62)}$$

$$= -\cos 45° = -\tfrac{1}{2}\sqrt{2}, \quad \textit{Ans.}$$

Ex. 2. Simplify $\cot(180° - A)$.

$$\cot(180° - A) = -\tan(90° - A),$$

$$= \cot(-A) = -\cot A, \quad \textit{Ans.}$$

64. Reduction Tables and General Rules. Some of the reductions made by the methods of the preceding articles are used so frequently that it is convenient to collect the results obtained by them, and arrange them in tables for future reference. Thus

$\sin(90° - x) = \cos x.$	$\sin(180° - x) = \sin x.$
$\cos(90° - x) = \sin x.$	$\cos(180° - x) = -\cos x.$
$\tan(90° - x) = \cot x.$	$\tan(180° - x) = -\tan x$
$\cot(90° - x) = \tan x.$	$\cot(180° - x) = -\cot x$
$\sec(90° - x) = \csc x.$	$\sec(180° - x) = -\sec x$
$\csc(90° - x) = \sec x.$	$\csc(180° - x) = \csc x$

Let the pupil form similar tables for the functions of $270° - x$, $360° - x$, $180° + x$, $270° + x$.

Or the following general rule may be used:

Each function of $180° \pm x$ *or* $360° \pm x$ *is equal in absolute value to the like-named function of x; but each function of* $90° \pm x$ *or* $270° \pm x$ *is equal in absolute value to the co-named function of x.**

For example, $\sin(180° + x)$ and $\sin x$ by the above rule are equal in absolute value. But it must also be remembered that they are opposite in sign. For if, for instance, x is acute, $180° + x$ is an angle in the third quadrant and therefore $\sin(180° + x)$ is negative. But x meantime would be an angle in the first quadrant, hence $\sin x$ would be positive. Hence, in general,

$$\sin(180° + x) = -\sin x.$$

Let the pupil show in like manner that, by the above rule,

$$\sin(360° - x) = -\sin x; \text{ also that } \sin(270° - x) = -\cos x.$$

In applying the above general rule to any particular example it will be found that *the algebraic sign of the result is the same as the sign of the original function.*

Thus, $\sin 330° = \sin(360° - 30°) = -\sin 30°$, the short way of determining the sign of $\sin 30°$ being to note that $\sin 330°$ is negative since $330°$ is in the fourth quadrant and that $\sin 30°$ must have the same sign as $\sin 330°$.

If geometrical proofs for the above reduction formulas are desired, such proofs may be obtained by following the methods of Art. 62. But in such proofs, when constructing an angle like $180° + x$, or $270° + x$ on the diagram, it is an advantage to construct the $180°$, or $270°$ first, beginning with the initial line, and then to annex the angle x to the $180°$, or $270°$, after it has been constructed.

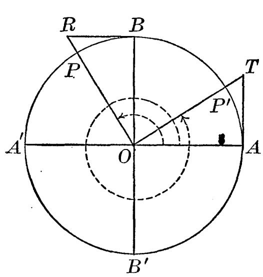


FIG. 50.

Thus, to prove that $\tan(270° + x) = -\cot x$ when x is an angle in the second quadrant (*i.e.* an obtuse angle) we first take (Fig. 50) the positive angle AOB' ($270°$) and annex to it $\angle B'OP'$ ($= x$ or $\angle AOP$). Then

* At this point it is often advantageous to have the class study the solution of Case I of oblique-angled triangles (Arts. 74, 79). This shows the pupil an important application of the preceding principle and introduces variety into the course of study.

$(270° + x) = \angle AOT$ (as indicated by the long bent arrow), and $\tan (270° + x) = AT$. Also $\cot x$ (or $\cot AOP) = BR$.

But $\angle B'OT = \angle AOR$ (*construction*)

Subtracting 90° from each of these angles we have

$\angle AOT = \angle BOP$. $\therefore \triangle AOT = \triangle BOP$. (*leg and acute* $\angle =$)

$\therefore AT = BR$, in absolute magnitude. (*hom. sides of* $=$ △s)

$\therefore \tan (270° + x)$ and $\cot x$ are equal in absolute magnitude.

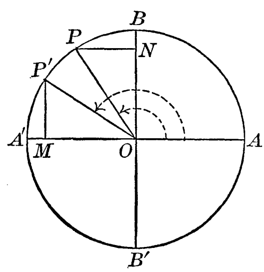


FIG. 51.

But AT and BR are opposite in sign.

$\therefore \tan (270° + x) = -\cot x$.

Similarly, to prove $\sin 270° - x = -\cos x$ when x is an angle in the second quadrant (Fig. 51) we take $\angle AOB'$ (270°) and from it deduct $\angle B'OP'$ ($= \angle AOP$ or x). Hence, $\sin (270° - x) = MP'$, while $\cos x = NP$.

Since $\triangle OMP' = \triangle ONP$, MP' and NP are equal in absolute magnitude. They are also opposite in sign.

$\therefore \sin (270° - x) = -\cos x$.

EXERCISE 27

Find the numerical value of:

1. $\sin (-225°)$.	**4.** $\cot (-210°)$.	**7.** $\sec (-240°)$.
2. $\tan (-300°)$.	**5.** $\tan (-600°)$.	**8.** $\tan (-150°)$.
3. $\cos (-120°)$.	**6.** $\sin (-900°)$.	**9.** $\sin (-135°)$.

Reduce the functions of the following negative angles to the functions of positive angles not greater than 45°:

10. $-119°$.	**13.** $-15°$.	**16.** $-900°$.
11. $-81°$.	**14.** $-253°$.	**17.** $-216° 43'$.
12. $-195°$.	**15.** $-1000°$.	**18.** $-307.24°$.

19. Show that $\sin 420° \cos 390° = 1 - \cos (-300°) \sin (-330°)$.

20. That $3 \tan (-60°) \cot (-210°) + 9 \sin (-240°) \cos (-150°) = \frac{9}{4}$.

By the general rule stated in Art. 64 reduce each of the following to a function of x:

21. $\cos (180° + x)$.	**23.** $\cos (270° - x)$.	**25.** $\sec (180° - x)$.
22. $\sin (270° + x)$.	**24.** $\tan (180° + x)$.	**26.** $\csc (270° + x)$.

Simplify the following expressions:

27. $5 \sin (90° - x) + 8 \cos (180° - x)$.

28. $a \sin (270° - x) - b \cos (270° - x) + c \tan (180° - x)$.

29. $m \cos (180° + A) + p \cot (180° - A) + q \tan (270° + A)$.

30. $\sin (270° + x) \cos (270° - x) \sin (180° - x)$.

31. $\sin (x - 90°) + \cot (x - 90°) + \tan (x - 180°)$.

65. General Solutions of Trigonometric Equations. If there be no limit to the size of an angle, an indefinite number of angles will satisfy every trigonometric equation (see Art. 38).

Ex. 1. Solve $\sin x = \frac{1}{2}$.

There are two angles less than 360° whose sine is $\frac{1}{2}$, viz.: 30° and 150°. If 360°, or any multiple of 360°, be added to, or subtracted from, each of these angles, the sine is unchanged.

Hence, in the above example, $x = 30° \pm n\,(360°),\ 150° \pm n\,(360°)$, where $n = 0$ or any positive integer.

Ex. 2. Solve $\tan x = \pm\sqrt{3}$.

$$x = \begin{cases} 60° \pm n(360°),\ 120° \pm n(360°), \\ 240° \pm n(360°),\ 300° \pm n(360°). \end{cases} \quad \textit{Ans.}$$

Ex. 3. Solve $\sin^2 x = \cos^2 x$.

$$1 - \cos^2 x = \cos^2 x.$$
$$2\cos^2 x = 1.$$
$$\cos x = \pm \tfrac{1}{2}\sqrt{2}.$$
$$x = \begin{cases} 45° \pm n(360°),\ 315° \pm n(360°), \\ 135° \pm n(360°),\ 225° \pm n(360°). \end{cases}$$

Or more briefly, $x = \pm n(180°) \pm 45°$. *Ans.*

The pupil should observe that the values of x in a trigonometric equation differ in an important respect from the values of x in an algebraic equation. Thus, in an algebraic equation the values of x are the roots of the equation and the number of values which x has equals the degree of the given equation. Whereas, for instance in Ex. 3 above, the roots are the values of $\cos x$, while the values of x are inferred from the values of $\cos x$ and may be unlimited in number no matter what the degree of the original trigonometric equation.

EXERCISE 28

Solve the following trigonometrical equations, for values of x or θ.

1. $\sin x = \frac{1}{2}$.
2. $\cos^2 x = \frac{3}{4}$.
3. $\tan^2 x = 1$.
4. $\tan x = \frac{1}{3} \cot x$.
5. $\sin x + \csc x = \frac{5}{2}$.
6. $\tan^2 x - \sec x = 1$.
7. $2 \cos^2 x - 3 \sin x = 0$.
8. $\tan x + \cot x = 2$.
9. $\cot x + \csc^2 x = 3$.
10. $2\sqrt{3} \cot \theta - \frac{3}{4} \csc^2 \theta = 1$.
11. $\tan \theta + \sec^2 \theta = 3$.
12. $\cos^2 \theta + \cot^2 \theta = 3 \sin^2 \theta$.
13. $\frac{1}{2} \cot \theta - \cos \theta + \sin \theta = \frac{1}{2}$.
14. $\sec^2 \theta \csc^2 \theta + 2 \csc^2 \theta = 8$.
15. $2\sqrt{3} \tan \theta = 3 \sec^2 \theta - 6$.
16. $4 \sec^2 \theta - 7 \tan^2 \theta = 3$.
17. $\cot \theta + 2 \tan \theta = \frac{5}{2} \sec \theta$.
18. $\sin \theta + \sqrt{3} \cos \theta = 2$.

19. A ship starting from a certain point sailed at the average rate of 9.25 mi. per hour on a course 22° 15′ [22.25°] north of east. At the end of 7 hr. 45 min., how far east of her starting point would she be? How far north?

20. If a railroad embankment is 11 ft. high, 76 ft. wide at the base, and 49 ft. wide at the top, and its two sides have the same slope, find the angle at which each side slopes.

21. In an oblique triangle ABC, $A = 127° 36'$ [127.6°], $AB = 472$ ft., $AC = 374$ ft. By dividing the triangle into right triangles and solving, find BC.

22. P is a spring of water, Q is a house, and R is a barn. If $QR = 217$ ft., $\angle PQR = 63° 40'$ [63.67°], $\angle PRQ = 58° 15'$ [58.25°], find the distance of the spring from the house and also from the barn, by solving right triangles only.

CHAPTER V

GONIOMETRY (Continued)

66. Formulas for $\sin (x+y)$ and $\cos (x+y)$. In Fig. 52 let AOQ be an angle x, and QOP an angle y, the sum of x and y being less than a right angle.

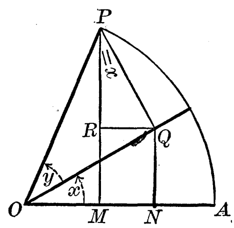


Fig. 52.

Let $OP = 1$. Draw $PM \perp OA$, $PQ \perp OQ$, $QR \perp PM$.

Then $\angle RPQ = \angle x$ (*sides* $\perp$),

$$PQ = \sin y, \; OQ = \cos y.$$

$$\sin (x+y) = PM = QN + PR.$$

In rt. $\triangle$ OQN, $QN = \sin x \; OQ$ (Art. 41) $= \sin x \cos y$.

In rt. $\triangle$ RPQ, $PR = \cos x \; PQ = \cos x \sin y$.

Hence, $\mathbf{\sin (x+y) = \sin x \cos y + \cos x \sin y}$.

Also on Fig. 52, $\cos (x+y) = OM = ON - RQ$.

In rt. $\triangle$ OQN, $ON = \cos x \; OQ = \cos x \cos y$.

In rt. $\triangle$ RPQ, $RQ = \sin x \; PQ = \sin x \sin y$.

Hence, $\mathbf{\cos (x+y) = \cos x \cos y - \sin x \sin y}$.

If x and y be acute angles whose sum is an obtuse angle, the above proofs will hold good without any change except that it is necessary to notice that in the statement $\cos (x+y) = OM = ON - RQ$, OM is a negative line and is obtained by subtracting the positive line RQ from the smaller positive line ON. See Fig. 53.

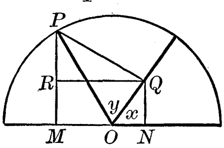


Fig. 53.

If either x or y is obtuse, the above formulas may be proved as follows:

Taking x and y as still acute,

$$\sin(90° + x + y) = \cos(x + y) \qquad \text{(Art. 62)}$$
$$= \cos x \cos y - \sin x \sin y.$$

But $\cos x = \sin(90° + x), - \sin x = \cos(90° + x).$ (Art. 62)

$$\therefore \sin(90° + x + y) = \sin(90° + x) \cos y + \cos(90° + x) \sin y.$$

Replacing $90° + x$ by x',

$\sin(x' + y) = \sin x' \cos y + \cos x' \sin y$, where x' is an obtuse angle.

In like manner the formula can be extended to the case where y is an obtuse angle. The formula for $\cos(x + y)$ may also be extended in like manner.

By successive additions of 90° to x and y, these angles may thus be made any angles however large. In like manner the formulas may be shown to be true when x and y are diminished by any integral multiple of 90°. Hence, the above formulas are true when x and y are any angles.

Ex. Taking the functions of 30°, 45°, 60° as known, find sin 75°.

$$\sin 75° = \sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30°$$
$$= \tfrac{1}{2}\sqrt{2} \cdot \tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}\sqrt{2} \cdot \tfrac{1}{2}$$
$$= \tfrac{1}{4}\sqrt{2}(\sqrt{3} + 1), \textit{ Ans.}$$

67. Formulas for sin $(x - y)$ and cos $(x - y)$. In Fig. 54 let AOQ be a positive acute angle x, and POQ a smaller angle y, subtracted from x.

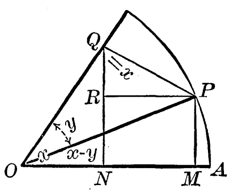


Fig. 54.

Then $\angle AOP = x - y$.

Let $OP = 1$; draw $PM \perp OA$, $PQ \perp OQ$, $QN \perp OA$, $PR \perp QN$.

Then $\angle RQP = \angle x$. (*sides* $\perp$)

Also $PQ = \sin y$, $OQ = \cos y$.

$$\sin(x - y) = PM = QN - RQ.$$

In rt. $\triangle\, OQN$, $QN = \sin x\, OQ = \sin x \cos y$.

In rt. $\triangle RQP$, $RQ = \cos x \, PQ = \cos x \sin y$.

Hence, $\mathbf{\sin (x-y) = \sin x \cos y - \cos x \sin y}$.

Also on Fig. 54,

$$\cos (x-y) = OM = ON + RP.$$

In rt. $\triangle OQN$, $ON = \cos x \, OQ = \cos x \cos y$.

In rt. $\triangle RQP$, $RP = \sin x \, PQ = \sin x \sin y$.

Hence, $\mathbf{\cos (x-y) = \cos x \cos y + \sin x \sin y}$.

By the same method as that used in Art. 66 these formulas can be proved true when x and y are any angles.

Ex. Obtain the numerical value of cos 15°.

$$\begin{aligned} \cos 15° &= \cos (45° - 30°), \\ &= \cos 45° \cos 30° + \sin 45° \sin 30° \\ &= \tfrac{1}{2}\sqrt{2} \cdot \tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}\sqrt{2} \cdot \tfrac{1}{2} \\ &= \tfrac{1}{4}\sqrt{6} + \tfrac{1}{4}\sqrt{2}, \quad \textit{Ans.} \end{aligned}$$

68. Formulas for tan $(x+y)$ and tan $(x-y)$. By Art. 66,

$$\tan (x+y) = \frac{\sin (x+y)}{\cos (x+y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}.$$

Divide both numerator and denominator of the last fraction by $\cos x \cos y$.

Then, $$\tan (x+y) = \frac{\dfrac{\sin x \cos y}{\cos x \cos y} + \dfrac{\cos x \sin y}{\cos x \cos y}}{\dfrac{\cos x \cos y}{\cos x \cos y} - \dfrac{\sin x \sin y}{\cos x \cos y}}.$$

or, $$\mathbf{\tan (x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}}.$$

Similarly, let the pupil show that

$$\mathbf{\tan (x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}},$$

and $$\mathbf{\cot (x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}}.$$

Ex. Find the numerical value of tan 105°.

$$\begin{aligned}\tan 105° &= \tan (60° + 45°)\\ &= \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}\\ &= \frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1} = \frac{1+\sqrt{3}}{1-\sqrt{3}} = -2-\sqrt{3}, \quad \textit{Result.}\end{aligned}$$

EXERCISE 29

1. If $\sin x = \frac{4}{5}$, $\cos x = \frac{3}{5}$, $\sin y = \frac{5}{13}$, $\cos y = \frac{12}{13}$, find the value of $\sin (x + y)$.

2. Also of $\sin (x - y)$, $\cos (x + y)$, and $\cos (x - y)$.

3. Find $\sin (x + 45°)$, $\cos (30° - x)$, and $\sin (x - 60°)$ in terms of $\sin x$ and $\cos x$.

4. If $\tan x = \frac{1}{2}$, and $\tan y = 2$, find the value of $\tan (x + y)$.

5. If $\cot x = -2$, and $\cot y = \frac{1}{2}$, find the value of $\cot (x - y)$.

Find the numerical value of:

6. cos 75°.	**8.** sin 105°.	**10.** sin 15°.
7. tan 75°.	**9.** cot 105°.	**11.** cos 105°.

12. Putting $90° = 60° + 30°$, find sin 90°; also cos 90°.

13. State in general language the formulas proved thus far in this chapter (thus for $\sin (x + y) = \sin x \cos y + \cos x \sin y$, say "the sine of the sum of two angles equals sine of the 1st angle times cosine of the 2d plus cosine of 1st times sine of 2d").

14. Find $\tan (45° + y)$, and also $\tan (45° - y)$, in terms of $\tan y$.

15. Find $\cot (60° + y)$, and also $\cot (30° + y)$, in terms of $\cot y$.

16. Show that $\sin (60° + 45°) + \cos (60° + 45°) = \cos 45°$.

Prove the following identities:

17. $\cot (45° + A) = \dfrac{\cot A - 1}{1 + \cot A}$.

18. $\cot (45° - A) = \dfrac{\cot A + 1}{\cot A - 1}$.

19. $\sin (60° + A) - \sin (60° - A) = \sin A$.

20. $\cos x - \sin x = \sqrt{2} \cos (x + 45°)$.

21. $\cos x + \sin x = \sqrt{2} \cos (x - 45°)$.

22. Find the smallest value of x which will satisfy the equation

$$\tan (x + 45°) + \cot (x - 45°) = 0.$$

69. Functions of the Double Angle. In the formula

$$\sin(x+y) = \sin x \cos y + \cos x \sin y,$$

let y have the value x;

then, $$\sin(x+x) = \sin x \cos x + \cos x \sin x$$

or, $$\mathbf{\sin 2x = 2 \sin x \cos x.}$$

Similarly from the formulas for $\cos(x+y)$, $\tan(x+y)$, and $\cot(x+y)$, let the pupil obtain

$$\cos 2x = \cos^2 x - \sin^2 x.$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}.$$

$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}.$$

Substituting $1 - \sin^2 x$ for $\cos^2 x$ in the formula for $\cos 2x$,

$$\cos 2x = 1 - 2\sin^2 x.$$

Substituting $1 - \cos^2 x$ for $\sin^2 x$ in the same formula,

$$\cos 2x = 2\cos^2 x - 1.$$

Ex. Find $\cos 120°$ from the functions of $60°$.

$$\begin{aligned} \cos 120° &= \cos 2 \times 60° \\ &= \cos^2 60° - \sin^2 60° \\ &= (\tfrac{1}{2})^2 - (\tfrac{1}{2}\sqrt{3})^2 \\ &= \tfrac{1}{4} - \tfrac{3}{4} = -\tfrac{1}{2}, \quad \textit{Ans.} \end{aligned}$$

EXERCISE 30

1. Given $\sin 30° = \frac{1}{2}$, and $\cos 30° = \frac{1}{2}\sqrt{3}$, find $\sin 60°$. Also $\cos 60°$.

2. Given $\tan 30° = \frac{1}{3}\sqrt{3}$, find $\tan 60°$.

3. By the formulas of Art. 69, find the value of $\sin 120°$ and $\tan 120°$.

Prove the following identities:

4. $\sin 2A = \dfrac{2 \tan A}{1 + \tan^2 A}.$

5. $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}.$

6. $\dfrac{\sin 2x}{\sin x} - \dfrac{\cos 2x}{\cos x} = \sec x.$

7. $\dfrac{1 + \sin 2\theta}{1 - \sin 2\theta} = \dfrac{(\tan \theta + 1)^2}{(\tan \theta - 1)^2}.$

8. State the formulas for $\sin 2x$ and $\cos 2x$ in general language.

9. Find $\sin 3x$ in terms of $\sin x$.

10. Find $\cos 3x$ in terms of $\cos x$.

11. Find $\tan 3x$ in terms of $\tan x$.

12. Prove $\sin 4\theta = 4 \sin\theta \cos\theta - 8 \sin^3\theta \cos\theta$.

13. Given $\tan\theta = \frac{5}{3}$, find $\tan 2\theta$.

14. Given $\cos\theta = \frac{3}{5}$, find $\cot 2\theta$.

In a right triangle, C being the right angle, prove:

15. $\tan B = \cot A$.

16. $\tan 2A = \dfrac{2ab}{b^2 - a^2}$.

17. $\sin(A - B) + \cos 2A = 0$.

18. Show that $\sin^2 x = \dfrac{1 - \cos 2x}{2}$, and $\sin^2 2x = \dfrac{1 - \cos 4x}{2}$.

19. Show that $\cos^2 x = \dfrac{1 + \cos 2x}{2}$, and $\cos^2 2x = \dfrac{1 + \cos 4x}{2}$.

20. Using the results of Exs. 18 and 19, transform $\sin^4 x$ into

$$\tfrac{1}{8} \cos 4x - \tfrac{1}{2} \cos 2x + \tfrac{3}{8}.$$

21. Also transform $\cos^4 x$ into an expression in terms of $\cos 2x$ and $\cos 4x$.

22. Also show that $\cos^6 x$ may be changed to the form

$$\tfrac{1}{16}(5 + 8 \cos 2x - 2 \sin^2 2x \cos 2x + 3 \cos 4x).$$

70. Functions of the Half Angle.

From Art. 69, $\cos 2A = 1 - 2 \sin^2 A$.

Hence, $2 \sin^2 A = 1 - \cos 2A$.

Let $A = \frac{1}{2}x$; then $2A = x$.

Hence, $2 \sin^2 \frac{1}{2}x = 1 - \cos x$.

$$\therefore \sin \tfrac{1}{2}x = \pm\sqrt{\frac{1 - \cos x}{2}}.$$

Similarly, from $\cos 2A = 2\cos^2 A - 1$,

we obtain,
$$\cos \tfrac{1}{2}x = \pm\sqrt{\frac{1 + \cos x}{2}}.$$

Also $$\tan \tfrac{1}{2} x = \frac{\sin \frac{1}{2} x}{\cos \frac{1}{2} x} = \pm \sqrt{\frac{1-\cos x}{1+\cos x}}.$$

$$\therefore \tan \tfrac{1}{2} x = \pm \sqrt{\frac{1-\cos x}{1+\cos x}}.$$

This formula may be reduced to another convenient form, thus:

$$\tan \tfrac{1}{2}x = \sqrt{\frac{(1-\cos x)^2}{(1+\cos x)(1-\cos x)}} = \sqrt{\frac{(1-\cos x)^2}{1-\cos^2 x}} = \frac{1-\cos x}{\sin x}.$$

$$\therefore \tan \tfrac{1}{2} x = \frac{1-\cos x}{\sin x}.$$

Similarly, $$\cot \tfrac{1}{2} x = \frac{1+\cos x}{\sin x}.$$

Ex. Find $\tan 22\frac{1}{2}°$ from the functions of $45°$.

$$\tan 22\tfrac{1}{2}° = \frac{1-\cos 45°}{\sin 45°} = \frac{1-\frac{1}{2}\sqrt{2}}{\frac{1}{2}\sqrt{2}} = \frac{2-\sqrt{2}}{\sqrt{2}} = \sqrt{2}-1, \quad \textit{Ans.}$$

EXERCISE 31

1. State the formulas for $\sin \frac{1}{2} A$, $\cos \frac{1}{2} A$, and $\tan \frac{1}{2} A$ in general language.

2. Given $\cos 30° = \frac{1}{2}\sqrt{3}$, find $\sin 15°$, $\tan 15°$, $\cos 15°$.

3. Given $\sin 45° = \frac{1}{2}\sqrt{2}$, find $\cot 22\frac{1}{2}°$, $\cos 22\frac{1}{2}°$, $\sin 22\frac{1}{2}°$.

4. Given $\cos 90° = 0$, find the functions of $45°$.

5. Given $\sin A = \frac{2}{3}$, and A acute, find $\cos \frac{1}{2} A$, $\cot \frac{1}{2} A$, $\tan \frac{1}{2} A$.

6. Given $\cos \theta = a$, find $\cos \frac{\theta}{2}$, $\cot \frac{\theta}{2}$, $\tan \frac{\theta}{2}$.

Prove the following identities:

7. $\tan \frac{1}{2} A = \dfrac{\sin A}{1+\cos A}$.

8. $\cot \frac{1}{2} A = \dfrac{\sin A}{1-\cos A}$.

9. $\sec^2 \dfrac{\theta}{2} = \dfrac{2\sec\theta}{\sec\theta + 1}$.

10. $\csc^2 \dfrac{\theta}{2} = \dfrac{2\sec\theta}{\sec\theta - 1}$.

11. $\sin \frac{1}{2} A + \cos \frac{1}{2} A = \sqrt{1+\sin A}$.

12. Express $\cos A$, $\sin A$, and $\cot A$, in terms of $\cos 2A$.

13. Find the value of $\dfrac{\tan \frac{1}{2} x + \sec x}{\cot \frac{1}{2} x + \cos x}$ if x is in the second quadrant and $\sin x = \frac{3}{5}$.

14. If x is in the fourth quadrant and $\csc x = -\frac{5}{4}$, find the numerical value of $\dfrac{\sin \frac{1}{2} x + \sec x}{\cot \frac{1}{2} x + \cos x}$.

15. In a right triangle show that $\tan \frac{1}{2} A = \sqrt{\dfrac{c-b}{c+b}}$.

16. By use of this formula solve the right triangle in which $c = 122$ and $a = 120$ (that is, the Ex. of Art. 46).

17. If the diagonal of a rectangle is 171 in. and one side of the rectangle is 13 ft. 7 in., find the angle between the diagonal and side.

18. Make up and solve a similar example for yourself.

71. Sum or Difference of Two Sines or of Two Cosines (Logarithmic Formulas).

Adding and subtracting the formulas of Art. 66, and also those of Art 67,

$$\sin(x+y) + \sin(x-y) = 2 \sin x \cos y \quad . \quad . \quad . \quad (a)$$

$$\sin(x+y) - \sin(x-y) = 2 \cos x \sin y \quad . \quad . \quad . \quad (b)$$

$$\cos(x+y) + \cos(x-y) = 2 \cos x \cos y \quad . \quad . \quad . \quad (c)$$

$$\cos(x+y) - \cos(x-y) = -2 \sin x \sin y \quad . \quad . \quad . \quad (d)$$

If we let $\quad x + y = A$, and $x - y = B$,

then $\quad x = \frac{1}{2}(A+B)$, and $y = \frac{1}{2}(A-B)$.

Hence, by substitution in (a), (b), (c), (d),

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B) \quad . \quad . \quad . \quad (1)$$

$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B) \quad . \quad . \quad . \quad (2)$$

$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B) \quad . \quad . \quad . \quad (3)$$

$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B) \quad . \quad . \quad (4)$$

These formulas enable us to convert the sum or difference of two sines, and also of two cosines, into a product of two functions, and hence open the way in certain examples for us to save labor by the use of logarithms.

Ex. Convert $\sin 50° + \sin 30°$ into a product.

By formula (1),

$$\sin 50° + \sin 30° = 2 \sin \tfrac{1}{2}(50° + 30°) \cos \tfrac{1}{2}(50° - 30°)$$
$$= 2 \sin 40° \cos 10°.$$

EXERCISE 32

Prove

1. $\sin 40° + \sin 10° = 2 \sin 25° \cos 15°$.
2. $\sin 60° + \sin 30° = \sqrt{2} \cos 15°$.
3. $\cos 80° - \cos 20° = - \sin 50°$.
4. $\dfrac{\sin 33° + \sin 3°}{\cos 33° + \cos 3°} = \tan 18°$.
5. $\dfrac{\cos 27° + \cos 3°}{\sin 27° + \sin 3°} = \cot 15°$.
6. $\dfrac{\sin 5x + \sin x}{\cos 5x + \cos x} = \tan 3x$.
7. $\dfrac{\cos 80° + \cos 20°}{\sin 80° - \sin 20°} = \sqrt{3}$.
8. $\dfrac{\sin A + \sin B}{\cos A - \cos B} = -\cot \tfrac{1}{2}(A - B)$.
9. $\dfrac{\cos 4x + \cos 2x}{\sin 2x + \sin 4x} = \cot 3x$.
10. $\dfrac{\sin A - \sin B}{\cos A - \cos B} = -\cot \dfrac{A + B}{2}$.
11. $\cos 20° + \cos 100° + \cos 140° = 0$.
12. $\sin x + \sin 3x + \sin 5x = \dfrac{\sin^2 3x}{\sin x}$.

13. Given $\sin A = \frac{1}{2}$ and $\sin B = \frac{1}{4}$, find $\sin(A + B)$, $\sin(A - B)$, $\cos(A + B)$, $\cos(A - B)$, $\sin 2A$, $\sin 2B$, $\cos 2A$, $\cos 2B$, when A and B are both in the first quadrant.

14. Find the numerical value of $\sin(60° + 30°)$. Also of $\sin 60° + \sin 30°$. Show geometrically why $\sin(60° + 30°)$ does not equal $\sin 60° + \sin 30°$.

Reduce each of the following to a form adapted to logarithmic computation (that is, to products or quotients):

15. $\dfrac{\sin 37° + \sin 22°}{\cos 38° - \cos 16°}$.
16. $\dfrac{\sin 4A - \sin 2A}{\cos 6A}$.
17. $\sin^2 A - \sin^2 B$.

18. Compute the value of the expression in Ex. 16 when $A = 14°$. Also of that of Ex. 17 when $A = 38°$ and $B = 24°$.

19. Make up for yourself an example similar to Ex. 17.

72. Complex Trigonometrical Identities. Besides those already arrived at, many other complex relations between the trigonometrical functions may be proved. Usually these relations are proved to the best advantage by reducing the two expressions, which are compared, to some common form, and hence inferring their identity by Ax. 1 (see Art. 31).

In most cases it is best to reduce given functions to sine and cosine.

Ex. 1. Prove that $\dfrac{1-\cos 2A}{\sin 2A}=\tan A.$

$$\frac{1-(1-2\sin^2 A)}{2\sin A\cos A}=\frac{\sin A}{\cos A}.$$

$$\frac{2\sin^2 A}{2\sin A\cos A}=\frac{\sin A}{\cos A}.$$

$$\frac{\sin A}{\cos A}=\frac{\sin A}{\cos A}.$$

Or if the teacher prefers, the proof may be put in the following form:

$$\frac{1-\cos 2A}{\sin 2A}=\frac{1-(1-2\sin^2 A)}{2\sin A\cos A}=\frac{2\sin^2 A}{2\sin A\cos A}=\frac{\sin A}{\cos A}=\tan A.$$

Ex. 2. Prove $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B.$

$$(\sin A\cos B+\cos A\sin B)(\sin A\cos B-\cos A\sin B)=\sin^2 A-\sin^2 B.$$

$$\sin^2 A\cos^2 B-\cos^2 A\sin^2 B=\sin^2 A-\sin^2 B.$$

$$\sin^2 A(1-\sin^2 B)-(1-\sin^2 A)\sin^2 B=\sin^2 A-\sin^2 B.$$

$$\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B=\sin^2 A-\sin^2 B.$$

$$\sin^2 A-\sin^2 B=\sin^2 A-\sin^2 B.$$

73. Functions of the Angles of a Triangle. If the sum of three angles is 180°, the functions of the angles have important relations.

Ex. If $A+B+C=180°$, prove that $\sin A+\sin B+\sin C$
$=4\cos\frac{1}{2}A\cos\frac{1}{2}B\cos\frac{1}{2}C.$

$A+B=180°-C$ and $\frac{1}{2}(A+B)=90°-\frac{1}{2}C.$

Hence $\sin \frac{1}{2}(A+B) = \sin(90^\circ - \frac{1}{2}C) = \cos \frac{1}{2} C.$

$$\begin{aligned}\sin A + \sin B + \sin C &= \sin A + \sin B + \sin[180^\circ - (A+B)]\\ &= \sin A + \sin B + \sin(A+B)\\ &= 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)\\ &\quad + 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A+B) \quad \text{(Arts. 69, 71)}\\ &= 2\sin\tfrac{1}{2}(A+B)[\cos\tfrac{1}{2}(A-B)+\cos\tfrac{1}{2}(A+B)]\\ &= 4\cos\tfrac{1}{2}C\cos\tfrac{1}{2}A\cos\tfrac{1}{2}B.\end{aligned}$$

EXERCISE 33

Prove the following identities:

1. $\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \dfrac{\sin 2\theta + 1}{\cos 2\theta}.$

2. $2\cos(45^\circ + \frac{1}{2}A)\cos(45^\circ - \frac{1}{2}A) = \cos A.$

3. $\cos(A+B)\cos(A-B) = \cos^2 B - \sin^2 A.$

4. $\tan(45^\circ + x) - \tan(45^\circ - x) = 2\tan 2x.$

5. $(\sqrt{1+\sin x} - \sqrt{1-\sin x})^2 = 4\sin^2 \frac{1}{2}x.$

6. $\dfrac{\cos(x+y)+\cos(x-y)}{\cos x \cos y} = \dfrac{\cos(x-y)-\cos(x+y)}{\sin x \sin y}.$

7. $\dfrac{\tan(45^\circ + \frac{1}{2}A) + \tan(45^\circ - \frac{1}{2}A)}{\tan(45^\circ + \frac{1}{2}A) - \tan(45^\circ - \frac{1}{2}A)} = \csc A.$

8. $\dfrac{\cos 3A}{\sin A} + \dfrac{\sin 3A}{\cos A} = 2\cot 2A.$

9. $\dfrac{\cos A - \sin A}{\cos A + \sin A} = \sec 2A - \tan 2A.$

10. $\tan\theta = \dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}.$

11. $\dfrac{\cot\theta - 1}{\cot\theta + 1} = \dfrac{1 - \sin 2\theta}{\cos 2\theta}.$

12. $\dfrac{1 - \tan^2\frac{1}{2}x}{1 + \tan^2\frac{1}{2}x} = \cos x.$

If $A + B + C = 180^\circ$, prove that

13. $\cos A + \cos B + \cos C = 1 + 4\sin\frac{1}{2}A\sin\frac{1}{2}B\sin\frac{1}{2}C.$

14. $\tan A + \tan B + \tan C = \tan A \tan B \tan C.$

15. $\cos(A+B+C) = -\cos 2C.$

EXERCISE 34. REVIEW

1. Given $\cos\theta = -\frac{3}{5}$ and θ is in the third quadrant, find $\csc\theta$, $\cot\theta$, $\sin\frac{1}{2}\theta$, $\tan(180° - \theta)$, $\sin(-\theta)$.

2. Given $\tan\frac{1}{2}x = 2$ (and x acute), find $\sin x$.

3. Given $\sin 2x = \frac{1}{2}\sqrt{3}$, find $\cot\frac{1}{2}x$.

4. Given $\cos\frac{1}{2}x = \frac{3}{4}$, find $\sin 2x$ and $\tan 2x$.

5. Given $\cot 30° = \sqrt{3}$, find $\cos 15°$, $\csc 15°$, and $\tan 15°$.

6. Given $\sin A = \frac{3}{5}$ and A acute, $\cos B = \frac{1}{2}$ and B acute, find (a) $\sin(A-B)$; (b) $\cos(A+B)$; (c) $\cos(A-B)$; (d) $\sin 2B$; (e) $\cos 2B$; (f) $\tan 2B$; (g) $\cot 2A$; (h) $\tan(A-B)$; (i) $\cot(A+B)$; (j) $\cos\frac{1}{2}B$.

7. Given $\cot\theta = -2$ and θ is the second quadrant, find (a) $\sec\theta$; (b) $\tan(180° - \theta)$; (c) $\cot(180° + \theta)$; (d) $\cos(-\theta)$.

8. Find sin, cos, tan, cot, of:

(a) $\left(x - \frac{\pi}{2}\right)$; ($b$) $(\pi - \theta)$; (c) $\left(x - \frac{3\pi}{2}\right)$; ($d$) $(\pi + x)$; where $\pi = 180°$.

Prove the following:

9. $\tan x = \dfrac{1 - \cos 2x}{\sin 2x}$.

10. $\tan\frac{1}{2}A = \dfrac{1 - \cos A}{\sin A}$.

11. $\dfrac{2\sin A - \sin 2A}{2\sin A + \sin 2A} = \dfrac{1 - \cos A}{1 + \cos A}$.

12. $\dfrac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$.

13. $\dfrac{\sin(A+B)}{\cos A\cos B} = \tan A + \tan B$.

14. $\dfrac{\sin 21° + \sin 5°}{\cos 21° + \cos 5°} = \tan 13°$.

15. $\dfrac{\cos 9\theta + \cos 5\theta + \cos\theta}{\sin 9\theta + \sin 5\theta + \sin\theta} = \cot 5\theta$.

16. $\cos^2 x\tan^2 x + \sin^2 x\cot x^2 = 1$.

17. $\dfrac{\cos 75° + \cos 15°}{\sin 75° - \sin 15°} = \sqrt{3}$.

18. $\dfrac{\sin A + \sin B}{\cos B - \cos A} = \cot\frac{1}{2}(A-B)$.

19. $\dfrac{\tan x + \cot x + 1}{\tan x + \cot x - 1} = \dfrac{2 + \sin 2x}{2 - \sin 2x}$.

20. $\dfrac{\cos 2x + 1}{\cos 2x - 1} = -\cot^2 x$.

21. $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{\cot x + \cot y}{\cot y - \cot x}$.

22. $\cos A = \dfrac{2}{\tan\left(\frac{\pi}{4} + \frac{A}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{A}{2}\right)}$.

23. $\dfrac{\sin(x+y)\sin(x-y)}{\cos^2 x\cos^2 y} = \tan^2 x - \tan^2 y$.

24. $\cos 5x + \cos 3x = 2\cos 4x\cos x$.

25. $\dfrac{\sin 2x + 1}{\sin 2x - 1} = \dfrac{2\tan x + \tan^2 x + 1}{2\tan x - \tan^2 x - 1}$.

26. $\sin(45^\circ + x) + \sin(45^\circ - x) = \sqrt{2}\cos x.$

27. $\dfrac{1+\cot^2\left(\dfrac{\pi}{4}+x\right)}{1-\cot^2\left(\dfrac{\pi}{4}+x\right)} = \csc 2x.$

28. $\dfrac{1-\cot^2\left(\dfrac{\pi}{4}-x\right)}{1+\cot^2\left(\dfrac{\pi}{4}-x\right)} = -\sin 2x.$

29. $\dfrac{1+\cos x+\cos 2x}{\cos x} = \dfrac{\sin x + \sin 2x}{\sin x}.$

30. $\cos 12x + \cos 6x + \cos 4x + \cos 2x = 4\cos 5x \cos 4x \cos 3x.$

31. $\tan\left(45^\circ + \dfrac{x}{2}\right) = \sqrt{\dfrac{1+\sin x}{1-\sin x}}.$

32. $(\sin x \cos y - \cos x \sin y)^2 + (\cos x \cos y + \sin x \sin y)^2 = 1.$

33. $\cos^2 \tfrac{1}{2}x\,(\tan \tfrac{1}{2}x - 1)^2 = 1 - \sin x.$

34. Find the value of $\dfrac{\csc\theta + \cos\theta}{\sec\theta + \sin\theta}$ when $\cot\theta = -\dfrac{1}{2}$, and θ is in quadrant II.

35. Find the value of $\dfrac{\tan\theta + \cos\theta}{\cot\theta + \sec\theta}$ when $\sin\,\theta = -\dfrac{4}{5}$ and θ is in the 3d quadrant.

36. Simplify $\cos 300^\circ - \cot\left(\dfrac{3\pi}{2} + 60^\circ\right) + \cot 150^\circ - \tan\left(-\dfrac{\pi}{4}\right)$.

37. Simplify $\sin 660^\circ + \tan\left(\dfrac{3\pi}{2} - 60^\circ\right) + \cot 330^\circ + \cos(-30^\circ)$.

38. Simplify:

$$(a-b)\sin\frac{\pi}{2} - (a+b)\tan 225^\circ + (a^2+b^2)\cot\frac{3\pi}{2} - a\cos\left(\frac{-3\pi}{2}\right).$$

39. If $\tan 2\theta = \frac{24}{7}$, find $\tan\theta$ and $\sin\theta$, θ being in the 3d quadrant.

40. Prove $\dfrac{\sin(A+B)}{\sin(A-B)} = \dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{\cot B + \cot A}{\cot B - \cot A}.$

41. If A is an angle in the second quadrant and $\sin A = \frac{3}{5}$, find the value of $\sin 2A + \cos 2A$.

If $A + B + C = 180^\circ$, prove:

42. $\sin A + \sin B - \sin C = 4 \sin \tfrac{1}{2}A \sin \tfrac{1}{2}B \cos \tfrac{1}{2}C.$

43. $\cot \tfrac{1}{2}A + \cot \tfrac{1}{2}B + \cot \tfrac{1}{2}C = \cot \tfrac{1}{2}A \cot \tfrac{1}{2}B \cot \tfrac{1}{2}C.$

44. $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C.$

45. $\cos 2A + \cos 2B + \cos 2C = -(4\cos A \cos B \cos C + 1).$

46. $\tan A - \cot B = \sec A \csc B \csc C.$

In a right triangle, C being the right angle, prove

47. $\sin^2 \dfrac{1}{2}B = \dfrac{c-a}{2c}.$

48. $\left(\cos \dfrac{1}{2}A + \sin \dfrac{1}{2}A\right)^2 = \dfrac{a+c}{c}.$

49. $\tan \dfrac{1}{2}A = \dfrac{a}{a+c}.$

50. $\cos^2 \dfrac{1}{2}A = \dfrac{b+c}{2c}.$

Using $\sin x \cos x = \dfrac{1}{2}\sin 2x$, $\sin^2 x = \dfrac{1-\cos 2x}{2}$, $\cos^2 x = \dfrac{1+\cos 2x}{2}$, transform:

51. $\sin^2 x \cos^2 x$ into $\frac{1}{8}(1 - \cos 4x)$.

52. $\sin^4 x \cos^2 x$ into $\frac{1}{16}(1 - \cos 4x) - \frac{1}{8}\sin^2 2x \cos 2x$.

53. $\sin^4 x \cos^4 x$ into an expression in terms of the cosines of even multiples of x.

54. $\sin^8 x$ into an expression of the same general kind as in Ex. 53.

55. What nation first used the formula for $\sin \frac{1}{2}A$?

56. What man discovered the formula for $\sin 2A$?

57. Who first published the formulas for $\sin(A - B)$ and $\cos(A - B)$, and at what date?

CHAPTER VI

OBLIQUE TRIANGLES

TRIGONOMETRIC PROPERTIES OF OBLIQUE TRIANGLES

74. Law of Sines in a triangle. *In any triangle the sides are to each other as the sines of the angles opposite.*

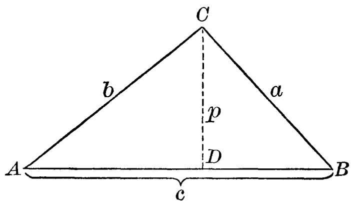


FIG. 55.

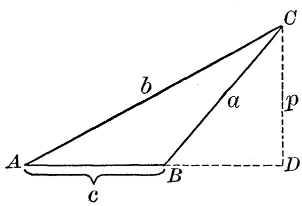


FIG. 56.

In Fig. 55 the angles A and B are both acute.
In Fig. 56 the angle A is acute, and angle ABC obtuse.
Let CD, denoted by p, be the altitude in each triangle.

In Fig. 55, in the rt. $\triangle ACD$, $p = b \sin A$; (Art. 41)

in the rt. $\triangle CBD$, $p = a \sin B$; (Art. 41)

$\therefore b \sin A = a \sin B$. (Ax. 1)

In Fig. 56, in the rt. $\triangle ACD$, $p = b \sin A$;

in the rt. $\triangle BCD$, $p = a \sin (180° - \angle ABC)$

$= a \sin \angle ABC$. (Art. 64)

Hence in $\triangle ABC$ in both figures, $b \sin A = a \sin B$,

or $a : b = \sin A : \sin B$.

In like manner, $b : c = \sin B : \sin C$,

and $a : c = \sin A : \sin C$.

Or, collecting results,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

75. Law of Tangents in a triangle. *In any triangle the sum of any two sides is to their difference as the tangent of half the sum of the angles opposite the given sides is to the tangent of half the difference of these angles.*

In a triangle ABC (Figs. 55 and 56),

$$a : b = \sin A : \sin B. \qquad \text{(Art. 74)}$$

By composition and division,

$$\frac{a+b}{a-b} = \frac{\sin A + \sin B}{\sin A - \sin B}$$

$$= \frac{2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)}{2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)}. \qquad \text{(Art. 71)}$$

Or, $$\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}.$$

In like manner,

$$\frac{b+c}{b-c} = \frac{\tan \frac{1}{2}(B+C)}{\tan \frac{1}{2}(B-C)},$$

and $$\frac{c+a}{c-a} = \frac{\tan \frac{1}{2}(C+A)}{\tan \frac{1}{2}(C-A)}.$$

It is also helpful to have a **geometric proof of the Law of Tangents.** This may be obtained as follows:

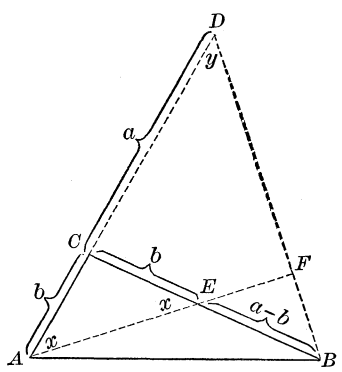


Fig. 57.

In a given triangle ABC ($CB > AC$), produce AC to D, making $CD = CB$ or a.

On CB mark off $CE = AC$ or b.

Draw the straight line DB.

Then $AD = CD + CA = a + b$.

Also $EB = CB - CE = a - b$.

$\angle DCB$, being an exterior angle of $\triangle ACE$, $= x + x = 2x$.

Also $\angle DCB$, being an exterior angle of $\triangle ACB$, $= A + B$ (of $\triangle ACB$).

$\therefore 2x = A + B$ (Ax. 1), or $x = \frac{1}{2}(A+B)$.

Also,

$$\angle FAB = A - x = A - \tfrac{1}{2}(A+B)$$
$$= \tfrac{1}{2}(A-B).$$

Also $\triangle$s ADF and EFB are similar (two $\angle$s equal).

$$\therefore \angle AFD = \angle EFB. \quad \therefore AF \perp DB.$$

In $\triangle$ AFD and EFB, $DF : FB = a + b : a - b$.

In $\triangle$ AFD and AFB,

$$\tan x : \tan \angle FAB = \frac{DF}{AF} : \frac{FB}{AF} = DF : FB.$$

By Ax. 1, $$a + b : a - b = \tan x : \tan \angle FAB$$
$$= \tan \tfrac{1}{2}(A + B) : \tan \tfrac{1}{2}(A - B).$$

76. Law of Cosines in a triangle.

In the triangle ABC, Fig. 55, by geometry,

$$a^2 = b^2 + c^2 - 2\,c \times AD.$$

But in the rt. $\triangle\, ACD$, $AD = b \cos A$.

$$\therefore a^2 = b^2 + c^2 - 2\,bc \cos A.$$

If A is an obtuse angle, Fig. 58, by geometry,

$$a^2 = b^2 + c^2 + 2\,c \times AD.$$

But in the rt. $\triangle\, ACD$,

$$AD = b \cos \angle CAD = b \cos (180^\circ - A) = -b \cos A.$$

$$\therefore a^2 = b^2 + c^2 - 2\,bc \cos A.$$

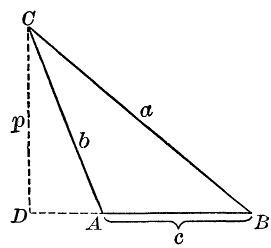


Fig. 58.

Hence in either case,

$$2\,bc \cos A = b^2 + c^2 - a^2,$$

or $$\cos \boldsymbol{A} = \frac{\boldsymbol{b^2 + c^2 - a^2}}{\boldsymbol{2\,bc}}.$$

In like manner it may be proved that

$$\cos \boldsymbol{B} = \frac{\boldsymbol{a^2 + c^2 - b^2}}{\boldsymbol{2\,ac}}, \quad \cos \boldsymbol{C} = \frac{\boldsymbol{a^2 + b^2 - c^2}}{\boldsymbol{2\,ab}}.$$

77. Formulas derived from the Cosine Formula. The formula for $\cos A$ in Art. 76 has a numerator which is primarily a sum and difference, hence logarithms cannot be used in computing numerical values from it. In order to put this formula in such a shape that its value can be computed by the aid of logarithms, it is necessary to transform the numerator of the fraction into a product. This is done

by the use of the formula for the cosine, or of that for the sine of a half angle (Art. 70). Thus:

$$2\cos^2 \tfrac{1}{2}A = 1 + \cos A = 1 + \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{2bc + b^2 + c^2 - a^2}{2bc} = \frac{(b+c)^2 - a^2}{2bc}$$

$$= \frac{(b+c+a)(b+c-a)}{2bc}.$$

Let $2s = a + b + c$; then, subtracting $2a$ from each member,

$$2s - 2a = b + c - a.$$

Hence,
$$2\cos^2 \tfrac{1}{2}A = \frac{2s(2s-2a)}{2bc},$$

or
$$\cos \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}.$$

In like manner,

$$\cos \tfrac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}},\quad \cos \tfrac{1}{2}C = \sqrt{\frac{s(s-c)}{ab}}.$$

Also from Art. 70,

$$2\sin^2 \tfrac{1}{2}A = 1 - \cos A = 1 - \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{2bc - b^2 - c^2 + a^2}{2bc} = \frac{a^2 - b^2 + 2bc - c^2}{2bc}$$

$$= \frac{a^2 - (b-c)^2}{2bc} = \frac{(a+b-c)(a-b+c)}{2bc}$$

$$= \frac{(2s-2c)(2s-2b)}{2bc} = \frac{4(s-b)(s-c)}{2bc}.$$

Hence, $\sin \tfrac{1}{2}A = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$.

In like manner,

$$\sin \tfrac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{ac}},\quad \sin \tfrac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{ab}}.$$

Dividing the formula for $\sin \tfrac{1}{2}A$ by that for $\cos \tfrac{1}{2}A$,

$$\tan \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

Similarly,

$$\tan \tfrac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} \text{ and } \tan \tfrac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}.$$

EXERCISE 35

1. Prove that the diameter of a circle circumscribed about a triangle is equal to any side of the triangle divided by the sine of the angle opposite that side.

2. By means of the property of sines, prove that the bisector of an angle of a triangle divides the opposite side into segments which are proportional to the sides forming the given angle.

3. In any triangle ABC, prove that $a = b \cos C + c \cos B$. State this property in words. Write the two similar formulas for b and c. What does the above formula become when $C = 90°$?

4. Prove that the radius of an inscribed circle of a triangle is equal to $\frac{c \sin \frac{1}{2} A \sin \frac{1}{2} B}{\cos \frac{1}{2} C}$ where c is one side of the triangle and A and B are the angles adjacent to c, and C is the angle opposite c.

5. Prove $\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$ if $s = \frac{a+b+c}{2}$.

6. Prove $\cos A = \frac{s(s-a)-(s-b)(s-c)}{bc}$.

7. Find the form to which the formula $\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$ reduces, and describe the nature of the triangle, when (I) $C = 90°$, (II) $A - B = 90°$, and $B = C$.

8. What does $a^2 = b^2 + c^2 - 2\,bc \cos A$ become when (I) $A = 90°$, (II) $A = 0°$, (III) $A = 180°$? What does the triangle become in each of these cases?

9. What does $\frac{a}{b} = \frac{\sin A}{\sin B}$ become when A is a right angle? When B is a right angle?

SOLUTION OF OBLIQUE TRIANGLES

78. Cases in the Solution of Oblique Triangles. Four cases occur in the solution of oblique triangles according as the parts given are

I. *One side and two angles.*

II. *Two sides and the included angle.*

III. *Three sides.*

IV. *Two sides and an angle opposite one of them.*

Case I. One Side and Two Angles Given

79. To solve Case I use the law of sines (Art. 74), thus:

Subtract the sum of the two given angles from 180°; *this will give the third angle.*

The unknown sides may then be found by the following proportion:

unknown side : known side = sine of angle opposite the unknown side : sine of angle opposite the known side.

In solving oblique triangles by the use of logarithms it is of special importance to make an outline or skeleton of the work before looking up any logarithms, and then to do all the work connected with the use of the tables together.

Ex. 1. Given $A = 67° 21'$, $B = 57° 48'$, $b = 367$. Solve the oblique triangle ABC.

Solution

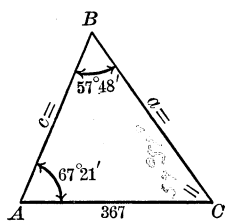


Fig. 59.

$$C = 180° - (67° 21' + 57° 48') = \mathbf{54° 51'}.$$

Then by the law of sines (Art. 74), (*Check*)

$$\frac{a}{367} = \frac{\sin 67° 21'}{\sin 57° 48'} \quad \Big| \quad \frac{c}{367} = \frac{\sin 54° 51'}{\sin 57° 48'} \quad \Big| \quad \frac{a}{c} = \frac{\sin 67° 21'}{\sin 54° 21'}$$

Before looking up any logarithms in the tables the pupil should outline the work as follows:

367 log	367 log	c log . . . .
67° 21′ log sin	54° 51′ log sin	67° 21′ log sin . . . .
57° 48′ colog sin	57° 48′ colog sin	54° 51′ colog sin . . . .
$a =$ . . . . . log	$c =$ . . . . . log	$a =$ . . . . . log . . .

The pupil can then look up all the logarithms at once and fill in the above tabulated form. (Any logarithm occurring more than once on being taken from the tables should be entered uniformly wherever it belongs.) Proceeding thus, he should obtain

$$\begin{array}{r|r}
367 \ \log 2.56467 & 367 \ \log 2.56467 \\
67^\circ\, 21' \ \log\sin 9.96541 - 10 & 54^\circ\, 51' \ \log\sin 9.91257 - 10 \\
57^\circ\, 48' \ \operatorname{colog}\sin 0.07253 & 57^\circ\, 48' \ \operatorname{colog}\sin 0.07253 \\
\hline
a = \mathbf{400.227} \ \log 2.60231 & c = \mathbf{354.625} \ \log 2.54947
\end{array}$$

(*Check*)

$$\begin{array}{r}
c \ \log 2.54947 \\
67^\circ\, 21' \ \log\sin 9.96541 - 10 \\
54^\circ\, 51' \ \operatorname{colog}\sin 0.08743 \\
\hline
a \ \log 2.60231
\end{array}$$

Ex. 2. Solve the triangle ABC, given $A = 18.29^\circ$, $B = 83.11^\circ$, and $b = 7641$.

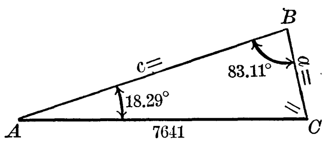


FIG. 60.

$$C = 180^\circ - (18.29^\circ + 83.11^\circ) = \mathbf{78.6^\circ}.$$

Then by the law of sines (Art. 74),

$$\begin{array}{r|r}
\dfrac{a}{7641} = \dfrac{\sin 18.29^\circ}{\sin 83.11^\circ} & \dfrac{c}{7641} = \dfrac{\sin 78.6^\circ}{\sin 83.11^\circ} \\
7641 \ \log 3.8832 & 7641 \ \log 3.8832 \\
18.29^\circ \ \log\sin 9.4967 - 10 & 78.6^\circ \ \log\sin 9.9913 - 10 \\
83^\circ\, 11' \ \operatorname{colog}\sin 0.0032 & 83.11^\circ \ \operatorname{colog}\sin 0.0032 \\
\hline
a = \mathbf{2416.11} \ \log 3.3831 & c = \mathbf{7546} \ \log 3.8777
\end{array}$$

(*Check*)

$$\frac{a}{c} = \frac{\sin 18.29^\circ}{\sin 78.6^\circ}$$

$$\begin{array}{r}
c \ \log 3.8777 \\
18.29^\circ \ \log\sin 9.4967 - 10 \\
78.6^\circ \ \operatorname{colog}\sin 0.0087 \\
\hline
a \ \log 3.3831
\end{array}$$

The accuracy of the work in Exs. 1 and 2 might also have been checked by use of the formula $a^2 = b^2 + c^2 - 2\,bc \cos A$, or of $\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}$.

In general in solving oblique triangles the accuracy of the work in any one case can be checked by applying to the results obtained one of the rules or formulas of the other cases.

EXERCISE 36

Find the remaining parts of the triangle, given:

1. $a = 12.632$, $A = 65°\ 35'$, $B = 73°\ 18'$.
2. $a = 300$, $B = 10°\ 18'$, $C = 35°\ 22'$.
3. $b = 1000$, $B = 49°\ 18'$, $C = 72°\ 50'$.
4. $c = 1640.22$, $C = 18°\ 25'$, $B = 52°\ 16'$.
5. $A = 66°\ 18'\ 36''$, $B = 43°\ 43'\ 48''$, $c = .87654$.
6. $C = 100°\ 18'\ 42''$, $B = 50°\ 40'\ 16''$, $c = 114.682$.
7. $C = 22°\ 18'\ 24''$, $B = 58°\ 12'\ 24''$, $a = 1.26984$.
8. $A = 68°\ 15'\ 20''$, $B = 43°\ 18'\ 36''$, $a = 1.8263$.
9. $B = 57°\ 23'\ 12''$, $A = 54°\ 21'\ 18''$, $c = .20814$.
10. Given $a = 5.267$, $A = 30°$, $B = 45°$, solve without using the tables.
11. Given $c = 1000$, $A = 60°$, $B = 45°$, find a and b without using tables.
12. In a parallelogram given a diagonal d, and the angles m and n which this diagonal makes with the sides, find the sides. Find the sides when $d = 14.632$, and $m = 38°\ 18'$, and $n = 12°\ 32'$.

Using four-place tables, find the unknown parts, having given:

13. $a = 14.26$, $A = 52.16°$, $B = 71.11°$.
14. $c = 200$, $C = 18.16°$, $B = 80.52°$.
15. $b = .7125$, $A = 116.18°$, $C = 38.25°$.
16. $a = 63.28$, $B = 63.28°$, $C = 36.82°$.
17. $b = 4000$, $B = 17.28°$, $C = 82.26°$.
18. $c = 8$, $A = 79.26°$, $B = 99.99°$.
19. $a = 19.28$, $B = 42.8°$, $C = 19.53°$.

20. $c = .2265$, $B = 71.28°$, $A = 52.85$.

21. $b = 176.8$, $C = 9.82°$, $B = 68.22°$.

22. $a = 4812$, $B = 75.6°$, $C = 48.71$.

23. $b = 14.267$, $C = 110.6°$, $A = 41.63°$.

24. $c = 712.8$, $B = 44.18°$, $A = 79.22$.

Without the use of tables, solve, having given:

25. $a = 100$, $B = 60°$, $A = 60°$.

26. $A = 120°$, $B = 30°$, $c = 200$.

27. $a = 500$, $A = 75°$, $B = 60°$.

28. $b = 200$, $A = 105°$, $c = 45°$.

Solve Exs. 29–31 by either set of tables.

29. A ship S can be seen from two points M and N on the shore. The distance MN is 700 ft., and the angles SMN and SNM are 57° 42′ [57.7°] and 75° 18′ [75.3°] respectively. Find the distance of the ship from M.

30. A balloon is directly over a straight road, and between two points on the road from which it is observed. The distance between the two points is 2652 yd., and the angles of elevation of the balloon as seen from the two points are 58° 50′ [58.83°] and 47° 24′ [47.4°] respectively. Find the distance of the balloon from each of the given points, and also the height of the balloon from the ground.

31. Which examples in Exercise 41 can be worked by Case I? Work such of these examples as the teacher may direct.

32. Make up some practical problem which can be solved by the method of Case I and solve it.

Case II. Two Sides and the Included Angle Given

80. To solve Case II we have the following method by the use of the law of tangents (Art. 75):

Subtract the given angle from 180°; *divide the remainder by* 2. *The result will be half the sum of the unknown angles.*

One half of their difference may then be found by the following proportion:

tan $\frac{1}{2}$ the difference of the unknown angles : tan $\frac{1}{2}$ their sum
= difference of the two given sides : their sum.

Then $\frac{1}{2}$ *sum of unknown* $\measuredangle$s $+\ \frac{1}{2}$ *their difference*

$$= \textit{greater unknown } \angle.$$

$\frac{1}{2}$ *sum of unknown* $\measuredangle$s $-\ \frac{1}{2}$ *their difference*

$$= \textit{smaller unknown } \angle.$$

The third side is found by Case I.

Ex. 1. Given $a = 4527$, $b = 3465$, $C = 66°\ 6'\ 28''$, solve the triangle.*

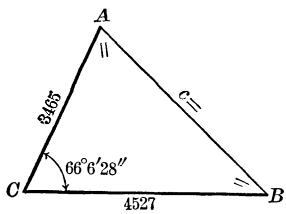


FIG. 61.

$$a + b = 7792.$$
$$a - b = 1062.$$
$$A + B = 180° - 66°\ 6'\ 28$$
$$= 113°\ 53'\ 32''.$$
$$\tfrac{1}{2}(A + B) = 56°\ 56'\ 46''.$$

By the law of tangents (Art. 75),

$$\tan \tfrac{1}{2}(A - B) : \tan \tfrac{1}{2}(A + B) = a - b : a + b,$$

that is,
$$\tan \tfrac{1}{2}(A - B) : \tan 56°\ 56'\ 46'' = 1062 : 7992.$$

$$\therefore\ \tan \tfrac{1}{2}(A - B) = \frac{1062 \tan 56°\ 56'\ 46''}{7992}.$$

$$\begin{array}{r}
1062 \log 3.02612 \\
56°\ 56'\ 46'' \log \tan 0.18659 \\
7992 \log 3.91266 - 10 \text{ colog } 6.09734 - 10 \\
\hline
\tfrac{1}{2}(A - B) = 11°\ 32'\ 28'' \log \tan 9.31005 - 10
\end{array}$$

$$\begin{array}{r}
\tfrac{1}{2}(A + B) = 56°\ 56'\ 46'' \\
\tfrac{1}{2}(A - B) = 11°\ 32'\ 28'' \\
\hline
A = \mathbf{68°\ 29'\ 14''} \\
B = \mathbf{45°\ 24'\ 18''}
\end{array}$$

The side c may now be found by Case I.

Thus we have
$$\frac{c}{3465} = \frac{\sin 66°\ 6'\ 28''}{\sin 45°\ 24'\ 18''}$$

---

* If only the third side, c, is required, and the numbers representing the other sides, a and b, are small, the solution may often be readily effected by the formula of Art. 76 without the use of logs.

Thus given $a = 5$, $b = 6$, $C = 60°$, find c.

$$c = \sqrt{a^2 + b^2 - 2ab \cos C} = \sqrt{25 + 36 - 60 \times \tfrac{1}{2}} = \sqrt{31} = 5.5775.$$

$$\begin{array}{rl}
& 3465 \log 3.53970 \\
& 66^\circ\ 6'\ 28'' \log \sin 9.96109 - 10 \\
45^\circ\ 24'\ 18'' \log \sin 9.85254 - 10 & \operatorname{colog} \sin 0.14746 \\
\hline
& c = \mathbf{4448.9} \log 3.64825
\end{array}$$

(What checks can you suggest for the work?)

Ex. 2. Given $c = 30.15$, $a = 18.159$, $B = 54.22^\circ$, solve the triangle.

$$\begin{aligned}
c + a &= 48.309. \\
c - a &= 11.991. \\
C + A &= 180^\circ - 54.22^\circ \\
&= 125.78^\circ. \\
\tfrac{1}{2}(C + A) &= 62.89^\circ.
\end{aligned}$$

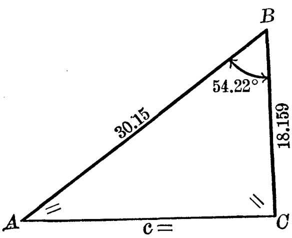


Fig. 62.

By Art. 75,

$$\tan \tfrac{1}{2}(C - A) : \tan \tfrac{1}{2}(C + A) = c - a : c + a;$$

that is, $$\tan \tfrac{1}{2}(C - A) : \tan 62.89^\circ = 11.991 : 48.309.$$

$$\therefore \tan \tfrac{1}{2}(C - A) = \frac{11.991 \tan 62.89^\circ}{48.309}.$$

$$\begin{array}{rl}
& 11.991 \log 1.0789 \\
& 62.89^\circ \log \tan 0.2908 \\
& 48.309 \log 1.6840 \ \operatorname{colog} 8.3160 - 10 \\
\hline
\tfrac{1}{2}(C - A) = 25.87^\circ & \log \tan 9.6857 - 10
\end{array}$$

$$\begin{array}{r}
\tfrac{1}{2}(C + A) = 62.89^\circ \\
\tfrac{1}{2}(C - A) = 25.87^\circ \\
\hline
C = \mathbf{88.76}^\circ \\
A = \mathbf{37.02}^\circ
\end{array}$$

The side b may now be found by Case I.

$$\frac{b}{18.591} = \frac{\sin 54.22^\circ}{\sin 37.02^\circ}$$

$$\begin{array}{rl}
& 18.159 \log 1.2591 \\
& 54.22^\circ \log \sin 9.9092 - 10 \\
37.02^\circ \log \sin 9.7797 - 10 & \operatorname{colog} \sin 0.2203 \\
\hline
& b = \mathbf{24.467} \log 1.3886
\end{array}$$

(What checks can you suggest for the work?)

EXERCISE 37

Using five-place tables, solve the following triangles, having given:

1. $a = 27.7$, $b = 18.6$, $C = 68°$.
2. $b = 400$, $c = 250$, $A = 68° 18'$.
3. $A = 30° 12' 20''$, $b = .24135$, $c = .35627$.
4. $B = 63° 35' 30''$, $a = .062788$, $c = .077325$.
5. $A = 123° 16' 30''$, $b = 2.1625$, $c = 3.1536$.
6. $A = 52° 6'$, $b = 420$, $c = 200$.
7. $C = 60°$, $b = 9$, $a = 7$. Find c only.

SUGGESTION. $c = \sqrt{a^2 + b^2 - 2\,ab \cos C}$.

8. $c = 26.369$, $b = 17.268$, $A = 32° 18' 30''$.
9. $B = 168° 18' 39''$, $c = 186.27$, $a = 132.91$.

Using four-place tables, solve the following triangles, having given:

10. $a = 200$, $b = 260$, $C = 51.82°$.
11. $b = 1.763$, $c = 1.112$, $A = 28.16°$.
12. $a = .3782$, $c = .412$, $B = 112.18°$.
13. $b = 11.65$, $a = 8.26$, $C = 12.12°$.
14. $a = 1720$, $c = 642$, $B = 78.63°$.
15. $b = 9$, $c = 6$, $A = 60°$. Find a only.

SUGGESTION. $a = \sqrt{b^2 + c^2 - 2\,bc \cos A}$.

16. $c = \sqrt{7}$, $b = \sqrt{11}$, $A = 1688°$. Find C, B, and a.
17. $b = 79.23$, $a = 100.6$, $C = 68.25°$.
18. $a = 1200$, $b = 2100$, $C = 43.18°$.

---

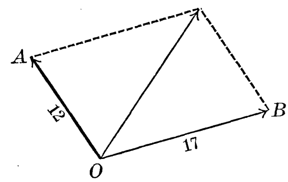
19. $a = 12$, $c = 15$, $B = 45°$. Find b without the use of tables.

Solve the following, using either set of tables:

20. Two trees M and P are on opposite sides of a pond. The distance of M from a point K is 159.6 ft., the distance of P from K is 216.8 ft., and the angle MKP is 75° 18′ [75.3°]. Find the distance between the trees.

21. The length of a lake subtends at a certain point an angle of 120°, and the distances of this point from the two extremities of the lake are 2 and 3 miles respectively. Find the length of the lake.

22. The point O is acted on by a force OA of 12 pounds and a force OB of 17 pounds, and the angle between the lines of direction of the two forces is 120° 43′ [120.72°]. What will be the resultant force and what angle will it make with each of the original forces? (Use the principle of the parallelogram of forces.)



23. Two trains leave the same station at the same time on straight tracks intersecting at an angle of 21° 12′ [21.2°]. If the trains travel at the rate of 40 and 50 miles an hour respectively, how far apart will they be in 10 minutes?

24. The sides of a parallelogram are 172.43 and 101.31 and the angle included by them is 61° 16′ [61.27°]. Find the two diagonals.

25. In Exercise 41 which examples can be worked by the methods of Case II? Work such of these as the teacher may direct.

26. Make up some practical problem which can be solved by the method of Case II and solve it.

Case III. Three Sides Given

81. The Solution of Case III is effected by the use of the formulas proved in Art. 77.

In case it is desired to find only one of the angles of a given triangle it will be best to use that one of the formulas of Art. 77 which will give the required angle most accurately. The cosine formula may be stated in general language thus:

The cosine of one half of any angle of a triangle is equal to the square root of one half the sum of the three sides multiplied by one-half the sum minus the side opposite, divided by the product of the other two sides. Thus

$$\cos\tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}},\ \cos\tfrac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}},\ \cos\tfrac{1}{2}C = \sqrt{\frac{s(s-c)}{ab}}.$$

Ex. 1. If in the triangle ABC, $a=123$, $b=113$, $c=103$, find the angle A.

$s=\frac{1}{2}(123+113+103)=169.5.$

$s-a=169.5-123=46.5.$

$$\cos\tfrac{1}{2}A=\sqrt{\frac{169.5\times 46.5}{113\times 103}}.$$

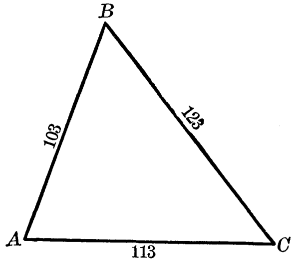


FIG. 63.

169.5	log	2.22917
46.5	log	1.66745
113	colog	7.94692−10
103	colog	7.98716−10
		2)19.83070−20
$\frac{1}{2}A=34°\ 37'\ 22''$	log cos	9.91535−10

$\therefore\ \angle A=$ **69° 14′ 44″**.

In case the half angle ($\frac{1}{2}A$) to be computed is small, it is best not to use the formula for $\cos\frac{1}{2}A$. Why?

In case the half angle to be computed is close to 90°, it is best not to use the formula for $\sin\frac{1}{2}A$. Why?

In case it is desired to find all three angles of a triangle, it is best to use the tangent formula of Art. 77. For it will be found that by that method it is necessary to employ the logarithms of but four different numbers, whereas by either of the other formulas it is necessary to use the logarithms of seven different numbers. It is a further advantage to transform the tangent formula thus:

$$\tan\tfrac{1}{2}A=\sqrt{\frac{(s-a)(s-b)(s-c)}{s(s-a)^2}}=\frac{1}{s-a}\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

Let $\sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}=r.$ Then

$$\tan\tfrac{1}{2}A=\frac{r}{s-a},\ \tan\tfrac{1}{2}B=\frac{r}{s-b},\ \tan\tfrac{1}{2}C=\frac{r}{s-c}.$$

To test the accuracy of the work add the angles obtained. Their sum should differ very slightly from 180°.

Ex. 2. If in the triangle ABC, $a=123$, $b=113$, $c=103$, find the three angles of the triangle.

$$\begin{array}{ll} s=169.5. & s-b=56.5. \\ s-a=46.5. & s-c=66.5. \end{array}$$

$$\therefore\ r=\sqrt{\frac{46.5\times 56.5\times 66.5}{169.5}}.$$

$$\begin{array}{rl} 46.5 \log & 1.66745 \\ 56.5 \log & 1.75205 \\ 66.5 \log & 1.82282 \\ 169.5 \text{ colog} & 7.77083-10 \\ & 2)\overline{3.01315} \\ r \log & 1.50658 \end{array}$$

$$\begin{array}{rl} r \log & 1.50658 \\ 46.5 \text{ colog} & 8.33255-10 \\ \tfrac{1}{2}A=34°37'22'' \log\tan & 9.83913-10 \end{array}$$

$$\begin{array}{rl} r \log & 1.50658 \\ 56.5 \text{ colog} & 8.24795-10 \\ \tfrac{1}{2}B=29°\ 36'\ 25'' \log\tan & 9.75453-10 \end{array}$$

$$\begin{array}{rl} r \log & 1.50658 \\ 66.5 \text{ colog} & 8.17718-10 \\ \tfrac{1}{2}C=25°\ 46'\ 15'' \log\tan & 9.68376-10 \end{array}$$

$$\begin{array}{rl} \text{Hence } A= & \mathbf{69°\ 14'\ 44''} \\ B= & \mathbf{59°\ 12'\ 50''} \\ C= & \mathbf{51°\ 32'\ 30''} \\ & 180°\ \ 0'\ \ 4''\quad (\textit{check}) \end{array}$$

The fact that the sum of the angles of the triangle as computed differs from 180° by four seconds is due to the fact that the logarithms used are only approximately correct in the last figure. When five-place tables are used, as in the above solution, the sum of the angles obtained should not differ from 180° by more than six or seven seconds.

Ex. 3. Find the three angles of the triangle in which $a=26.16$, $b=29.15$, $c=32.24$.

$$\begin{array}{ll} s=43.775 & s-b=14.625 \\ s-a=17.615 & s-c=11.535 \end{array}$$

$$\therefore\ r=\sqrt{\frac{17.615\times 14.625\times 11.535}{43.775}}.$$

$$\begin{array}{rl} 17.615 \log & 1.2459 \\ 14.625 \log & 1.1651 \\ 11.535 \log & 1.1620 \\ 43.775 \text{ colog} & 8.3587-10 \\ & 2)\overline{1.8317} \\ r \log & 0.9159 \end{array}$$

$$\begin{array}{rl} r \log & 0.9159 \\ 17.615 \text{ colog} & 8.7541-10 \\ \tfrac{1}{2}A=25.07° \log\tan & 9.6700-10 \end{array}$$

$$\begin{array}{rl} r \log & 0.9159 \\ 11.535 \text{ colog} & 8.9280-10 \\ \tfrac{1}{2}C=35.54° \log\tan & 9.8539-10 \end{array}$$

$$\begin{array}{rl} r \log & 0.9159 \\ 14.625 \text{ colog} & 8.8349-10 \\ \tfrac{1}{2}B=29.39° \log\tan & 9.7508-10 \end{array}$$

$$\begin{array}{rl} A= & \mathbf{50.14°} \\ B= & \mathbf{58.78°} \\ C= & \mathbf{71.08°} \\ & 180°\quad (\textit{check}) \end{array}$$

EXERCISE 38

By use of five-place tables solve each of the following triangles, having given:

1. $\begin{cases} a=54, \\ b=47, \\ c=38. \end{cases}$

2. $\begin{cases} a=2.6, \\ b=3.7, \\ c=2.8. \end{cases}$

3. $\begin{cases} a=.117, \\ b=.261, \\ c=.217. \end{cases}$

4. $\begin{cases} a=122.6, \\ b=169.4, \\ c=95.2. \end{cases}$

5. $\begin{cases} a=100, \\ b=125, \\ c=140. \end{cases}$

6. $\begin{cases} a=1.57, \\ b=1.7, \\ c=1.266. \end{cases}$

7. $\begin{cases} a=17.03, \\ b=12.585, \\ c=11.085. \end{cases}$

8. $\begin{cases} a=113, \\ b=147, \\ c=48. \end{cases}$

9. $\begin{cases} a=\sqrt{14}, \\ b=\sqrt{19}, \\ c=\sqrt{33}. \end{cases}$

10. $\begin{cases} a=4.1409, \\ b=4.9935, \\ c=1.8181. \end{cases}$

11. $\begin{cases} a=2.6, \\ b=5.7, \\ c=7.8. \end{cases}$

12. $\begin{cases} a=17.51, \\ b=12.575, \\ c=23.645. \end{cases}$

13. $\begin{cases} a=79.38, \\ b=48.16, \\ c=50. \end{cases}$

14. $\begin{cases} a=2, \\ b=3, \\ c=4. \end{cases}$ Find the largest angle.

15. The sides of a triangle are 10, 17, and 25. Find the smallest angle in the triangle.

16. The sides of a triangle are 3, 4, and 5.5. Find the sine of the smallest angle.

17. The sides of a triangle are 1.1, 1.3, 1.6. Find the cosine of the largest angle.

18. The sides of a triangle are 18, 21, and 25 ft. Find the length of the perpendicular from the vertex of the largest angle to the opposite side.

19. By use of four-place tables solve Exs. 1–18.

20. The distances between three towns, P, Q, R, are as follows: $PQ=51$, $QR=65$, $PR=20$. If R is due east from P, what is the direction of each place from every other place? If R is N.E. from P, what would each of these directions be?

21. What angle is subtended by an island 2 miles long as viewed from a point 3 miles distant from one end of the island and 4 miles from the other end?

22. Make up two practical problems which can be solved by the method of Case III and solve them.

CASE IV. GIVEN TWO SIDES AND AN ANGLE OPPOSITE ONE OF THEM

82. The **Solution of Case IV,** like that of Case I, is effected by the use of the law of sines (Art. 74). But it has been shown in geometry that when two sides and an angle opposite one of them are given, several special cases arise in the construction of the triangle.

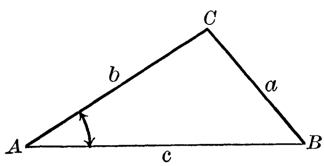


FIG. 64.

Thus in the triangle ABC (Fig. 64) let the given parts be the angle A and the sides a and b. Then under the following conditions the following triangles may be constructed:

I. If **given** $\angle A$ is **obtuse**

and 1. *side opp.* $A >$ *side adj.* . . . . . . *one* $\triangle$.

2. *side opp.* $A <$ *side adj.* . . . . . . *no* $\triangle$.

II. If **given** $\angle A$ is **right** (same results as in I).

III. If **given** $\angle A$ is **acute**

and 1. *side opp.* $>$ *side adj.* . . . . . . *one* $\triangle$.

2. *side opp.* $=$ *side adj.* . . . *one isosceles* $\triangle$.

3. *side opp.* $<$ *side adj.*

The case last mentioned (3) subdivides into three special cases as follows:

(1) *side opp.* $>$ (*side adj.*) $\times$ (*sin given* $\angle$) . . . *two* $\triangle$s.

(2) *side opp.* $=$ (*side adj.*) $\times$ (*sin given* $\angle$) . . *one right* $\triangle$.

(3) *side opp.* $<$ (*side adj.*) $\times$ (*sin given* $\angle$) . . . . *no* $\triangle$.

In practice, the cases of no solution and of one right triangle or one isosceles triangle as the solution do not often occur. Hence we usually need merely a method of discriminating between the cases where one oblique triangle or two

oblique triangles form the solution. We may state this test in the form of question and answer thus:

Q. *In general, when are there two solutions in Case IV?*

Ans. *When the side opposite the given angle is less than the other given side.*

Q. *In this case, how may the two triangles be constructed?*

Ans. *Take the vertex between the two given sides as a center, and describe an arc, using the smaller side as radius.*

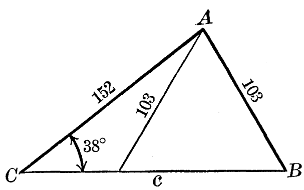


FIG. 65.

It is usual so to letter the figure that the vertex of the given angle comes at the left end of the unknown base. Thus given $\angle C = 38°$, $b = 152$, $c = 103$, we have Fig. 65.

Hence, in solving examples in Case IV,

Observe whether the side opposite the given angle is less than the other given side; if it is, there are, in general, two solutions, which construct by taking the vertex between the given sides as a center and describing an arc with the smaller side as radius.

In either case find the unknown angle opposite the known side by the use of the following proportion:

sine of unknown $\angle$ opp. known side : sine of known $\angle$
$=$ side opp. unknown $\angle$: side opp. known $\angle$.

In case there are two solutions, use in one triangle the angle obtained from the table, and in the other triangle the supplement of this angle.

Find the third angle and third side by Case I.

Ex. 1. Given $a = 84$, $b = 48.5$, $A = 21° 31'$, solve the triangle.

Since the side opposite the given angle, 84, is greater than the other given side, 48.5, there is but one solution.

$$\frac{\sin B}{\sin 21^\circ 31'} = \frac{48.5}{84}.$$

$$\therefore \sin B = \frac{48.5 \sin 21^\circ 31'}{84}.$$

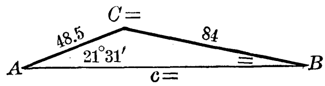


FIG. 66.

$$\begin{array}{rl} 48.5 & \log 1.68574 \\ 21^\circ 31' & \log\sin 9.56440 - 10 \\ 84 \log 1.92428 & \text{colog } 8.07572 - 10 \\ \hline B = \mathbf{12^\circ 13' 33''} & \log\sin 9.32586 - 10. \end{array}$$

$$C = 180^\circ - (A + B)$$
$$= \mathbf{146^\circ 15' 27''}.$$

By Case I we find $c = \mathbf{127.211}$.

Ex. 2. $a = 22$, $b = 34$, $A = 30^\circ 20'$, solve the triangle.

Since the side a opposite the given angle A is less than the other given side (A being acute, and $22 > 34 \sin 30^\circ 20'$) there are two solutions to the given triangle. In this case it is well to draw the smaller triangle separately as well as the general figure.

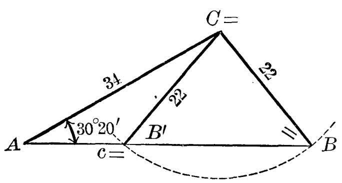


FIG. 67.

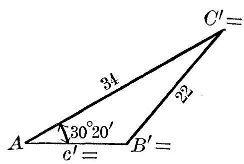


FIG. 67*a*.

By the law of sines (Art. 74),

$$\frac{\sin B}{\sin 30^\circ 20'} = \frac{34}{22}. \qquad \therefore \sin B = \frac{34 \sin 30^\circ 20'}{22}.$$

$$\begin{array}{rl} 34 & \log 1.53148 \\ 30^\circ 20' & \log\sin 9.70332 - 10 \\ 22 \log 1.34242 & \text{colog } 8.65758 - 10 \\ \hline B = \mathbf{51^\circ 18' 27''} & \log\sin 9.89238 - 10 \end{array}$$

$\therefore$ on Fig. 67a, $B' = 180^\circ - 51^\circ 18' 27''$
$= \mathbf{128^\circ 41' 33''}$.

To complete the solution of $\triangle ACB$,

$$\angle ACB = 180^\circ - (\angle A + \angle ABC)$$
$$= 180^\circ - 81^\circ 38' 27''$$
$$= \mathbf{98^\circ 21' 33''}.$$

Hence by Case I we find
$c = \mathbf{43.098}$.

To complete the solution of $\triangle\ AC'B'$ (Fig. 67*a*).

$$C' = 180^\circ - (A + B')$$
$$= 180^\circ - 159^\circ 1' 33'' = \mathbf{20^\circ 58' 27''}.$$

Then by Case I we find $c' = \mathbf{15.5926}$.

(What checks can be used in the case of each of the two triangles ?)

Ex. 3. Given $a=22$, $b=34$, $A=30.33°$, solve the triangle.

Since the side a opposite the given angle A is less than the other given side (A being acute and $22>34 \sin 30.33°$), there are two solutions. In this case it is well to draw the smaller triangle separately as well as the general figure.

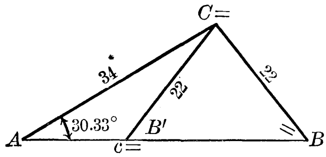


Fig. 68.

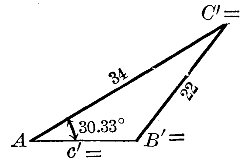


Fig. 68a.

By the law of sines (Art. 74),

$$\frac{\sin B}{\sin 30.33°}=\frac{34}{22}. \qquad \therefore \ \sin B=\frac{34 \sin 30.33°}{22}.$$

34 log 1.5315	To complete the solution of $\triangle ACB$,
30.33° log sin 9.7033 − 10	$\angle ACB=180°-(30.33°+51.32°)$
22 log 1.3424 colog 8.6576 − 10	$=$ **98.35**°.
$B=$ **51.32**° log sin 9.8924 − 10	Hence by Case I, obtain $c=$ **43.1**.

$$\therefore \ \angle B'=180°-51.32°=\mathbf{128.68}°.$$

To complete the solution of $\triangle AC'B'$ (Fig. 68a),

we have $\quad C'=180°-(30.33°+128.68°)=\mathbf{20.99}°.$

Hence, by Case I, find $c'=$ **15.6**.

EXERCISE 39

State the number of solutions for each of the following and construct a figure for each example, lettering it according to the method specified in Art. 82:

1. $A=30°$, $b=50$, $a=60$.
2. $B=30°$, $a=100$, $b=70$.
3. $C=45°$, $a=60$, $c=60$.
4. $A=60°$, $b=12$, $a=10$.
5. $C=80°$, $b=16$, $c=15.5$.
6. $B=54°$, $a=23$, $b=36$.
7. $C=30°$, $a=18$, $c=9$.
8. $B=50°$, $a=50$, $b=37$.
9. $A=75.16°$, $c=18$, $a=17.6$.

Using five-place tables, solve the following triangles, having given:

10. $A=38° \, 18'$, $b=120.6$, $a=138.7$.
11. $A=61° \, 18'$, $c=23.7$, $a=21.25$.

12. $C = 104° 13' 48''$, $b = 115.72$, $c = 165.28$.

13. $B = 22° 22'$, $a = .6728$, $b = .81434$.

14. $A = 47° 19'$, $a = 100$, $c = 120$.

15. $B = 15° 30' 12''$, $a = 1200$, $b = 590$.

16. $C = 78° 18' 18''$, $a = .26725$, $c = .37926$.

17. $B = 26° 18' 36''$, $a = 28.604$, $b = 12.678$.

18. $A = 131° 18' 24''$, $a = .8888$, $c = .4128$.

19. $C = 31° 31' 15''$, $b = 11.111$, $c = 8.267$.

Using four-place tables, solve the following triangles, having given:

20. $B = 32.37°$, $b = 126.6$, $a = 138.7$.

21. $A = 57.366°$, $c = 22.7$, $a = 20.672$.

22. $B = 105.273°$, $b = 306.72$, $c = 241.8$.

23. $C = 26.223°$, $a = 66.35$, $c = 82.59$.

24. $B = 14.3°$, $a = 20.17$, $b = 17.8$.

25. $A = 22.37°$, $c = 300$, $a = 200$.

26. $B = 63.31°$, $c = 7.67$, $b = 9.54$.

27. $C = 49.31°$, $b = .17634$, $c = .15678$.

---

28. In a parallelogram, one side is 167, one diagonal is 295.6, and the angle included by the diagonals is $24° 18'$ [$24.3°$]. Find the other side and other diagonal, and also the angles of the parallelogram.

29. If the angle between two forces is $154° 20'$ [$154.33°$], one of the forces is 960 pounds, and the resultant of the two forces is 440.46 pounds, find the other force.

AREA OF AN OBLIQUE TRIANGLE

83. I. **Given two sides and the included angle,** to find the area of a triangle, use the rule:

The area of a triangle equals one half the product of any two sides multiplied by the sine of the angle included by these sides.

For let the given sides be a and c.

In Fig. 69a, let $\angle B$ be acute; in Fig. 69b, let $\angle ABC$ be obtuse.

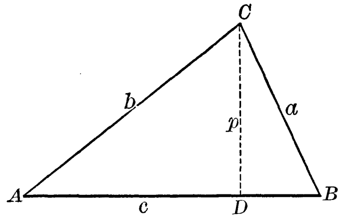


FIG. 69a.

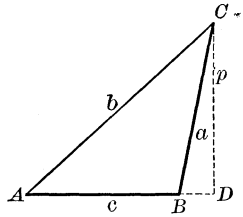


FIG. 69b.

Let p be the perpendicular from C to AB or AB produced.

In each figure, the area of $\triangle\, ABC = \frac{1}{2} c \times p$.

In Fig. 69a, in the rt. $\triangle\, CBD$, $p = a \sin B$. (Art. 41)

In Fig. 69b in the rt. $\triangle\, CBD$, $p = a \sin (180° - \angle\, ABC)$
$= a \sin ABC$. (Art 64)

Hence, in each figure, if we denote area of $\triangle\, ABC$ by K,

$$\boldsymbol{K = \tfrac{1}{2} ac \sin B.}$$

In case the given parts are a, b, C, or b, c, A, let the pupil state what the formula becomes.

Let the pupil also state these formulas in general language.

Ex. 1. $A = 66°\ 4'\ 19''$, $b = 21.66$, $c = 36.94$, find the area of the triangle ABC.

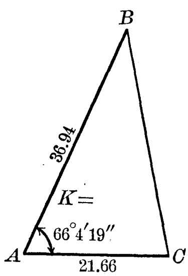


FIG. 70.

By the formula $K = \frac{1}{2} bc \sin A$,

$K = \frac{1}{2}(21.66 \times 36.94 \times \sin 66°\ 4'\ 19'')$.

$\therefore \log K = \log 21.66 + \log 36.94 + \log \sin 66°\ 4'\ 19'' + \text{colog } 2.$

21.66	log	1.33566
36.94	log	1.56750
66° 4′ 19″	log sin	9.96097 − 10
2	colog	9.69897 − 10
Area = **365.682**	log	2.56310

Ex. 2. Given $A = 66.07°$, $b = 21.66$, $c = 36.94$, find the area of the triangle ABC.

By the above rule,

$$K = \tfrac{1}{2}(21.66 \times 36.94 \times \sin 66.07°).$$

$$\therefore \log K = \log 21.66 + \log 36.94 + \log \sin 66.07° + \text{colog } 2.$$

21.66	log	1.3357
36.94	log	1.5675
66.07°	log sin	9.9610 − 10
2	colog	9.6990 − 10
Area = **365.75**	log	2.5632

84. II. **Given two angles and a side,** find the third angle as usual. Let the given side be a, then a second side c may be determined as follows:

$$c : a = \sin C : \sin A.$$

$$\therefore c = \frac{a \sin C}{\sin A} = \frac{a \sin C}{\sin [180° - (B+C)]} = \frac{a \sin C}{\sin (B+C)}.$$

Substituting this result in the formula for K in Art. 83,

$$\boldsymbol{K = \frac{a^2 \sin B \sin C}{2 \sin (B+C)}}.$$

Hence the area may be found by substituting directly in this last formula.

85. III. **Given three sides.** In this case we know from plane geometry that

$$\boldsymbol{K = \sqrt{s(s-a)(s-b)(s-c)}}.$$

86. IV. In case **two sides and an angle opposite one** of them are given, to find the area it is necessary to find the log sin of the angle included between the two given sides by the method of Case IV (Art. 82), and then proceed as in Art. 83. In some cases two answers may occur (see Art. 82).

EXERCISE 40

Using either five-place or four-place tables, find the area of the following triangles, having given:

1. $a = 16.7$, $b = 21.6$, $C = 36° 18' 24''$ [36.31°].
2. $a = .86$, $B = 52° 18'$ [52.3°], $C = 66° 42'$ [66.7°].

3. $a = 18$, $b = 14$, $c = 24$.

4. $b = 200$, $c = 150$, $A = 72° 18' 30''$ [72.31°].

5. $b = 600$, $A = 18° 26'$ [18.43°], $C = 31° 44'$ [31.73°].

6. $b = 14.7$, $a = 18.6$, $A = 74° 18'$ [74.3°].

7. $a = .8167$, $b = .68256$, $c = .72623$.

8. $a = 100$, $c = 125$, $B = 170° 16'$ [170.27°].

9. $b = 62.8$, $c = 47.2$, $A = 60°$.

10. Given $A = 29° 32' 16''$ [29.54°], $b = 500$, and $a = 300$, find the difference in area between the two triangles which contain these parts.

11. In a parallelogram, given two adjacent sides, c and d, and the included angle A, obtain a formula for the area of the parallelogram in terms of the given parts.

12. Prove that the area of any quadrilateral is equal to one half the product of its diagonals and the sine of their included angle.

13. Two sides of a parallelogram are 30 and 40 respectively, and their included angle is 60°. Find the area of the parallelogram without the use of tables.

14. The diagonals of a quadrilateral are 17.6 and 20.5, intersecting at an angle of 36° 18' [36.3°]. Find the area of the quadrilateral.

CHAPTER VII

PRACTICAL APPLICATIONS

87. Instruments for Measuring Angles. In order to determine unknown heights or distances it is important to have an instrument for measuring angles either in the horizontal or in the vertical plane. Horizontal angles can be measured by the Surveyor's Compass. Both horizontal and vertical angles can be measured by the Transit Instrument.

88. An **angle of elevation** is the angle between a line drawn from the eye of the observer to the point observed and the horizontal plane through the eye of the observer, when this angle is above the horizontal plane.

Thus, on Fig. 71, ACB is the angle of elevation of A as viewed from C.

An **angle of depression** is the angle between a line drawn from the eye of the observer to the point observed and the horizontal plane through the eye of the observer, when this angle is below the horizontal plane.

Thus, on Fig. 71, DAC is the angle of depression of C as viewed from A.

89. I. To determine the Height of an Accessible Object above a Horizontal Plane.

In Fig. 71 let AB be the object whose altitude is sought, and EF the horizontal plane, and C the point of observation.

In the right triangle ABC, what line shall we measure? What angle? How then can AB be computed?

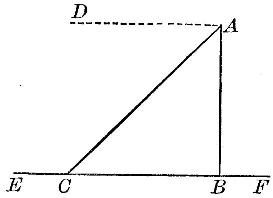


FIG. 71.

90. II. To find the Distance on a Horizontal Plane to an Inaccessible Object whose Height is Known. In Fig. 71, let AB be the inaccessible object whose height is known; let EF be the horizontal plane and C the position of the observer. In the right triangle ABC, what side is known? What angle can be measured? How then can BC be computed?

91. III. To determine the Height of an Inaccessible Object above a Horizontal Plane.

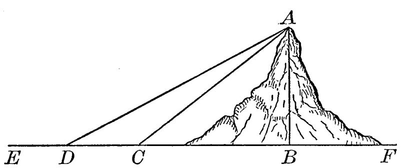


FIG. 72.

Let AB, Fig. 72, be the altitude which is to be measured, and EF the horizontal plane. Place the transit instrument at D and measure the angle of elevation ADB. Measure the distance DC toward B, and measure the angle ACB. By solving the triangle ACD the line AC is found. By solving the right triangle ACB, AB is found.

In case it is desired to compute AB by means of right triangles alone, the solution may be effected by dropping a perpendicular CP from C to AD and solving the right triangles DCP, CPA, and CAB (let the pupil supply the exact steps in this process).

Or we may proceed by the use of natural tangents thus:

On Fig. 72, in $\triangle DAB$, $DB = AB \tan \angle DAB$,

in $\triangle CAB$, $CB = AB \tan \angle CAB$.

Subtracting, $DB - CB$,

$$\text{or} \qquad DC = AB(\tan \angle DAB - \tan \angle CAB).$$

$$\text{Hence} \qquad AB = \frac{DC}{\tan \angle DAB - \tan \angle CAB}.$$

In case it is not possible to move directly from D toward B, we may proceed as follows: Measure $\angle ADB$ (Fig. 73).

Measure the line DC in the horizontal plane in any convenient direction from D. Measure ∡ BDC and DCB.

Then in the triangle DCB, DB may be computed (How?). Afterward in the triangle ADB compute AB (How?).

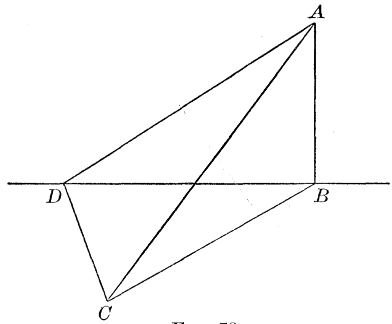


Fig. 73.

92. IV. To determine the Height of an Inaccessible Object on an Inclined Plane.

Let DF (Fig. 74) be the horizontal plane, DB the inclined plane, and AB the object whose height is sought. If we measure the ∡ ADC and ACB, and the distance DC, we may then compute AC (How?). If we then measure $\angle BDF$, we may compute $\angle CAB$ (How?). Then AB may be computed (How?).

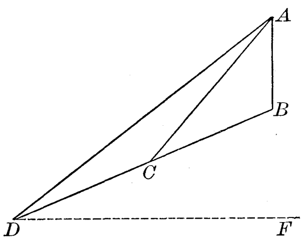


Fig. 74.

93. V. To find the Distance of an Inaccessible Object.

Let A (Fig. 75) be the position of the observer and let it be required to determine the distance from A to B.

Let the pupil determine what measurements and computations are necessary in accordance with the figure.

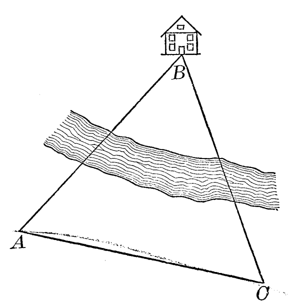


Fig. 75.

94. VI. To find the Distance between two Objects separated by an Impassable Barrier (and possibly invisible to each other).

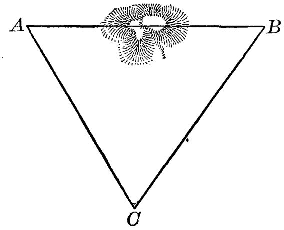


Fig. 76.

Let it be required to find the distance between A and B (Fig. 76), which are separated by a swamp or a mountain for instance. Take a station C from which both A and B are visible. Measure the angle C and the lines CA and CB. In the triangle ABC, compute AB (How ?).

95. VII. To find the Distance between two Objects, both Inaccessible and lying in the Horizontal Plane.

Let A and B (Fig. 77) be two inaccessible objects (as two islands off the shore CD). Measure the line CD and the ∠s ACD, BCD, ADC, BDC. In the triangle ACD, compute AC; in the triangle BCD, compute BC; in the triangle ABC, compute AB.

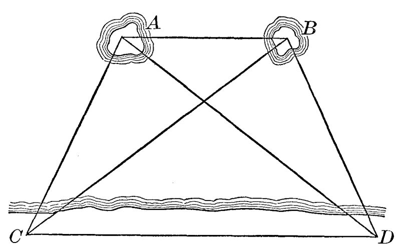


Fig. 77.

96. Range Finders. In war, both on land and sea, the use of a range finder to determine the distance of an enemy is becoming general. The essential principle of such an instrument is the finding of the distance of an inaccessible object by the solution of a triangle in which a side (called a base line) and the two angles which include the side are known (see Art. 93). On land a convenient base line is taken and measured. In naval warfare, the distance between two points on the vessel is utilized as a base line. In the range finder the triangle employed is not usually solved by numerical computation, but by some mechanical method, which gives the result sought much more expeditiously.

97. Coast and Geodetic Survey. The essential parts of the work of the coast and geodetic survey are as follows:

1. The measurement of a base line AB (Fig. 78) at least 4 or 5 miles long, so accurately that the error shall not exceed $\frac{1}{10}$ of an inch per mile.

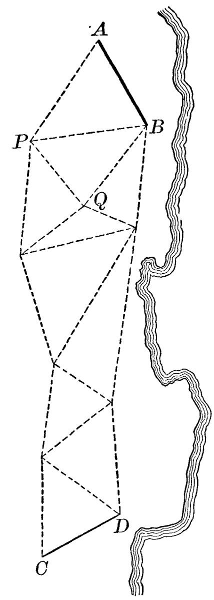


Fig. 78.

2. The choice of a convenient station P and the measurement of the angles PAB and PBA, and the computation of PA and PB in the triangle PAB.

3. The choice of another station Q, the measurement of the angles QBP and QPB, and hence the computation of PQ and QB.

4. Proceeding in like manner from station to station till convenient points, C and D, are reached, and the length of the line CD computed.

5. The careful measurement of CD and the comparison of its computed length with the result of the measurement. This final measurement of CD serves as a test of the accuracy of all the intervening work. By carrying these measurements far enough, a considerable arc of a great circle of the earth may be measured, and from this arc the radius or diameter of the earth computed.

98. Distance of the Sun and Stars. The usual method of determining the distance of the sun from the earth consists essentially in taking a line (AB, Fig. 79) nearly equal to the diameter of the earth as a base line, and observing from each end of AB the angle made by a line drawn to some convenient planet P. The distance of the planet may then be computed by Art. 93. The ratio of the distance of the sun to that of the planet from the earth being

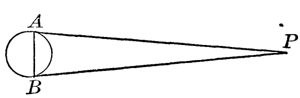


Fig. 79.

known by an astronomical law, the distance of the sun is readily determined. The distance of the sun from the earth is thus found to be approximately 93,800,000 miles.

The distances of the fixed stars are found by taking the diameter of the earth's orbit as a base line, measuring the angles made by this line with lines drawn from its ends to a fixed star, and making the necessary computations.

Thus the trigonometrical solution of a triangle in which a side and the two angles adjacent to it are known is seen to have very wide practical applications.

99. Application to Navigation. Trigonometry also has many applications to different departments of applied science. As an illustration of these applications we will briefly indicate its method of use in navigation.

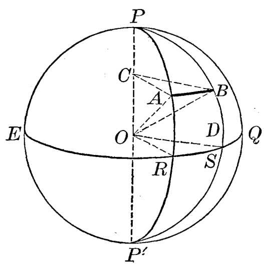


FIG. 80.

If a ship should sail from R to B on the diagram (Fig 80), crossing each meridian at the same angle, for certain purposes the $\triangle ARB$ (AB being the arc of a parallel of latitude) could be regarded as a plane triangle and solved, when necessary, by the methods of plane trigonometry. This form of navigation is called Plane Sailing.

The *departure* between two meridians is the arc of a parallel of latitude comprehended between the two meridians. Thus, AB is a departure between PAP' and PBP'. Evidently the departure between two given meridians diminishes with the distance from the equator.

The *difference of longitude* between two places is the angle at the pole (or the arc on the equator) included between the meridians of the two given places. Thus the difference of longitude for A and D is the angle RPS, or arc RS.

In *Parallel Sailing* a vessel sails due east or west (*i.e.* on a parallel of latitude) as from A to B. The difference of

longitude corresponding to the course sailed may be found by the formula

$$\textit{diff. of longitude} = \textit{departure} \times \textit{sec. latitude}.$$

For on Fig. 80,

$$\text{diff. long.} : \text{dep.} = \text{arc } RS : \text{arc } AB = OR : CA = OA : CA = \frac{OA}{CA} : 1$$
$$= \text{sec. lat} : 1.$$
$$\therefore \text{diff. long.} : \text{departure} = \text{sec. lat.} : 1.$$

In *Middle Latitude Sailing* a ship sails between two places in a course oblique to a parallel of latitude. For short distances (especially near the equator) sufficient accuracy is obtained by regarding the departure as measured on the parallel of latitude midway between the parallels of the two places, and computing the difference of longitude by the formula

$$\textit{diff. long.} = \textit{departure} \times \textit{sec. mid. lat.}$$

EXERCISE 41

1. In Exercise 22 point out the examples which are solved by the method of Art. 89.

2. Also those which are solved by the method of Art. 90.

3. Also those solved by principles contained or implied in Art. 91.

4. The angle of elevation of the top of a tree measured from a point 213.5 ft. from its foot is observed to be 18°. Find the height of the tree.

5. A water tower 92.5 ft. high stands on a horizontal plane. An observer finds the angle of elevation of the top of the tower to be 52°. Find the distance of the observer from the base of the tower.

6. Pike's Peak when viewed from a certain point on the Colorado plain has an angle of elevation of 15° 48′ [15.8°]. Two miles farther off the angle of elevation is 11° 59′ [11.98°]. What is the altitude of the mountain above the Colorado plain? If the Colorado plain is 5176 ft. above sea level, what is the altitude of Pike's Peak above sea level?

7. From the top of a hill 350 ft. high the angle of depression of the top of a tower which is known to be 150 ft. high is 57°. What is the distance from the foot of the tower to the top of the hill?

8. A man standing west of a tree, on the same horizontal plane, observes its angle of elevation to be 48°; he goes north 50 yd. and finds its angle of elevation to be 41°. Find the height of the tree.

9. The angle subtended by a tower on an inclined plane, is at a certain point on the plane 56°; 200 ft. further down it is 28°. The inclination of the plane is 7°. Find the height of the tower.

10. From the top and bottom of a castle which is 75 ft. high the angles of depression of a ship at sea are 19° and 15° respectively. Find the distance of the ship from the bottom of the castle.

11. A monument 70 ft. high and a tower stand on the same horizontal plane. The angle of elevation of the top of the tower at the top of the monument is 20° 40′ 12″ [20.67°], at the base of the monument it is 53° 31′ 12″ [53.52°]. Find the height of the tower and its distance from the monument.

12. The three angles of a triangle are to each other as 11 : 13 : 6 and the longest side is 11. Find the other two sides.

13. Two mountains, A and B, are respectively 12 and 16 mi. from a point C, and the angle ACB is 72° 18′ [72.3°]. Find the distance between the mountains.

14. In a parallelogram one side is 16.9 and a diagonal is 30.72, and the angle included by the diagonals is 26° 36′ [26.6°]. Find the other side and the other diagonal, also the angles of the parallelogram.

15. A flagstaff 50 ft. in height stands on a tower. From a position near the base of the tower, and on the same horizontal plane, the angles of elevation of the top and bottom of the flagstaff are 41° 36′ [41.6°] and 22° 18′ [22.3°], respectively. Find the distance and height of the tower.

16. The diagonals of a parallelogram are 12.5 and 12.8 ft. respectively, and their included angle is 52° 16′ [52.27°]. Find the sides of the parallelogram.

17. The sides of a triangle are 11, 13, and 16. Find the cosine of the largest angle.

18. From a point 4 mi. from one end of an island and 7 mi. from the other, the island subtends an angle of 33° 33′ 33″ [33.56°]. Find the length of the island.

19. Two buoys are 1500 yd. apart. The angles formed by lines from a boat to each buoy form angles with the line between the buoys of 77° 18′ [77.3°] and 51° 16′ [51.27°], respectively. Find the distance of the boat from the nearer buoy.

20. Two straight roads cross each other at an angle of $48° 24'$ [$48.4°$] at the point M. Four miles from M on one road is the town of P, and 6 miles from M on the other road is the town of K. How far apart are P and K? (Two answers.)

21. The diagonals of a quadrilateral are 47.6 and 61.23 rd., respectively, and the angle included by the diagonals is $43° 10'$ [$43.17°$]. Find the area of the quadrilateral.

22. To find the distance between two trees T and T', on opposite sides of a river, a line TK and the angles $T'TK$ and $T'KT$ are measured and found to be 412 ft., $62° 30'$ [$62.5°$], and $57° 32'$ [$57.53°$], respectively. Find the distance TT'.

23. Two objects which are invisible from each other on account of a hill are visible from a station whose distances from the objects are 367 yd. and 514 yd., respectively, and the angle at the station subtended by the distance between the objects is $57° 36'$ [$57.6°$]. Find the distance between the objects.

24. Given a circle with radius 19.8 ft. Find the area inclosed between two parallel chords on opposite sides of the center whose lengths are 25.6 and 31.7.

25. Wishing to find the distance between two trees T and T', separated by a marsh, I take TK on the prolongation of TT' through T, 89 yd. in length, and then take KP, 165 yd. in length, at right angles to KT. The angle $T'PT$ is found to be $33° 36' 36''$ [$33.61°$]. Find the distance from T to T'.

26. Two yachts start at the same time from the same point, and sail one due west at the rate of 9.75 mi. per hour, and the other due northwest at the rate of 11.5 mi. per hour. How far apart will they be at the end of 2 hr. sail?

27. In order to find the distance from a rock R to a buoy B, distances RK and KP are measured to points K and P from which both rock and buoy can be seen, the distance RK being 2500 m., and KP being 3600 m. The following angles are then measured: $\angle BKR = 38° 48'$ [$38.8°$], $\angle BKP = 75° 54'$ [$75.9°$], and $\angle BPK = 79° 30'$ [$79.5°$]. Find the distance from the rock to the buoy.

28. A ship sails due east 416 mi. in latitude $40° 23'$. Find the difference in longitude which she makes.

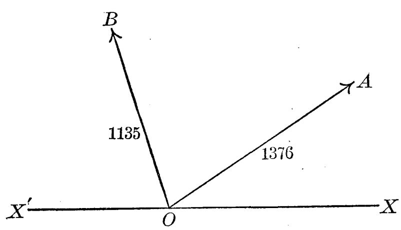
29. A ship leaves latitude $30° 16'$ N., longitude $43° 17'$ W., and sails N.E. 350 mi. Find the difference of latitude and departure which she makes.

Hence find her new latitude and longitude.

30. A flagstaff 30 ft. high stands on the top of a building. From a point on the ground, the angles of elevation of the top and bottom of the flagstaff are observed to be 41° and 36° respectively. Assuming the ground to be level, find the height of the building.

31. A tower stands on a hillside whose inclination to the horizon is 11°; a line is measured straight up the hill from the base of the tower 110 ft. in length and, at the upper extremity of the line, the tower subtends an angle of 52°. Find the height of the tower.

32. A rock 60 ft. high stands on the top of a hill whose side is inclined 21° to the horizon. An observer standing on the hillside below the rock finds the angle of elevation of the top of the rock to be 64°, and a second observer, farther down the slope, and in direct line with the first observer, finds the angle of elevation of the top of the rock to be 42°. Find the distance between the observers, and the distance from the first observer to the base of the rock.



33. A point at O is acted on by a force which gives a velocity of 1376 ft. per second along OA, and by another force which gives O a velocity of 1135 ft. per second along OB. $\angle AOX = 30°$, $\angle BOX = 101°$. What will be the magnitude and direction of the resultant velocity?

34. Show that the projection of OA plus the projection of OB on $X'OX$ equals the projection of the resultant of OA and OB on $X'OX$.

35. If, in the figure of Ex. 33, $OA = 200$ and the resultant $= 300$, find OB, the angles being unchanged.

36. A tower 190 ft. high stands on the seashore. From its top the angle of depression of two boats are 8° and 11° respectively. From the bottom of the tower the angle subtended by the distance between the boats is 101°. Find the distance between the boats.

37. A man on the opposite side of a river from two trees P and Q wishes to determine the distance between the trees. He measures a distance AB, 287 ft. He also measures the angles PAB, QAB, PBA, and PBQ and finds them 31°, 36°, 51°, and 42°, respectively. Find the distance between the trees.

38. Two straight paths cross each other at an angle of 68°. A line is drawn so as to inclose, with the two paths, an acre of ground. This line cuts one of the paths at a distance of 52 yd. from the point of

intersection of the two paths. What angle does this line make with each path?

39. A tower 135 ft. high stands at one corner of a triangular garden. From the top of the tower the angles of depression of the other two corners of the garden are $56° 18'$ [$56.3°$] and $19° 36'$ [$19.6°$], respectively. The side of the garden opposite the tower subtends, from the top of the tower, an angle of $66°$. Find the length of the sides of the garden.

40. Two towers are 144 ft. apart. The angle of elevation of one observed from the base of the other is twice that of the first observed from the base of the second; but from a point midway between the towers, the angles of elevation of the tops of the towers are complementary. Find the height of the towers. (Do not use logarithms.)

41. A railroad embankment is 9 ft. high. The length of the slope of the embankment on each side is 14 ft. Find the angle which the slope makes with the horizontal, and also find the width of the embankment at the base if the top is 8 ft. wide.

42. Given the triangle ABC, whose sides are $AB = 87.6$ yd., $AC = 112.7$ yd., and $BC = 121.6$ yd. A point D is taken on the line AC produced through C, so that the angle BDC is $18° 37' 48''$ [$18.63°$]. Find the distance DC.

43. The area of a triangle is 3 acres and two of its sides are 92.6 and 26.72 rd. Find the angle between these sides.

44. A shooting star is observed at two places 200 mi. apart on the earth's surface; the angle of elevation of the star at one station is $27°$ and at the other is $63°$, the star being in the same plane with the two stations and the center of the earth. Taking the radius of the earth as 3956 mi. find the height of the shooting star above the earth's surface and hence the height of the earth's atmosphere. (What is a shooting star? What causes its light?)

45. Show how to solve each of the cases in oblique triangles by dividing the oblique triangle into right triangles and using the methods of solving right triangles given in Chapter III. Why do we not ordinarily use this method of solving oblique triangles?

46. Make up (or collect) all the different examples you can showing practical applications of trigonometry, each example being distinct in principle or in field of application from the other examples.

CHAPTER VIII

CIRCULAR MEASURE. GRAPHS OF TRIGONOMETRIC FUNCTIONS

100. Radians, or the Circular Measure of Angles. The method of measuring angles by taking a right angle as the unit, dividing the right angle into 90 degrees, dividing each degree into 60 minutes, etc., is called the *sexagesimal* method and originated in Babylonia (see Art. 127) in very early times. It continues to be generally used in spite of its awkwardness because of the extensive tables and large number of results stated in terms of it which have been accumulated.

However, the advantages of the decimal division of any unit are so great that it is a growing custom to divide the degree of angle into tenths and hundredths instead of minutes and seconds (see many examples in this book).

Also within the past century it has become customary in many kinds of work (especially algebraic or theoretic work) to use a unit of angle different from the right angle, called the radian, and to divide this unit decimally.

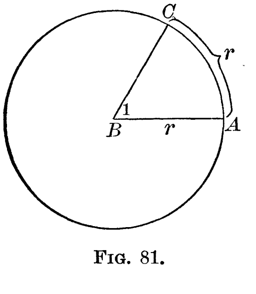


Fig. 81.

A **radian** is the angle which, when its vertex is placed at the center of a circle, intercepts an arc equal to the radius of the circle.

Thus if the arc AC (Fig. 8) equals the radius AB, the angle ABC is a radian, or the unit angle in the so-called circular method of measuring angles.

Hence, to determine the number of radians in an angle whose arc and radius are given, we have the relation

no. of radians in an angle $= \dfrac{arc}{radius}$, or,

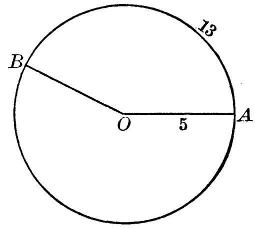


FIG. 82.

denoting the number of radians in an angle by ρ, the subtended arc by a, and the radius of the circle by R, $\rho = \dfrac{a}{R}$.

Ex. 1. Find the number of radians in an angle AOB whose arc is 13 and radius 5.

We have, $\angle AOB = \frac{13}{5} = 2.6$ radians, *Ans.*

From the above relation it follows that

Any two of the three quantities, number of radians in an angle, arc, and radius, being given, the other may be found.

Ex. 2. An angle containing 2.4 radians subtends an arc 14 in. long. Find the radius.

Substituting for ρ and a in the formula $\rho = \dfrac{a}{R}$,

$$2.4 = \frac{14 \text{ in.}}{R}. \qquad \therefore R = \frac{14 \text{ in.}}{2.4} = 5.83^{+} \text{ in.}, \textit{ Ans.}$$

101. I. Converting Degrees into Radians.

The number of radians about a point in a plane

$$= \frac{\text{circumference}}{\text{radius}}$$

$$= \frac{2\pi R}{R} = 2\pi.$$

$\therefore 360° = 2\pi$, or 6.2832 radians. $\quad 45° = \dfrac{\pi}{4}$, or 0.7854 radians.

$180° = \pi$, or 3.1416 radians.

$90° = \dfrac{\pi}{2}$, or 1.5708 radians. $\quad 30° = \dfrac{\pi}{6}$, or 0.5236 radians.

$60° = \dfrac{\pi}{3}$, or 1.0472 radians. $\quad 1° = \dfrac{\pi}{180}$, or .01745 radians.

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Hence to convert degrees into radians

Multiply the given number of degrees by $\frac{\pi}{180}$ (*or by* $.01745^+$).

Ex. 1. How many radians in $26° 17' 36''$?

$$\begin{aligned} 26° 17' 36'' &= 26.293^{+°} \\ &= (26.293^+)(.01745) \text{ radians.} \\ &= 0.45882^+ \text{ radians, } \textit{Ans.} \end{aligned}$$

Ex. 2. Simplify $\sin\left(\frac{\pi}{6}+x\right)$.

$$\begin{aligned} \sin\left(\frac{\pi}{6}+x\right) &= \sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x && \text{(Art. 66)} \\ &= \tfrac{1}{2}\cos x + \tfrac{1}{2}\sqrt{3}\sin x, \textit{ Ans.} && \text{(Art. 33)} \end{aligned}$$

Where the meaning is evident from the context, it is customary to abbreviate "π radians" into "π." Thus also we abbreviate "$\sin\frac{\pi}{6}$ radians" into "$\sin\frac{\pi}{6}$" and similarly for other expressions.

102. II. **Converting Radians into Degrees.**

Since
$$\begin{aligned} 2\pi \text{ radians} &= 360° \\ 1 \text{ radian} &= \frac{180°}{\pi}, \end{aligned}$$
or
$$\begin{aligned} 1 \text{ radian} &= 57.29579^{+°} \\ &= 57° 17' 45'' \\ &= 206265''. \end{aligned}$$

Hence to convert radians into degrees

Multiply the given number of radians by $\frac{180°}{\pi}$ (or $57.3°-$).

Ex. Convert 2.5 radians into degrees, minutes, and seconds.

$$\begin{aligned} 2.5 \text{ radians} &= 2.5 \times (57.2958°-) \\ &= 143.2395° \\ &= 143° 14' 22'', \textit{ Ans.} \end{aligned}$$

Hence, if the number of degrees in an angle be denoted by A, the number of radians in it by ρ, etc., any two of the

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four quantities A, ρ, a, R being given (provided one of them is a or R), the other two may be found by substitution of the two given quantities in the two equations

D. B. C.

$$\rho = \frac{a}{R} \quad \text{and} \quad A = \rho\left(\frac{180^\circ}{\pi}\right).$$

103. The solution of a right triangle containing an angle less than 2° may often be conveniently effected by the use of radians. For the sine or tangent of a small angle may be taken as equivalent to the number of radians in the angle (*i.e.* the circular measure of the angle) without appreciable error (see Art. 115).

Thus $\sin A = A$ (in radians) when A is a small angle, is an approximation frequently used in Physics, and the result is accurate to within the probable degree of error in measurement.

Ex. If a railroad track has a rise of 1 ft. in every 2000 ft. in its length, what angle does it make with the horizontal?

Denoting the required angle by A,

$$\sin A = \frac{1}{2000} = \text{no. radians in } A \text{ approximately.}$$

$$\therefore A = \frac{1}{2000} \times 206265'' = 103^{+''} = 1'\,43'', \textit{ Ans.}$$

EXERCISE 42

1. Reduce the following angles to circular measure, expressing the results as fractions of π:

$$30^\circ, 135^\circ, 60^\circ, 90^\circ, 210^\circ, 270^\circ, 225^\circ, 72^\circ, 315^\circ.$$

2. Express the following angles in degrees:

$$\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{15}.$$

3. What decimal part of a radian is 1°? $16''$? $2'\,15''$? $5^\circ\,14'$?

4. How many degrees (minutes and seconds) in 2 radians? 3.2 radians? .003 radians?

5. A circle has a radius of 14 inches. How many radians are there in an angle at the center subtended by an arc 21 in. long? By an arc 7 in. long?

6. In a circle of radius R, an arc 3 ft. 6 in. subtends an angle of 1.5 radians. Find R.

7. One angle of a triangle is 30°, and the circular measure of another angle is 1.5 radians. Find the third angle in degrees. Also in radians.

8. The difference between two angles is $\frac{\pi}{6}$ and their sum is 110°. Find the angles in degrees; in radians.

9. Find both in radians and degrees the complement and supplement of the following angles:

$$\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{9}, \frac{5\pi}{18}.$$

10. Write out the trigonometric ratios of the following angles:

$$\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{6}, \frac{7\pi}{4}.$$

11. How many radians in an angle whose arc is 12 and radius 10? How many degrees?

12. Show that $\sin(x + \frac{1}{3}\pi) + \sin(x - \frac{1}{3}\pi) = \sin x$.

Supply the two missing quantities in each of the following:

	ρ	a	R	A
13	2.5	10 in.	—	—
14	.25	—	50 in.	—
15	—	12 ft.	1 ft. 6 in.	—
16	—	—	42 in.	1° 30′
17	—	100	—	37°

18. If a railroad track has a rise of 1 ft. in 750 ft., what angle does the track make with the horizontal?

19. If a railroad makes an angle of 1° 30′ with the horizontal, what is its rise in one half mile?

20. An irrigating ditch should have a fall of at least $\frac{1}{4}$ in. per rod. What angle does the bottom of the ditch make with the horizontal?

21. If the moon is at a distance of 240,000 mi. from the earth and the radius of the moon subtends an angle of 16′ as seen from the earth, what is the radius of the moon in miles?

22. If the sun is at a distance of 92,800,000 mi. from the earth, and the diameter of the sun subtends an angle of 32.4′ as viewed from the earth, what is the radius of the sun in miles?

23. The planet Mars has a diameter of 4200 miles. When Mars is nearest the earth, its diameter subtends an angle of 24.5″ as seen from

the earth. What is the distance of Mars from the earth at such a time?

24. Find the numerical value of $3 \sin \frac{\pi}{4} - 4 \cos \frac{\pi}{6} \tan \frac{\pi}{3} + \cot \frac{\pi}{2}$.

25. Make up two practical problems in each of which a right triangle is solved by the use of radians as in Exs. 17–21.

We shall now illustrate the use of radians, or the circular measure of angles, (1) in tracing the graphs of trigonometric functions, (2) in solving trigonometric equations.

GRAPHS OF TRIGONOMETRIC FUNCTIONS

104. Graph of sin x. To form what is called the graph of sin x use the equation $y = \sin x$ and also a pair of rectangular axes (see Art. 54). In the equation $y = \sin x$, *let x have convenient successive values and find the corresponding values of y. Lay off each corresponding pair of values of x and y as the abscissa and ordinate of a point. Draw a continuous curve through the terminal points thus located.*

It is usually convenient to make the scale of the drawing such that a unit space of the cross-section paper stands for $\frac{\pi}{6}$ or $.5236^+$.

Thus, if we desire to make a graph of $y = \sin x$ we may take the following corresponding values of x and y:

$x = 0,\ y = 0.$

$x = \frac{\pi}{6},\ y = \frac{1}{2} = .5.$ $\qquad x = -\frac{\pi}{6},\ y = -\frac{1}{2} = -.5.$

$x = \frac{\pi}{3},\ y = \frac{1}{2}\sqrt{3} = .86^+.$ $\qquad x = -\frac{\pi}{3},\ y = -\frac{1}{2}\sqrt{3} = -.86^+.$

$x = \frac{\pi}{2},\ y = 1.$ $\qquad x = -\frac{\pi}{2},\ y = -1.$

$x = \frac{2\pi}{3},\ y = \frac{1}{2}\sqrt{3} = .86^+.$ $\qquad x = -\frac{2\pi}{3},\ y = -\frac{1}{2}\sqrt{3} = -.86^+.$

$x = \frac{5\pi}{6},\ y = \frac{1}{2} = .5.$ $\qquad x = -\frac{5\pi}{6},\ y = -\frac{1}{2} = -.5.$

$x = \pi,\ y = 0,$ etc. $\qquad x = -\pi,\ y = 0,$ etc.

Using these results, the curve $AOBCDE$ (Fig. 83) is obtained as the graph of $\sin x$. Such a figure shows at a glance the changes in the values of $\sin x$ as x changes in value.

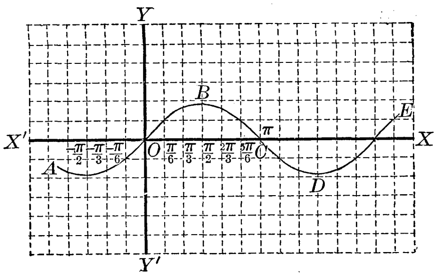

FIG. 83.

105. Graphs of Other Trigonometric Functions. By treating the equations $y = \cos x$, $y = \tan x$, $y = \sec x$, etc, similarly, the graphs of the other trigonometric functions may be constructed.

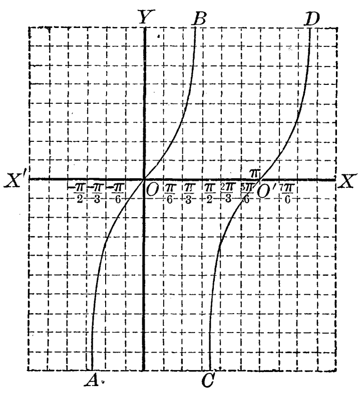

FIG. 84.

It is important to observe in constructing the graph of $\tan x$, that, as $x \doteq \frac{\pi}{2}$, $y \doteq$ either $+\infty$ or $-\infty$. For as we proceed from $x = 0$ and make $x \doteq \frac{\pi}{2}$, $y \doteq +\infty$; but as we proceed from $x = \pi$ and make $x \doteq \frac{\pi}{2}$, $y \doteq -\infty$. Hence we

obtain as part of the graph of $\tan x$ the curve AOB, $CO'D$ of Fig. 84.

EXERCISE 43

Graph each of the following:

1. $y = \sin x$.
2. $y = \cos x$.
3. $y = \tan x$.
4. $y = \cot x$.
5. $y = \sec x$.
6. $y = \csc x$.
7. $y = \sin \frac{1}{2} x$.
8. $y = \sin 2x$.
9. $y = \tan \frac{1}{2} x$.
10. $y = \sin x + \cos x$.
11. $y = \sin x - \cos x$.
12. $y = \sqrt{\sin x}$.
13. $y = \sin^2 x$.
14. $y = 1 + \sin x$.
15. $y = 1 - \cos x$.
16. $y = x + \sin x$.

106. Solutions of Trigonometric Equations. Answers not greater than 360°, *i.e.* than 2π radians.

Ex. 1. Find the values of x less than 2π radians which shall satisfy the equation $\sin x = \frac{1}{2}$.

Since $\sin 30° = \frac{1}{2}$, and also $\sin 150° = \frac{1}{2}$,

$$x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \text{ radians, } Ans.$$

Ex. 2. Solve $4 \cos x - 3 \sec x = 0$ for values of x less than 2π.

$$4 \cos x - \frac{3}{\cos x} = 0.$$

$$4 \cos^2 x - 3 = 0.$$

$$\cos x = \pm \tfrac{1}{2}\sqrt{3}.$$

Hence, $$x = 30°,\ 150°,\ 210°,\ 330°,$$

or $$x = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{7\pi}{6},\ \frac{11\pi}{6} \text{ radians, } Ans.$$

107. Answers Unlimited.

Ex. 1. Solve the equation $\cos x = \frac{1}{2}$.

One value of x is 60° and another value is $-60°$. But if 360° be added to or subtracted from the value of an angle, the value of the function is unchanged.

Hence, $x = 2n\pi \pm \frac{\pi}{3}$ radians, where n is zero or any positive or negative integer.

Ex. 2. Solve the equation $\sin x - \csc x + \frac{3}{2} = 0$.

Solving the equation, we obtain,

$$\sin x = -2,\ \tfrac{1}{2}.$$

Since the sine of an angle cannot be greater than 1, no angle corresponds to the value -2.

For $$\sin x = \tfrac{1}{2},$$

$$x = 2n\pi + \frac{\pi}{6},\ (2n+1)\pi - \frac{\pi}{6},\ \textit{Ans.}$$

EXERCISE 44

Solve each of the following equations, expressing the answers in radians, by use of π.

1. $\cot^2 \theta = -3$.
2. $\tan^2 \theta = 3$.
3. $\cot^2 \theta = 1$.
4. $\sin^2 \theta = \frac{3}{4}$.
5. $\cot \theta = 2 \cos \theta$.
6. $\cos \theta + \sec \theta = \frac{5}{2}$.
7. $3 \sin^2 x + \cos^2 x = \frac{3}{2}$.
8. $3 \cot^2 x + \tan^2 x = 4$.
9. $\cos x = \sin 2x$.
10. $\cos 2x + \sin x = 4 \sin^2 x$.
11. $\sin 2x = \tan^2 x$.
12. $\dfrac{\cot x + 1}{\cot x - 1} = \cos 2x$.
13. $2 \sin^2 x - \sin x = \sin 2x - \cos x$.
14. $\cos 2x + \cos x = 0$.
15. $\tan(45° + x) + \tan(45° - x) = 4$.
16. $2 \csc^2 x - \sqrt{3} \cot x = 5$.
17. $\sin 3x = \sin 5x + \sin x$.
18. $\cos 3x + \cos x = \cos 2x$.
19. $\sin 5x - \sin x = \cos 3x$.
20. $\cos 3x - \cos x = -\sin 2x$.
21. $\sin 5x + \sin 3x + \sin x = 0$.
22. $\cos 5x + \cos 3x + \cos x = 0$.

108. Simultaneous Trigonometric Equations.

Ex. 1. Solve $\left.\begin{aligned} x \sin y &= a \\ x \cos y &= b \end{aligned}\right\}$ for x and y.

Dividing the first equation by the second,

$$\tan y = \frac{a}{b}. \quad \therefore y = \angle \text{ whose tan is } \frac{a}{b},\ \textit{Ans.}$$

(For a briefer way of expressing this result see Chapter IX.)

From this result the value of y may be obtained. When y is known x can be obtained from either of the original equations.

$$\text{Thus } x = \frac{a}{\sin y}, \text{ or } x = \frac{b}{\cos y}.$$

Ex. 2. Solve for x and y the equations,

$$\begin{cases} x \cos A + y \sin A = a. & (1) \\ x \sin A - y \cos A = b. & (2) \end{cases}$$

Multiply equation (1) by $\cos A$, then

$$x \cos^2 A + y \sin A \cos A = a \cos A. \qquad (3)$$

Multiply equation (2) by $\sin A$, then

$$x \sin^2 A - y \sin A \cos A = b \sin A \qquad (4)$$

Add (3) and (4), using the fact that $\sin^2 A + \cos^2 A = 1$,

then $\quad x = a \cos A + b \sin A,$

and similarly, $\quad y = a \sin A - b \cos A.$ *Ans.*

EXERCISE 45

Solve for x and θ, or for x and y:

1. $\begin{cases} x \cos \theta = 86.65, \\ x \sin \theta = 50. \end{cases}$

2. $\begin{cases} x \sin \theta = 118.96, \\ x \cos \theta = 160.78. \end{cases}$

3. $\begin{cases} x \tan \theta = 816.95, \\ x \sin \theta = 426.3. \end{cases}$

4. $\begin{cases} x \sin y = 4, \\ x \cos y = 8. \end{cases}$

5. $\begin{cases} x \sin 30° + y \cos 45° = 53.28, \\ x \cos 30° + y \sin 45° = 71.58. \end{cases}$

6. $\begin{cases} x \sin 48° + y \cos 19° = 2634.1, \\ x \cos 48° + y \sin 19° = 1320.3. \end{cases}$

7. $\begin{cases} \sin x + \sin y = 1.573, \\ \cos x + \cos y = 1.207. \end{cases}$ [Use Art. 71.]

8. $\begin{cases} \sin x - \sin y = .2154, \\ \cos x - \cos y = -.1231. \end{cases}$

9. $\begin{cases} x \sin (\theta - 21.5°) = 771.1, \\ x \cos (\theta - 32.5°) = 766. \end{cases}$

10. $\begin{cases} x \cos A - y \sin A = a, \\ x \sin A + y \cos A = b. \end{cases}$

CHAPTER IX

INVERSE TRIGONOMETRIC FUNCTIONS

109. Anti-sine. If y is an angle and x its sine, the relation between x and y may be expressed in either of two ways:

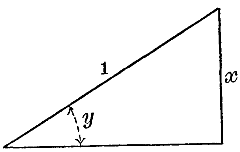

FIG. 85.

(1) $x = \sin y$, or

(2) $y = \sin^{-1} x$, which reads "y is the angle whose sine is x" or "y is the anti-sine of x."

One or the other of methods (1) or (2) is used according as *the angle*, or *its sine*, has the leading place in the discussion. Thus if the angle, or y, is more prominent, $x = \sin y$ is used; but if the sine, x, is more prominent, $y = \sin^{-1} x$ is used.

The pupil should carefully discriminate between $\sin^{-1} x$ and the -1 power of $\sin x$. The latter is expressed thus, $(\sin x)^{-1}$. Thus, $\frac{1}{\sin x} = (\sin x)^{-1}$, and not $\sin^{-1} x$. But $(\sin x)^{-2}$ may be written $\sin^{-2} x$.

110. Other Anti-trigonometric Functions. Similarly $\cos^{-1} x$ means "the angle whose cosine is x"; $\tan^{-1} x$ means "the angle whose tangent is x." Let the pupil state the meaning of $\cot^{-1} x$, $\csc^{-1} x$, $\text{vers}^{-1} x$.

It is evident that $\sin(\sin^{-1} x) = x$, since the sine of the angle whose sine is x must be x. Similarly $\cos(\cos^{-1} x) = x$, etc.

Hence there is a similarity in form between $a(a^{-1})x = x$, and $\sin(\sin^{-1} x) = x$. It is because of this similarity that the system of symbols described above is used to express the anti-trigonometric functions.

A much better symbolism for "y equals the angle whose sine is x" would seem to be "$y = \angle \sin x$," and if the pupil has difficulty in grasping the principles of this chapter, it may be well for him to use this latter method of writing inverse functions till he becomes familiar with their nature.

111. Values of Inverse Trigonometric Functions. The direct and inverse trigonometric functions have an important difference with reference to the number of values which satisfy them.

Thus, if $y = \sin 30°$, y has a single value, $\frac{1}{2}$; but if $x = \sin^{-1}\frac{1}{2}$, x can have an indefinite number of values, viz.: 30°, 150°, 390°, 510°, etc.; or

$$x = 2n\pi + \frac{\pi}{6},\ (2n+1)\pi - \frac{\pi}{6}. \quad \text{(See Art. 107, Ex. 2.)}$$

For many purposes it is customary to limit the values of an inverse circular function to the smallest value that will satisfy a given expression.

Thus, if $\theta = \tan^{-1} 1$, we take $\theta = 45°$.

112. Given an Anti-trigonometric Function, to find the other Related Functions.

Ex. 1. Given $\theta = \tan^{-1}\frac{2}{3}$, find $\sin\theta$; that is, find $\sin(\tan^{-1}\frac{2}{3})$.

$\theta = \tan^{-1}\frac{2}{3}$ may be converted into the form $\tan\theta = \frac{2}{3}$ for which a diagram may be constructed (Fig. 86).

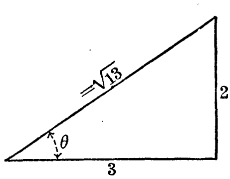

FIG. 86.

$$\therefore \sin\theta = \frac{2}{\sqrt{13}} = \tfrac{2}{13}\sqrt{13}.$$

$$\therefore \sin(\tan^{-1}\tfrac{2}{3}) = \tfrac{2}{13}\sqrt{13} \quad \textit{Ans.}$$

Ex. 2. Find $\sin 2(\cos^{-1}\frac{1}{3})$.

Let x be the angle whose cosine is $\frac{1}{3}$.

Then $\cos x = \frac{1}{3}$, $\sin x = \sqrt{1 - \frac{1}{9}} = \frac{2}{3}\sqrt{2}$.

$$\therefore \sin 2x = 2\sin x \cos x = 2(\tfrac{2}{3}\sqrt{2})\tfrac{1}{3} = \tfrac{4}{9}\sqrt{2}.$$

Hence, $\sin 2(\cos^{-1}\frac{1}{3}) = \frac{4}{9}\sqrt{2}$, *Ans.*

Ex. 3. If $\theta = \tan^{-1} a$, express the direct and inverse functions of θ in terms of a.

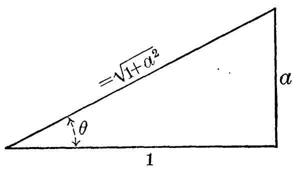

FIG. 87.

$$\begin{aligned}
\tan\theta &= a, & \text{hence } \theta &= \tan^{-1} a.\\
\cot\theta &= \frac{1}{a}, & \theta &= \cot^{-1}\frac{1}{a}.\\
\sec\theta &= \sqrt{1+a^2}, & \theta &= \sec^{-1}\sqrt{1+a^2}.\\
\cos\theta &= \frac{1}{\sqrt{1+a^2}}, & \theta &= \cos^{-1}\frac{1}{\sqrt{1+a^2}}.\\
\sin\theta &= \frac{a}{\sqrt{1+a^2}}, & \theta &= \sin^{-1}\frac{a}{\sqrt{1+a^2}}.\\
\csc\theta &= \frac{\sqrt{1+a^2}}{a}, & \theta &= \csc^{-1}\frac{\sqrt{1+a^2}}{a}.
\end{aligned}$$

Ordinarily only the positive value of each radical is used.

113. Inverse Trigonometric Functions of Two Angles.

Ex. 1. Find $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{2}{3}\right)$.

Let $x = \sin^{-1}\frac{1}{2}$.

$\therefore \sin x = \frac{1}{2}$,

$\cos x = \frac{1}{2}\sqrt{3}$.

Let $y = \cos^{-1}\frac{2}{3}$.

$\cos y = \frac{2}{3}$,

$\therefore \sin y = \frac{1}{3}\sqrt{5}$.

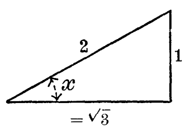

FIG. 88.

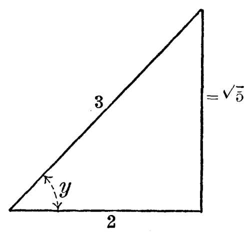

FIG. 89.

$$\begin{aligned}
\text{Then } \sin\left(\sin^{-1}\tfrac{1}{2} + \cos^{-1}\tfrac{2}{3}\right) = \sin(x+y) &= \sin x\cos y + \cos x \sin y\\
&= \tfrac{1}{2}\cdot\tfrac{2}{3} + \tfrac{1}{2}\sqrt{3}\cdot\tfrac{1}{3}\sqrt{5}\\
&= \tfrac{1}{6}(2+\sqrt{15}), \textit{ Ans.}
\end{aligned}$$

Ex. 2. Prove that $\sin^{-1} a + \cos^{-1} a = \dfrac{\pi}{2}$.

Using the method of Ex. 1, show that

$$\sin(\sin^{-1} a + \cos^{-1} a) = 1 = \sin\tfrac{\pi}{2}.$$

Ex. 3. Show that $\tan^{-1} a + \tan^{-1} b = \tan^{-1}\dfrac{a+b}{1-ab}$.

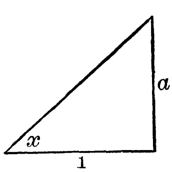

FIG. 90.

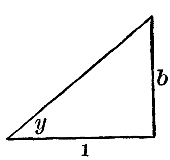

FIG. 91.

Let $x = \tan^{-1} a$.

$\therefore a = \tan x$,

$y = \tan^{-1} b$.

$\therefore b = \tan y$.

But

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}.$$

$\therefore \tan(\tan^{-1} a + \tan^{-1} b) = \dfrac{a+b}{1-ab}$, or $\tan^{-1} a + \tan^{-1} b = \tan^{-1} \dfrac{a+b}{1-ab}$.

114. Solution of Trigonometric Equations by Use of Inverse Trigonometric Functions. It is sometimes useful to express the answer obtained by solving a trigonometric equation in terms of an inverse function.

Ex. Solve $6\cos^2 x - \cos x = 2$.

Factoring, $(2\cos x + 1)(3\cos x - 2) = 0$.

$\therefore \cos x = -\frac{1}{2}, \frac{2}{3}$.

$\therefore x = \cos^{-1}(-\frac{1}{2}), \cos^{-1}\frac{2}{3}$, *Ans.*

EXERCISE 46

If the pupil has any difficulty in grasping any one of the following problems, it will be well for him to translate the symbols of the problem into general language before attempting the solution. Thus Ex. 2 would read "find the cosine of the angle whose cotangent is $\frac{3}{4}$," and might be written in the form "find $\cos\angle\cot\frac{3}{4}$" (see Art. 110).

Express the following angles first in degrees and then in radians:

1. $\cos^{-1}\frac{1}{2}\sqrt{2}$, $\tan^{-1}\sqrt{3}$, $\sin^{-1}\frac{1}{2}$, $\sec^{-1}\sqrt{2}$, $\csc^{-1}\frac{2}{3}\sqrt{3}$, $\cot^{-1}\sqrt{3}$, $\cos^{-1}\frac{1}{2}$, $\sec^{-1} 2$, $\sin^{-1}\frac{1}{2}\sqrt{3}$, $\cot^{-1}\frac{1}{3}\sqrt{3}$, $\tan^{-1}\frac{1}{3}\sqrt{3}$.

Find the value of:

2. $\cos(\cot^{-1}\frac{3}{4})$.
3. $\tan(\sin^{-1}\frac{5}{13})$.
4. $\sec(\tan^{-1}\frac{8}{15})$.
5. $\sin(\cot^{-1} a)$.
6. $\cot\left(\cos^{-1}\dfrac{a}{b}\right)$.
7. $\tan(2\sin^{-1}\frac{1}{2})$.
8. $\sin(2\tan^{-1}\frac{5}{12})$.
9. $\cos(2\sec^{-1}\frac{17}{8})$.
10. $\sin(\frac{1}{2}\cos^{-1}\frac{1}{3})$.
11. $\cot(\frac{1}{2}\tan^{-1}\frac{15}{8})$.
12. $\sin(3\sin^{-1}\frac{1}{2})$.
13. $\sin(\sin^{-1}\frac{1}{2} - \cos^{-1}\frac{2}{3})$.
14. $\tan(\tan^{-1} 2 + \cot^{-1} 3)$.

Show that:

15. $\tan^{-1}\dfrac{1}{2} + \tan^{-1}\dfrac{1}{3} = \dfrac{\pi}{4}$.
16. $\tan^{-1} 2 + \tan^{-1}\dfrac{1}{2} = \dfrac{\pi}{2}$.
17. $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{77}{85}$.
18. $\cos^{-1}\frac{3}{5} + \cos^{-1}\frac{5}{13} = \cos^{-1}(-\frac{33}{65})$.
19. $\tan^{-1}\frac{3}{4} + \tan\frac{8}{15} = \tan^{-1}\frac{77}{36}$.
20. $\cot^{-1} a + \cot^{-1} b = \cot^{-1}\dfrac{ab-1}{b+a}$.

Prove that:

21. $\sin(\sin^{-1}\frac{4}{5}+\cot^{-1}\frac{4}{3})=1.$

22. $(\cos^{-1}\frac{15}{17}+\tan^{-1}\frac{5}{12})=\sin^{-1}\frac{171}{221}.$

23. $\sin(2\tan^{-1}x)=\dfrac{2x}{1+x^2}.$

24. $\sin^{-1}x=\cot^{-1}\dfrac{\sqrt{1-x^2}}{x}.$

25. $\cos^{-1}a-\cos^{-1}b=\cos^{-1}(ab+\sqrt{1-a^2-b^2+a^2b^2}).$

26. $3\cos^{-1}x=\cos^{-1}(4x^3-3x).$

27. $3\sin^{-1}x=\sin^{-1}(3x-4x^3).$

28. $\tan^{-1}a-\tan^{-1}b=\dfrac{a-b}{1+ab}.$

29. $\sin^{-1}a+\sin^{-1}b=\cos^{-1}(\sqrt{1-a^2-b^2+a^2b^2}-ab).$

Express the value of each of the following in its most general form:

30. $\sin^{-1}\frac{1}{2}.$

31. $\tan^{-1}\frac{1}{3}\sqrt{3}.$

32. $\cos^{-1}\frac{1}{2}\sqrt{2}.$

33. $\cot^{-1}\frac{1}{3}\sqrt{3}.$

34. $\sin^{-1}\frac{1}{2}\sqrt{3}.$

35. $\cos^{-1}\frac{1}{2}\sqrt{3}.$

36. $\tan^{-1}\infty.$

37. $\cot^{-1}\sqrt{3}.$

38. $\sec^{-1}\sqrt{2}.$

39. $\sin^{-1}(-\frac{1}{2}).$

40. Prove that $\tan(2\tan^{-1}a)=\dfrac{2a}{1-a^2}.$

41. Prove $\sin(2\tan^{-1}a)=\dfrac{2a}{1+a^2}.$

42. If $\cos^{-1}x=2\cos^{-1}x$, find x.

43. Express the following angles in the inverse notation: 30°, 60°, 90°, 45°, 0°; n 180°, n 90°.

Can each of these angles be expressed in more than one way in the inverse notation?

44. Who first, and at what time, brought inverse circular functions into use in their present form (see p. 173)?

45. At what time did the circular method of measuring angles come into use (see p. 167)?

CHAPTER X

COMPUTATION OF TABLES

TRIGONOMETRIC SERIES

115. Limiting values of $\frac{\sin x}{x}$ and $\frac{\tan x}{x}$. It is important to determine the values which $\frac{\sin x}{x}$ and $\frac{\tan x}{x}$ approach when $x \doteq 0$, x being the value of an angle expressed in circular measure (radians).

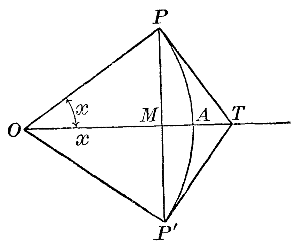

FIG. 92.

Take any angle AOP (Fig. 92) less than $90°$ and denote it by x; construct the angle AOP' equal to AOP, and draw the tangents PT and $P'T$. These tangents will meet at T on OA produced. Draw PP'.

Then OT is $\perp$ to PP' at its middle point M.

By geometry, arc $PP' >$ chord PP';

also arc $PP' < PT + P'T$.

Hence arc $PA > PM$, and arc $PA < PT$.

$$\therefore \frac{\text{arc } PA}{OP} > \frac{PM}{OP}, \text{ and } \frac{\text{arc } PA}{OP} < \frac{PT}{OP}.$$

$$\therefore x > \sin x, \text{ and } x < \tan x.$$

$$\therefore \frac{x}{\sin x} > 1, \text{ and } \frac{x}{\sin x} < \frac{1}{\cos x}.$$

$$\therefore \cos x < \frac{\sin x}{x} < 1.$$

As $x \doteq 0$, $\cos x \doteq 1$, hence $\frac{\sin x}{x} \doteq 1$, since $\frac{\sin x}{x}$ lies between $\cos x$ and 1.

Hence as $x \doteq 0$, limit $\left(\frac{\sin x}{x}\right) = 1$.

This result may also be stated thus, as $x \doteq 0$, $\sin x \doteq x$.

Also $\frac{\tan x}{x} = \frac{\sin x}{x \cos x} = \left(\frac{\sin x}{x}\right)\left(\frac{1}{\cos x}\right)$.

But as $x \doteq 0$, $\frac{\sin x}{x} \doteq 1$, and $\frac{1}{\cos x} \doteq \frac{1}{1}$ or 1.

Hence $\frac{\tan x}{x} \doteq 1 \times 1$, or 1.

Or, as $x \doteq 0$, limit $\left(\frac{\tan x}{x}\right) = 1$.

Since the number of radians in $x = \frac{\text{arc } AP}{OA}$, it follows that as the angle $x \doteq 0$, the *number of radians in* $x \doteq \sin x$, *and also* $\doteq \tan x$.

In practical work, when $x < 2°$, $\sin x$ *and* $\tan x$ *may be taken as* $= \rho$ *without appreciable error.*

116. Computation of the Tables of Trigonometric Functions. Since, as $x \doteq 0$, $\sin x$ and x approach equality (Art. 115), the circular measure of a small angle is the same as the sine of that angle to a large number of decimal places. By the use of methods which are beyond the scope of this book it is found that the value of $\sin 1'$ and the circular measure of $1'$ coincide for the first fourteen decimal places. Hence in constructing tables which are to be correct for the first five decimal places, there will be no error in taking

$$\sin 1' = 1' \text{ (in radians)}.$$

But, by Art. 101,

$$1' = \frac{3.141592^{+}}{180 \times 60} \text{ radians} = .0002908882^{+} \text{ radians}.$$

Hence $\sin 1' = .0002908882^{+}$.

But $\cos 1' = \sqrt{1-\sin^2 1'} = \sqrt{1-(.0002908882^+)^2}$
$= .9999999577^+.$

$$\sin 2' = 2 \sin 1' \cos 1' = 2 \times (.0002909^-)(.9999999577^+)$$
$$= .000582^+.$$

$$\sin 3' = \sin (2' + 1') = \sin 2' \cos 1' + \cos 2' \sin 1'.$$

From this the value of $\sin 3'$ may be computed.

In like manner the sines of all angles less than 90° may be obtained.

The cosines of these angles may be obtained similarly, or by use of the formula $\cos x = \sin (90° - x)$.

The tangents of these angles may be computed by the use of the formula $\tan x = \frac{\sin x}{\cos x}$. To obtain the cotangents, the formula $\cot x = \tan (90° - x)$ may be used.

The above method of computing sines and cosines may be abbreviated thus:

$$\sin (x+y) + \sin (x-y) = 2 \sin x \cos y. \qquad \text{(Art. 71)}$$

Let $x = a + 2b$, and $y = b$. Then, by substitution,

$$\sin (a+3b) + \sin (a+b) = 2 \sin (a+2b) \cos b.$$

Whence

$$\sin (a+3b) = 2 \sin (a+2b) \cos b - \sin (a+b). \quad . \quad . \quad (1)$$

In like manner,

$$\cos (a+3b) = 2 \cos (a+2b) \cos b - \cos (a+b). \quad . \quad (2)$$

Let $b = 1'$ in (1) and (2).

$$\sin (a+3') = 2 \sin (a+2') \cos 1' - \sin (a+1'). \quad . \quad . \quad (3)$$
$$\cos (a+3') = 2 \cos (a+2') \cos 1' - \cos (a+1'). \quad . \quad . \quad (4)$$

Letting $a = -1', 0, 1', 2', \ldots$ in succession, we obtain from (3)

$$\sin 2' = 2 \sin 1' \cos 1'.$$
$$\sin 3' = 2 \sin 2' \cos 1' - \sin 1'.$$
$$\sin 4' = 2 \sin 3' \cos 1' - \sin 2', \text{ etc.}$$

Similarly from (4),

$$\cos 2' = 2 \cos 1' - 1.$$
$$\cos 3' = 2 \cos 2' \cos 1' - \cos 1'.$$
$$\cos 4' = 2 \cos 3' \cos 1' - \cos 2', \text{ etc.}$$

117. Computation by the Use of Series. The computation of the numerical values of the trigonometric functions is, however, performed much more expeditiously by the use of certain trigonometric series than by the above method. The demonstration of these series lies beyond the scope of this work. The series are as follows:

$$\sin x = x - \frac{x^3}{\underline{|3}} + \frac{x^5}{\underline{|5}} - \frac{x^7}{\underline{|7}} + \cdots$$

$$\cos x = 1 - \frac{x^2}{\underline{|2}} + \frac{x^4}{\underline{|4}} - \frac{x^6}{\underline{|6}} + \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2\,x^5}{15} + \frac{17\,x^3}{315} + \cdots$$

The student is aided in recalling these series by the fact that $\sin(-x) = -\sin x$ (Art. 63); hence $\sin x$ must equal a series composed of odd powers of x. The same is true of $\tan x$. But since $\cos(-x) = \cos x$, $\cos x$ must equal a series composed of even powers of x.

118. Analytical Trigonometry. Theory of Functions. When trigonometry is treated in the way indicated in certain preceding articles, it ceases to be merely an instrument for solving triangles and becomes the theory of quantities varying in certain periodic or rhythmic ways.

Also by the use of the so-called imaginary quantities, the subject of trigonometry is still further extended. Thus, for instance, denoting $\sqrt{-1}$ by the symbol i, it is shown that

$$(\cos x + i \sin x)^n = \cos nx + i \sin nx$$

(called De Moivre's Theorem).

By the aid of this theorem and similar principles, trigonometry gains much additional power. This branch of the subject is termed *analytical trigonometry* (though it is sometimes treated as a part of higher algebra).

When trigonometry is extended in these various ways, it is also looked upon as a part of the larger subject, *the theory of functions*.

EXERCISE 47

1. By use of De Moivre's Theorem obtain the formulas for $\sin 3x$ and $\cos 3x$.

By use of this theorem we obtain

$$(\cos x + i \sin x)^3 = \cos 3x + i \sin 3x.$$

But

$$(\cos x + i \sin x)^3 = \cos^3 x + 3i \sin x \cos^2 x + 3i^2 \sin^2 x \cos x + i^3 \cos^3 x.$$

$$\therefore \cos 3x + i \sin 3x = \cos^3 x - 3 \sin^2 x \cos x + i(3 \cos^2 x \sin x - \sin^3 x).$$

By a theorem of algebra, in an identical equation containing both real and imaginary quantities, the sum of the reals in one member is equal to the sum of the reals in the other member, and so with imaginaries. Hence,

$$\cos 3x = \cos^3 x - 3 \sin^2 x \cos x = 4 \cos^3 x - 3 \cos x$$
$$\sin 3x = 3 \cos^2 x \sin x - \sin^3 x = 3 \sin x - 4 \sin^3 x.$$

In like manner, by De Moivre's Theorem, prove:

2. $\begin{cases} \sin 4x = 2 \sin 2x\,(1 - 2 \sin^2 x), \\ \cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1. \end{cases}$

3. $\begin{cases} \sin 5x = 16 \sin^5 x - 20 \sin^3 x + 5 \sin x, \\ \cos 5x = 16 \cos^5 x - 20 \cos^3 x + 5 \cos x. \end{cases}$

4. $\sin 7x = 7 \sin x - 56 \sin^3 x + 112 \sin^5 x - 64 \sin^7 x.$

5. $$\cos nx = \cos^n x - \frac{n(n-1)}{\underline{|2}} \cos^{n-2} x \sin^2 x + \frac{n(n-1)(n-2)(n-3)}{\underline{|4}} \cos^{n-4} x \sin^4 x + \cdots.$$

6. $$\sin nx = n \cos^{n-1} x \sin x - \frac{n(n-1)(n-2)}{\underline{|3}} \cos^{n-3} x \sin^3 x + \frac{n(n-1)(n-2)(n-3)(n-4)}{\underline{|5}} \cos^{n-5} x \sin^5 x + \cdots.$$

7. $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

8. Find the value of $\sin 225°$ by use of the formula for $\sin 5x$ in Ex. 3.

CHAPTER XI

HISTORY OF TRIGONOMETRY

119. Epochs in the History of Trigonometry. The beginnings, or germs, of Trigonometry are found in the Rhind Papyrus, now preserved in the British Museum. This papyrus, the oldest known mathematical document, was written by a scribe named Ahmes about 1400 B.C., and is a copy, so the writer states, of a more ancient work, dating, say, 3000 B.C., or several centuries before the time of Moses. In dealing with pyramids, Ahmes makes use of two of the trigonometrical ratios, viz.: that between a lateral edge of a pyramid and diagonal of the base, corresponding to the cosine of an angle; and another which corresponds to the trigonometrical tangent of the angle made by the lateral face of a pyramid with the plane of the base.

This use of ratios is, however, too crude to be regarded as scientific trigonometry. We have the following principal epochs in the scientific development of Trigonometry:

1. **Greek** (at Island of Rhodes and Alexandria), 150 B.C.–200 A.D.
2. **Arab** (in western Asia and in Spain), 650 A.D.–1200 A.D.
3. **Hindoo**, 450 A.D.–1100 A.D.
4. **European**, 1200 A.D. —

We shall also find the three following principal stages in the development of trigonometry:

I. (150 B.C.–1400 A.D.) Spherical Trigonometry studied as a part of Astronomy, with incidental use of Plane Trigonometry.

II. (1400 A.D.–1700 A.D.) Plane and Spherical Trigonometry studied as a part of Geometry.

III. (1700 A.D.–) Trigonometry as an independent science.

PRINCIPAL MAKERS OF TRIGONOMETRY

120. Hipparchus. The founder of trigonometry as a science was Hipparchus, a Greek, born about 180 B.C. in Bithynia in the northern part of Asia Minor. Hipparchus studied at Alexandria and afterward retired to the Island of Rhodes, where he did his principal work. He was primarily an astronomer and determined, for instance, the length of the year to within six minutes. He created trigonometry as a tool or aid in his astronomical work. Hence the trigonometry used by him was almost exclusively spherical.

121. Ptolemy (87 A.D.–165 A.D.). The next great name in the history of trigonometry is that of Ptolemy, also a Greek. He lived and did his work in Egypt at Alexandria. Like Hipparchus, Ptolemy was primarily an astronomer and used trigonometry merely as an aid in his astronomical investigations. He wrote a treatise on mathematical and astronomical topics, now known as the *Almagest*,* which was the standard authority in astronomy for 1200 years. The *Almagest* contains thirteen books, the first of which treats mainly of trigonometry.

122. Regiomontanus (or Johann Müller, 1436–1476 A.D.) was a German and studied at the University of Vienna. After doing important work in Germany he was called to Rome by the Pope to reform the calendar and was assassinated while in that city. The ephemerides calculated by

* Ptolemy entitled his work μεγίστη μαθηματικὴ συντάξις, or "Greatest Mathematical Collection." The book was translated by the Arabs into their language and used by them as a text-book. The name *Almagest* comes from a blending of the Arabic article "al" (the) with the Greek word μεγίστη (greatest).

Regiomontanus were used by Columbus in crossing the Atlantic. Regiomontanus wrote a text-book entitled *De Triangulis,* in which he freed the subject of trigonometry from its astronomical bondage. Though he made trigonometry a part of geometry, he presented the subject essentially in the form in which it is customary even yet to make a first presentation of it to pupils.

Several other Germans, as Pitiscus, Rheticus, and several French and English mathematicians made important contributions to the development of trigonometry, but the thinker who first put the subject on a firm modern basis was

123. Euler (1707–1783), born in Basle, Switzerland. Euler's life as a scientific worker was spent mainly at St. Petersburg and Berlin. Through his writings and influence trigonometry was established as an independent science.

Since Euler, a large number of mathematicians have made contributions to trigonometry in the larger sense, that is, considered as a branch of the theory of functions, which has been mentioned merely in an incidental way in this book.

HISTORY OF TRIGONOMETRICAL FUNCTIONS AND THEIR NOTATION

124. Sine. During all the early history of trigonometry, the trigonometric functions were regarded as lines, not as ratios.

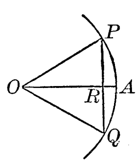

FIG. 93.

Hipparchus (120 B.C.) used but one trigonometric function. This was the chord subtended by double the angle, and it therefore corresponded in a general way to the sine of an angle. Thus, the angle AOP was regarded as determined by the chord PQ.

Ptolemy (150 A.D.) treated angles by the same method as Hipparchus, that is, by use of the chord of the double angle.

This method introduced unnecessary labor in two ways: first, it made it necessary to double each angle dealt with, in

order to get the required chord; second, it made it necessary to divide by two each angle obtained as the result of a process.

The **Hindoos** regarded an angle as determined by the semichord of twice the angle; thus by them in the above figure the angle AOP would be regarded as determined by PR. This is the method which is used at present when the sine is regarded as a line.

The **Arabs** also determined the angle by the semichord of twice the angle, one of their writers remarking that the use of the semichord "saves the continual doubling" mentioned above.

Rheticus (Germany, 1514–1576) was the first to consider the right triangle OPR as independent of any arc or circle. He defined the trigonometric functions as ratios of the sides of the right triangle, but this improvement was not adopted by other mathematicians until the time of Euler.

Euler also defined the sine and other trigonometric functions as ratios between the sides of a right triangle. He was thus able to make them functions of the angle only and to treat them as pure numbers. In this way, trigonometry became an independent science.

125. Other Functions. The **Egyptians** used the *cosine* and *cotangent*, in effect.

Hero, of Alexandria (110 B.C.), in effect, used a table of *cotangents* by which to determine the areas of regular polygons.

The **Hindoos** used the *versed sine* and *cosine* as well as the sine.

The **Arabs** invented the *tangent, cotangent,* and *secant,* though these functions were afterward neglected and reinvented in Europe.

Regiomontanus rediscovered the *tangent* and *cotangent.*

Rheticus, using the simple right triangle, had the *secant* and *cosecant* suggested to him by it.

126. Notation of the Trigonometric Functions. The Egyptians used the word *segt* for both the ratios employed by them (cosine and tangent).

The Hindoos called the chord *jiva;* the semi-chord, or sine, *ardhajya,* and later, *jiva* also; the cosine they termed *katijya,* and the versed sine *utkramajya.*

The Hindoo word for sine, *jiva,* the Arabs transliterated as *jiba,* which resembled an Arabic word, *jaib,* meaning an indentation or gulf. The Arabs in time substituted the latter familiar word for the former artificial one. Hence, when the Arabic mathematical works were translated into Latin, the term *jaib* was designated by the Latin word *sinus* (which means "gulf").

Later, Rheticus, in his use of the right triangle, termed the sine the *perpendicular,* and the cosine the *basis.*

By others the cosine was sometimes termed the *sinus rectus secundus,* and sometimes the *complementi sinus.*

Gunter (England, 1580–1626) was the first to use the word *cosine,* which he obtained by contracting the words "complementi sinus."

The Arabs called the tangent *umbra,* and the secant, *diameter umbrae,* as a result of their use of these functions in connection with the shadows of tall objects.

Later in Europe the tangent was sometimes spoken of as the *umbra recta,* and the cotangent as the *umbra versa.*

The words *tangent* and *secant* for the corresponding trigonometric functions were first used by Thomas Finck (Denmark, 1583).

Gunter, who invented the word *cosine,* also invented the word *cotangent.*

Girard (Holland, 1590–1633) was the first to use the abbreviations sin, tan, sec, etc. These abbreviations, however, were not generally accepted till they were taken up (1748) by Simpson in England and Euler in Germany.

HISTORY OF TRIGONOMETRICAL TABLES

127. History of Methods of Measuring Angles. The division of the circumference of a circle into 360 degrees, each degree into 60 minutes, and each minute into 60 seconds, is due to the Babylonians. This system of angular measurement was transmitted from the Babylonians to the Greeks, Hindoos, and Arabs. The terms *minutes* and *seconds* are derived from their Latin names which were in full "partes minutæ primæ" and "partes minutæ secundæ."

This so-called sexagesimal notation also came to be applied to other lines and quantities than the circumference of a circle as we shall see later.

The Hindoos developed the Babylonian sexagesimal method into a rude form of the circular method of measuring angles (see Art. 128). The circular method in its present form (use of radians, etc.) came into use in the early part of the eighteenth century.

The inventors of the metric system of weights and measures at the time of the French Revolution proposed to divide the right angle into 100 equal parts called "grades," and to subdivide the grade decimally, but this system never came into practical use. At present the custom of dividing a right angle into 90 degrees, and then dividing each degree decimally (instead of into minutes and seconds), is growing in favor.

128. Notation used in Trigonometric Tables. As decimal fractions in their present form are a comparatively modern invention, in the early history of Trigonometry the values of the trigonometrical functions were necessarily expressed in some other way. Thus the Greeks used sexagesimal fractions in expressing the lengths of the lines which were their trigonometrical functions. Ptolemy divided the diameter of the circle into 120 equal parts, each of these parts into 60 minutes, and each minute into 60 seconds.

For instance, where we would write sine $18° = .3090$, Ptolemy wrote chord $36° = 37° \; 4' \; 55''$.

The Hindoos divided the radius of the circle into 3148

equal parts, 3148 being the number of minutes in an arc equal to the radius. Hence the Hindoos made an approach to the circular measure for angles, the number denoting the radius, however, in their use of the relations being determined by the angle rather than the unit angle by the radius.

Regiomontanus in forming his tables first used a radius of 600,000, but later he used a purely decimal scale, 10,000,000 being the radius. Hence his work may be regarded as a transition from the sexagesimal to the decimal scale.

129. Computation of Trigonometrical Tables. Hipparchus (120 B.C.) computed a table of chords for different angles. This table, however, has been lost.

Ptolemy in his *Almagest* gives a table of chords (computed in sexagesimal fractions carried out to a point equivalent to 5 decimal places) for every $\frac{1}{2}°$ of the quadrant, the table being remarkably accurate.

Hero of Alexandria (110 B.C.) gives a table of cotangents calculated for $\cot\left(\frac{2\pi}{n}\right)$ when $n = 3, 4, \ldots 12$.

The Hindoos (530 A.D.) computed a table of sines for every $3\frac{3}{4}°$ of the quadrant.

The Arabs (Bagdad, 980 A.D.) formed a table of sines for every $\frac{1}{2}°$, and also a table of tangents and cotangents.

The printing press was invented about the year 1450. Shortly afterward the Germans took up the problem of computing very full and exact trigonometric tables, and to their industry we owe our tables essentially in their present form.

Peuerbach (1423–1461), teacher of Regiomontanus, computed a table of sines for every $10'$ with 600,000 as a radius (*i.e.* six-place tables).

Regiomontanus constructed a table of sines with 6,000,000 and another with 10,000,000 as the radius.

Regiomontanus also constructed a table of tangents for every $1'$ with 100,000 as a radius.

Apian (1495–1552) made a table of sines for every 1′ with a radius equal to 100,000.

Rheticus computed tables of sines, tangents, and secants for every 10″ with radius equal to 10,000,000,000; and later a table of sines with radius equal to 10^{15}. He began tables of tangents and secants on the same scale, but died before completing them. In this work he employed several computers for twelve years and spent large sums of money. When completed by his pupil, Otho, and published, these tables made a volume of 1468 pages.

Pitiscus (1561–1613) computed tables of sines, tangents, secants, cosines, cotangents, cosecants, with radius equal to 10^{25}. By annexing tables of proportional parts, he facilitated interpolations.

It is to be remembered that each time we use trigonometric tables we use again the labor of these indefatigable workers; or, to put it another way, by a species of kindly foresight on the part of these men we find a large part of our work already done for us by them.

Lord Napier of Scotland published his invention of logarithms in 1614. Immediately upon this invention, logarithmic tables of sines, cosines, tangents, and cotangents were formed. These tables were printed in 1633.

130. Methods of Computing Trigonometric Tables. Hipparchus and Ptolemy in constructing their tables of chords used the theorem of geometry which reads "If a quadrilateral be inscribed in a circle, the product of the diagonals equals the sum of the products of the opposite sides;" *i.e.* (Fig. 94) $AC \times BD = BC \times AD + CD \times AB$. By means of this theorem, if the chords of two arcs are known (as of 45°, 30°), the chords of the sum and of the difference of those arcs (*i.e.* of 75° and 15°) can be com-

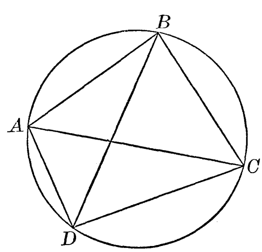

FIG. 94.

puted. Hence the theorem in a rough way is equivalent to the trigonometrical formulas for sin $(A \pm B)$ and cos $(A \pm B)$ (Art. 71). The theorem was also applied by Ptolemy to the problem of finding the chord of half an arc when the chord of the whole arc was known.

Both the Hindoos and Germans in computing their tables of trigonometric functions used methods which were essentially the same as those given in Art. 116. As has been said, much more expeditious methods are now at the service of the computer, and these methods have been used in verifying and correcting the tables as at first computed.

SOLUTION OF TRIANGLES

131. Greeks (see Ptolemy's *Almagest,* Book 1) made spherical trigonometry primary and fundamental. Plane trigonometry was developed only as a part or detail of spherical trigonometry. The methods of solving spherical triangles used by the Greeks were mainly geometrical and comparatively awkward. These methods are derived from the principles of projection, and when applied to right spherical triangles become equivalent to four of the ten formulas which are included in Napier's Rule for Circular Parts.

In plane trigonometry, as treated by the Greeks, a right triangle was solved by inscribing the triangle in a circle. An oblique triangle was solved by resolving it into right triangles. The fundamental principle in the solution of plane oblique triangles, viz. that the sides are to each other as the double chords of double the angles opposite (*i.e.* as sines of angles opposite) was used implicitly by Ptolemy, but was not stated by him in so many words. In one of the examples solved in the *Almagest,* three sides of an oblique triangle are given, and the triangle is solved by finding the segments of one of the sides made by a perpendicular on it from the opposite vertex.

To show how spherical trigonometry led the Greeks to plane trigonometry, we may mention one of the problems occurring in their treatment of the former subject, viz: To divide a given arc into two parts so that the chords of the doubles of those arcs shall have a given ratio.

Stated in terms of modern notation this problem is, Given $x+y=$ a given angle (j), to find x and y so that $\frac{\sin x}{\sin y}=\frac{a}{b}\cdot$ Stated with reference to the triangle ABC, this problem becomes one in Case II of oblique plane triangles; for $\angle C = 180° - (x+y) = 180° - j$, $\angle A = x$, $\angle B = y$; $BC = a$; $AC = b$.

The **Hindoos**, like the Greeks, made use of trigonometry only as an aid in the study of astronomy. They solved both plane and spherical triangles, but treated plane trigonometry as a mere detail of spherical trigonometry.

132. The **Arabs** also gave spherical trigonometry the leading place in the study of the subject. They simplified Ptolemy's method of solving spherical triangles, discovered that in spherical triangles $\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$, and to the four of the ten formulas included in Napier's Rule for Circular Parts, which Ptolemy had implicitly known, added two others, viz.:

$$\cos B = \cos b \sin A, \quad \cos c = \cot A \cot B.$$

The Arabs, however, developed no general theory for the solution of plane or spherical triangles.

133. Regiomontanus separated plane from spherical trigonometry and made plane trigonometry primary. In his treatise he begins with the right triangle, solves it by using the sine function only, and then solves equilateral and isosceles triangles by resolving them into right triangles. He also solves oblique triangles much as is done at present. His treatment of spherical trigonometry, however, is far less general and satisfactory.

Romanus (Belgium, 1561–1625) condensed the twenty-six cases of spherical trigonometry then in use into six cases.

134. Lord Napier (Scotland, 1550–1617) reduced the solution of right spherical triangles to ideal simplicity by his Rule for Circular Parts. This has been commended as perhaps "the happiest example of artificial memory that is known." He also simplified the solution of oblique spherical triangles by his discovery of the formulas known as Napier's Analogies.

135. Notation of Triangles. To Euler is due the method of denoting the angles of a triangle by the capital letters A, B, C, and the sides opposite by the small letters a, b, c.

136. The theory of the **complete spherical triangle,** that is, of the triangle in which the length of the sides is not necessarily less than 180°, was developed by Gauss (Germany, 1777–1855) and Moebius (Germany, 1790–1868), but such triangles are not much used in practice.

137. Spheroidal trigonometry, that is, the theory of triangles on the surface of a spheroid has great practical importance because of its use in surveying large portions of the earth's surface, as in the coast and geodetic surveys in different countries.

DEVELOPMENT OF GONIOMETRY

138. Greeks. As has been stated (Art. 130), the geometrical methods used by the Greeks in constructing tables of chords were in a rough way equivalent to a use of the formulas for $\sin(A \pm B)$, $\cos(A \pm B)$, and $\sin \frac{1}{2} A$.

139. The **Hindoos** knew the identical equation

$$\sin^2 A + \cos^2 A = 1.$$

They also used the formula $\sin \frac{1}{2} A = \sqrt{1719(3438 - \cos A)}$, where 3438 is the radius of the circle. This is equivalent to the formula $\sin \frac{1}{2} A = \sqrt{\frac{1 - \cos A}{2}}$.

In computing trigonometric tables they appear to have used the formula

$$\sin(n+1)a - \sin na = \sin na - \sin(n-1)a - \sin na \operatorname{cosec} a.$$

This formula is not quite accurate and was probably arrived at inductively.

140. The **Arabs** knew the relations

$$\tan\phi = \frac{\sin\phi}{\cos\phi},\ \cot\phi = \frac{\cos\phi}{\sin\phi},$$

and were also able to solve an equation like $\tan\phi = a$, obtaining $\sin\phi = \frac{a}{\sqrt{1+a^2}}$.

141. Rheticus obtained the formulas

$$\sin 2A = 2\sin A\cos A,$$
$$\sin 3A = 3\sin A - 4\sin^3 A.$$

Romanus discovered the formula for $\sin(A+B)$.

The formulas for $\sin(A-B)$ and $\cos(A \pm B)$ were published by Pitiscus (1599).

142. Vieta (France, 1540–1603) gave the general formulas for $\sin nA$ and $\cos nA$ in terms of $\sin A$ and $\cos A$.

OTHER PROCESSES

143. Trigonometrical Series. The series for $\sin x$ and $\cos x$ in terms of powers of x and for $\sin^{-1} x$ in terms of $\sin x$ were known to Sir Isaac Newton before the year 1669.

Those for $\tan x$ and $\sec x$ in terms of powers of x and for $\tan^{-1} x$ in terms of powers of $\tan x$ were discovered by Gregory (England, 1638–1675) in 1670.

144. Inverse Circular Functions in their general form were introduced by John Bernouilli (1667–1748).

145. Use of $\sqrt{-1}$ or i. John Bernouilli first treated trigonometry as a branch of analysis. Among other algebraic methods he introduced the use of $\sqrt{-1}$, or i, into trigonometry and obtained real results by its use. For instance, by employing $\sqrt{-1}$ he obtained a series for $\tan n\phi$ in term of powers of $\tan \phi$.

This use of i was followed up by Euler, who among other results obtained the formula

$$(\sin x + i \cos x)^n = \sin nx + i \cos nx$$

known as De Moivre's Theorem.

EXERCISE 48. GENERAL REVIEW

1. Simplify $\log_2 4 + 5 \log_3 9 + \frac{1}{2} \log_{10} .1 - \log_{10}\sqrt{.001}$.

2. Compute the value of x from the equation $5x^3 = \sqrt[4]{.2784}$.

3. Also from $\cos x = (.9387)^{\frac{3}{2}}$.

4. Also from $\tan x = \dfrac{(7.605)^3\sqrt{14.82}}{(27.32)^{\frac{5}{2}}}$.

5. If x is an angle in the first quadrant and $\cos x = \frac{8}{17}$, find the value of $\dfrac{\sin x + \tan x}{\cos x - \cot x}$.

6. If x is an angle in the first quadrant and $2 \cos x = 2 - \sin x$, find the value of $\tan x$.

7. If $\tan x = \dfrac{a}{b}$, find $\sin 2x$.

8. If $\sin y = a$ and $\tan y = b$, prove that $(1 - a^2)(1 + b^2) = 1$.

9. $ABCD$ is a square. D is joined to E, the midpoint of AB. Find the trigonometric ratios of $\angle ECD$.

10. Determine the numerical value of $\sin 18°$ by use of the geometric method of inscribing a regular decagon in a circle.

11. If A is an angle in the first quadrant and $\tan A = \dfrac{p}{q}$, find the value of $\dfrac{p \cos A - q \sin A}{p \cos A + q \sin A}$.

12. Which of the following statements are possible and which impossible:

(1) $16 \sin x = 1$. (2) $4 \sec \theta = 1$. (3) $7 \tan y = 30$.

13. Prove that $\sec x + \tan x = \dfrac{\sec^2 x + \sec x \tan x}{\tan x + \sec x} + \tan x$.

14. Prove that $\dfrac{\text{vers}^2 x}{\sin x} = \dfrac{2 \sin x}{1 + \cos x} - \sin x$.

15. Find the numerical value of $3 \tan^3 30° \sec^3 60° \sin^2 90° \tan^2 45° + 5 \cos 90°$.

16. If $\tan^2 45° - \cos^2 60° = y \sin 45° \cos 45° \tan 60°$, find y.

17. If $x \sin \dfrac{\pi}{6} \cos^2 \dfrac{\pi}{4} = \dfrac{\cos^2 \frac{\pi}{6} \sec \frac{\pi}{3} \tan \frac{\pi}{4}}{\csc^2 \frac{\pi}{4} \cos \frac{\pi}{6}}$, find x.

Solve each of the following right triangles, given:

18. $A = 36° 18' 6''$ [36.3°], $b = 217.9$ ft.

19. $b = 315.92$ ft., $c = 814.23$ ft.

20. $c = 900$, $b = 887$.

21. $B = 12° 15'$ [12.25°], $c = 1001.4$.

22. $A = 1° 20'$ [1.33°], $c = 872.56$.

23. In a right triangle $b = 426$, $A = 38° 45'$ [38.75°]. Find $a + c$ and the area.

24. The hypotenuse of a right triangle is 5 ft. and one angle of the triangle is 30°. Solve the triangle and find the area without the use of tables.

25. The area of a regular polygon of 11 sides is 80. Find the side, radius, and apothem of the polygon.

26. In an isosceles triangle the leg is 21.7 and the area 32.51. Solve the triangle.

27. The legs of a right triangle are to each other as 5 : 9. Find the angles of the triangle.

28. On the steepest part of the Mt. Washington railway (Jacob's Ladder), there is a rise of $13\frac{1}{2}$ inches for every 3 ft. of track. What angle does the track make with the horizontal? At this rate what would be the rise in one mile of track?

Show that in a right triangle:

29. $\cos 2A = \dfrac{b^2 - a^2}{c^2}$.

30. $\sin 3A = \dfrac{3ac^2 - a^3}{c^3}$.

31. $(\sin A - \sin B)^2 + (\cos A + \cos B)^2 = 2$.

32. Find the other trigonometric functions of A, when $\cos A = -\frac{1}{2}$ and A lies between 540° and 630°.

33. Given $\sec x = -\frac{5}{4}$ and x in the third quadrant, find the value of $\dfrac{\sin x + \tan x}{\cos x + \cot x}$.

34. Find the trigonometric functions of $180° + x$ and of $270° - x$ when $\tan x = \frac{1}{4}$.

35. For what values of x between 0° and 360° is $\sin x + \cos x$ positive, and for what values is it negative?

36. Find the numerical value of

$$3 \sin^2 225° + 4 \sin(-120°) \tan 150° - \tfrac{1}{2} \cos^2 330° \cot 750° + 5 \sin^2 180°.$$

37. For each of the following angles state which of the three principal trigonometric ratios are positive:

(1) 460°. (2) $-220°$. (3) $-1200°$. (4) $\dfrac{13\pi}{6}$.

38. Trace the changes in sign and magnitude of

$\sin A$ between 0° and 360°.
$\csc A$ between 0 and π.
$\cos x$ between π and 2π.
$\tan A$ between $-90°$ and $-270°$.

39. If A is in the third quadrant and $\tan A = \frac{5}{12}$, find the value of $\sin 2A$.

40. Express the cosine of an angle in the second quadrant in terms of (a) each of the other trigonometric functions of the given angle, (b) the cosine of the complement of the angle.

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41. If $\sin A = \frac{12}{13}$ and $\sin B = \frac{3}{5}$, and A and B are both acute, find the numerical value of $\tan(A + B)$; also of $\tan(A - B)$.

42. If x is an angle in the second quadrant and $\sin x = \frac{3}{5}$, find the value of $\sin 2x + \cos 2x$.

43. Express $2 \cos \dfrac{2\theta}{3} \cos \dfrac{5\theta}{3}$ as a sum or difference.

44. If $\sin \frac{1}{2} x = \frac{1}{4}$, find the numerical value of $\cos x$. Also of $\tan x$.

Prove that:

45. $\sin^2(A + B) - \sin^2(A - B) = \sin 2A \sin 2B$.

46. $\dfrac{\sin 4x + \sin 3x}{\cos 3x - \cos 4x} = \cot \frac{1}{2} x$.

47. $\sin 50° + \sin 10° = \sin 70°$.

48. $\sin^2 15^\circ + \cos^2 15^\circ = 1.$

49. $\cos 55^\circ + \sin 25^\circ = \sin 85^\circ.$

50. $\dfrac{\sin A + \sin 2A + \sin 3A}{\cos A + \cos 2A + \cos 3A} = \tan 2A.$

51. $\dfrac{1 - \tan^2(45^\circ - x)}{1 + \tan^2(45^\circ - x)} = \sin 2x.$

52. $\dfrac{\cos\left(\dfrac{\pi}{4} - \theta\right) - \cos\left(\dfrac{\pi}{4} + \theta\right)}{\sin\left(\dfrac{2\pi}{3}\theta +\right) - \sin\left(\dfrac{2\theta}{3} - \theta\right)} + \sqrt{2} = 0.$

Solve each of the following oblique triangles, given:

53. $A = 30^\circ\ 18'\ 12''$ [39.3°], $b = 3294$, $c = 2846$.

54. $A = 76^\circ\ 24'\ 36''$ [76.41°], $B = 48^\circ\ 42'$ [48.7°], $c = 1012$.

55. $a = 850$, $b = 760$, $c = 590$.

56. $B = 46^\circ\ 18'$ [46.3°], $b = 213.76$, $a = 192.72$.

57. $b = 927$, $A = 79^\circ$, $B = 21^\circ\ 17'\ 12''$ [21.29°].

58. $a = \sqrt{3}$, $b = \sqrt{2}$, $c = \sqrt{5}$.

59. $A = 51^\circ\ 30'$ [51.5°], $a = 294.6$, $b = 301.7$.

60. $a = 926.8$, $b = 842.5$, $C = 46^\circ\ 27'$ [46.45°].

61. Solve the triangle in which $K = 20.602$, $a = 214.2$, and $b = 315.8$.

62. The diagonals of a parallelogram are 347 and 264 ft., and the area of the parallelogram is 40.437 sq. ft. Find the sides and angles of the parallelogram.

63. The diagonals of a quadrilateral are 34 and 56, and they intersect at an angle of 67°. Find the area of the quadrilateral.

Solve the following equations for answers not greater than 360° or less than 0°:

64. $\sec x + \tan x = \pm\sqrt{3}.$

65. $\sec^2 x + \cot^2 x = \frac{13}{3}.$

66. $\sin 2x = \sqrt{3}\cos x.$

67. $2\sin x \sin 3x - \sin^2 2x = 0.$

68. $\sin 2\theta + \sin\theta = \cos 2\theta + \cos\theta.$

69. $\sin 2y + \sqrt{3}\cos 2y = 1.$

70. $\sin(60^\circ - x) - \sin(60^\circ + x) = \frac{1}{2}\sqrt{3}.$

71. Give the answers to Exs. 64–70, in the unlimited form.

72. If $2\cos^2 x - 7\cos x + 3 = 0$, show that there is only one value for $\cos x$.

73. Find the least possible positive value of θ which will satisfy the equation $2\sqrt{3}\cos^2\theta = \sin\theta$.

74. Solve $\sin x + \sin 2x + \sin 3x = 1 + \cos x + \cos 2x$.

75. If $\sin 3x + \sin 2x = \sin x$, find $\tan x$.

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76. Find the length of an arc intercepted by an angle of 2.2 radians at the center of a circle whose radius is 5 ft. How many degrees in this angle?

77. Two angles of a triangle are .5 and .4 radians. Find the third angle in radians and in degrees.

78. The sum of two angles is 2 radians, their difference is 10°. How many radians are there in each of these angles?

79. Prove $\cos\left(\frac{3\pi}{2} + x\right) - \cos\left(\frac{3\pi}{2} - x\right) = 2\sin x$.

80. Find the numerical value of $\frac{3}{2}\sin^2\frac{\pi}{6} + 4\cos^2\frac{5\pi}{4} - \frac{1}{3}\tan^2\frac{3\pi}{4}$.

81. If $\sin\left(x + \frac{\pi}{6}\right)\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$, find x.

82. Simplify $\tan\left(\frac{7\pi}{4} - x\right) + \tan\left(\frac{3\pi}{4} + x\right)$.

83. An angle of 30° at the center of a circle subtends an arc AB of length $\frac{\pi}{3}$ ft. Find the length of the perpendicular dropped from A on BC.

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84. Express each of the following angles in degrees:
$\sin^{-1}\frac{1}{2}$; $\cos^{-1}\frac{1}{2}\sqrt{2}$; $\tan^{-1}(-1)$; $\sin^{-1}(-1)$; $\cos^{-1}(-\frac{1}{2}\sqrt{3})$.

85. Find $\tan(\cot^{-1}\frac{1}{2})$.

86. Prove that $\tan^{-1}2 + \tan^{-1}\frac{1}{2} = \frac{\pi}{2}$.

87. Find the value of x, if $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$.

88. How many degrees in $\sin^{-1}(-\frac{1}{2}\sqrt{2})$? How many radians?

89. Prove $\sin^{-1}a = \sec^{-1}\frac{1}{\sqrt{1-a^2}}$.

90. Solve the following for x and y:

$$\sin^{-1} x + \sin^{-1} y = 120^\circ. \qquad \cos^{-1} x - \cos^{-1} y = 60^\circ.$$

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91. At a point 50 ft. from the base of a tower the angle of elevation of the top of the tower was found by the use of a transit instrument to be 68° 18′ [68.3°]. If the height of the instrument above the ground was 4.75 ft., what was the height of the tower?

92. If the railway up Pike's Peak rises 7552 ft. in $8\frac{3}{4}$ mi., what angle does the railway make with the horizon on the average?

93. Two towers are 240 and 80 ft. high, respectively. From the foot of the second the angle of elevation of the top of the first is 60°. Find, without the use of tables, the angle of elevation of the second from the foot of the first.

94. An unknown force combined with one of 128 lb. produces a resultant force of 200 lb. The resultant makes an angle of 18° 24′ [18.4°] with the known force. Find the magnitude of the unknown force and the angle which it makes with the known force.

95. A tree 82 ft. high stands at one corner of a garden which is in the form of an equilateral triangle. The distance from the top of the tree to the midpoint of the opposite side of the garden is 112 ft. Find a side of the garden.

96. If the earth's radius (3956 mi.) as viewed from the sun subtends an angle of 8.8″, find the distance of the earth from the sun.

97. In a circle whose radius is 13.7, find the area of a segment whose angle is $\frac{4\pi}{11}$ radians.

98. In order to determine the breadth of a river, a base line of 500 yd. was measured on one shore, and at each end of the base line the angle included between the base line and a line to a rod on the other bank was measured. These angles were found to be 53° and 79° 12′ [79.2°], respectively. What was the breadth of the river?

99. If a barn is 40×80 ft., and the pitch of the roof is 45°, find the length of the rafters and the area of the entire roof, the horizontal projection of the cornice being 1 ft.

100. If the sun's angle of elevation is 60°, what angle must a stick make with the horizontal in order that its shadow on a horizontal plane may be the largest possible.

101. If a railroad rises 1 ft. for every 1000 ft. of its length, what angle does it make with the horizontal?

102. In surveying a circular railroad curve successive chords of 100 ft. each are laid off. Find the radius of the curve, if the angle between two successive chords is 177°.

103. If the diagonal of a regular pentagon is 32.835, what is the radius of the circumscribed circle?

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104. The angle x is in the third quadrant and $\cos x = -\frac{3}{5}$; find the value of $\csc x$, $\tan x$, $\sin \frac{1}{2} x$, $\tan (180° - x)$, and $\sin - x$.

105. Find all the values of x between 0° and 360° which satisfy the equation $\sin (30° - x) = \cos (30° + x)$.

106. If x is an angle in the second quadrant, prove geometrically that $\tan (270° + x) = - \cot x$.

107. One angle of a rhombus is 60° and the opposite diagonal is 5 inches. Without the use of tables find the sides of the rhombus and its area.

108. Give a general formula for all angles whose sine is $\frac{1}{2}$. Is $-\frac{1}{2}$. Is -1.

109. Express $\cos 2x$ in terms of each of the functions of x.

110. Express $\cos A \cos B$ as a sum.

111. If $\cos A = h$, and $\tan A = k$, find the equation connecting h and k.

112. How many radians in each interior angle of a regular hexagon? In each exterior angle? How many degrees in each of these angles?

113. Prove that $\cos^{-1} \frac{63}{65} + .2 \tan^{-1} \frac{1}{5} = \sin^{-1} \frac{3}{5}$.

114. If $\sin x = \dfrac{2}{3}$, find $\dfrac{\tan^2 x + \cos^2 x}{\tan^2 x - \cos^2 x}$.

115. In the isosceles right triangle ABC, D is the midpoint of AC. Prove without the use of tables that $\cot \angle ABD : \cot \angle DBC = 2 : 3$.

116. If θ lies between 180° and 270°, and $3 \tan \theta = 4$, find the value of $2 \cot \theta = - 5 \cos \theta + \sin \theta$.

117. Is it possible to have an angle whose tan is 503? Whose cos is $\frac{4}{3}$? Whose secant is $\frac{1}{3}$? Whose sine is 23?

118. Show that $\cos 80° + \cos 40° - \cos 20° = 0$.

119. That $2 \sin \left(x + \dfrac{\pi}{4}\right) \sin \left(x - \dfrac{\pi}{4}\right) = \sin^2 x - \cos^2 x$.

120. If $\sin(60° - x) - \sin(60° + x) = \frac{1}{2}\sqrt{3}$, find $\tan 2x$.

121. Express $2 \sin 9A \sin A$ in the form of a sum or difference.

122. Find the value of $\sin^{-1}\frac{1}{2} + 3\tan^{-1}\frac{1}{3}\sqrt{3} - 2\cot^{-1}1 + \sec^{-1}1$, using values between 0° and 90°.

123. If $\tan 2x = \frac{24}{7}$, find $\tan x$ and $\sin x$, it being given that x is an angle in the third quadrant.

124. Find by inspection one value of x when

$$\cos(10° + A)\cos(10° - A) + \sin(10° + A)\sin(10° - A) = \cos x.$$

125. A surveyor standing on a bank of a river observes the angle subtended by a flagpole on the opposite bank to be 33° 10′ [33.17°] and when he retires 120 ft. from the bank he finds the angle to be 18° 16′ [18.27°]. Find the width of the river.

126. Develop $\cos(270° - x - y)$ in the shortest way.

127. What is the angle of elevation of the sun when the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole?

128. If $\tan A = \frac{3}{4}$ and $\sin B = \frac{12}{13}$, and A is in the third quadrant and B in the second, find $\sin(A + B)$, $\cos(A + B)$, $\tan(A + B)$.

129. At the Panama Canal the Gatun dam has three different slopes: the ratio of the horizontal to the vertical near the base is 16 to 1; in the middle of the dam this ratio is 8 to 1; and at the top the ratio is 4 : 1. What three different angles does the surface of the dam make with the horizontal?

130. If A is an angle in the first quadrant, and $\sec^2 A \csc^2 A - 4 = 0$, find the numerical value of $\cot A$.

131. If θ is an angle in the third quadrant, and $\sec^2\theta = 2 + 2\tan\theta$, find $\sin\theta$.

132. Find all the values of x between 0° and 500° which satisfy the equation $\tan(45° - x) + \cot(45° - x) = 4$.

133. Graph $y = \sin^{-1}x$. **134.** Also, $y = \tan^{-1}x$.

135. From the top of a mountain 3 mi. high, the angle of depression of the horizon is 2° 13′ 50″ [2.23°]. Hence determine the diameter of the earth.

136. Can an angle exist such that $9\sin 2x + 3\sin x = 20$? Why?

137. Find the numerical value of $\tan^2\frac{2\pi}{3} + \cos^2\frac{7\pi}{4} + \sin^2\frac{\pi}{6}$.

138. Find the sines of all angles less than 2π whose tangents are equal to $\cos 135°$.

139. Given $\cos\left(\frac{\pi}{2}+x\right)=a$, find $\cot\left(\frac{3\pi}{2}+x\right)$.

140. What is the most general value of x which satisfies both of the equations $\cot x = -\sqrt{3}$ and $\csc x = -2$.

141. Show that $2\sin\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}+B\right)=\cos(A+B)+\sin(A-B)$.

142. Find the length of a circular arc whose radius is 5 ft. and whose subtending angle is 3 units of circular measure.

143. In the triangle ABC, B is 45°, and C is 120°, and a is 40. Without the use of tables find the length of the perpendicular drawn from A to BC produced.

144. Prove that $\dfrac{\sin x+\sin 2x}{1+\cos x+\cos 2x}=\tan x$.

145. When $y=\frac{11\pi}{4}$, find the numerical value of

$$\sin^2 y-\cos^2 y+2\tan y-\sec^2 y.$$

146. Prove the identity $\sin^{-1}y+\tan^{-1}y=\sin^{-1}\dfrac{y(1+\sqrt{1-y^2})}{\sqrt{1+y^2}}$.

147. Is $\sin x-2\cos x+3\sin x-6=0$ a possible equation?

148. A vertical pole stands at the center of a circular millpond and rises 100 ft. above the surface of the water. From a point on the shore the angle of elevation of the top of the pole is 20°. Find the area of the pond.

149. When the planet Venus is most brilliant, its diameter subtends an angle of 40″ as seen from the earth. If the diameter of the planet is 7600 mi., what is the distance of the planet from the earth at such a time?

150. Verify the statement

$$\frac{4}{3}\cot^2\frac{\pi}{6}+3\sin^2\frac{\pi}{3}-2\csc^2\frac{\pi}{3}-\frac{3}{4}\tan^2\frac{\pi}{6}=\frac{10}{3}.$$

151. Find the value of $\sin x$, if $\tan\left(\frac{\pi}{3}+x\right)\tan\left(\frac{\pi}{3}-x\right)+2=0$.

152. What sign has $\sin x\cos x$ for the following values of x: 140°, 278°, −356°, −1125°?

153. If $1+\sin^2 x=3\sin x\cos x$, find $\tan x$.

154. If i denotes the angle of incidence of a ray of light falling on water, and r the angle of refraction, and $\dfrac{\sin i}{\sin r}=1.423$, find r when $i=34.37°$.

155. When is $\sin x = \frac{a^2+b^2}{2ab}$ possible, and when impossible?

156. Show that

$$\sin(2\alpha - \beta)\cos(\alpha - 2\beta) - \cos(2\alpha - \beta)\sin(\alpha - 2\beta) = \sin(\alpha + \beta).$$

157. Solve $\sin 2x - \cos 2x - \sin x + \cos x = 0$.

158. Solve $x = \sin^{-1}\frac{1}{2} + \tan^{-1} 1$.

159. Trace the changes in sign and magnitude of $\frac{\sin 3\theta}{\cos 2\theta}$ as x increases from 0 to $\frac{\pi}{2}$.

160. Two trains leave a railroad crossing at the same time on straight tracks, including an angle of $21° 12'$ $(21.2°)$. If they travel at the rate of 40 and 50 mi. per hour respectively, how far apart will they be in 45 min.?

161. Show that $\frac{\cos 2B + \cos 2A}{\cos 2B - \cos 2A} = \cot(A+B)\cot(A-B)$.

162. In a right triangle show that $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{2\sin A}{\sqrt{\cos 2B}}$.

163. Prove $$\frac{\tan\left(\frac{\pi}{4} + \frac{1}{2}A\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}A\right)}{\tan\left(\frac{\pi}{4} + \frac{1}{2}A\right) - \tan\left(\frac{\pi}{4} - \frac{1}{2}A\right)} = \csc A.$$

164. In any triangle prove that $c = a\cos B + b\cos A$, and hence show that $\sin(A+B) = \sin A\cos B + \cos A \sin B$.

165. Determine the angles in a right triangle in which $a > b$, and $c - a = a - b$.

166. Prove $\cos^2(x-y) - 2\cos(x-y)\cos x \cos y = \sin^2 x - \cos^2 y$.

167. If $\sin x - \cos x + 4\cos^2 x = 2$, find the ratio of $\tan x$ to $\sec x$.

168. If $A + B = 225°$, prove that $\left(\frac{\cot A}{1+\cot A}\right)\left(\frac{\cot B}{1+\cot B}\right) = \frac{1}{2}$.

169. The shadow of a tower is found to be 60 ft. larger when the sun's altitude is 30° than when it is 45°. Find the height of the tower without the use of tables.

170. A workman is told to make a triangular enclosure having 50, 41, and 21 yd. as its sides. If he makes the first side one yard too long, of what length must he make the other two sides in order to inclose the required area, and keep the perimeter of the triangle unchanged?

171. If $\sin A$ is a geometric mean between $\sin B$ and $\cos B$, prove $\cos 2A = 2\sin(45° - B)\cos(45° + B)$.

172. If the diameter of the earth's orbit about the sun is 186,000,000 miles, and this diameter when viewed from the nearest fixed star subtends an angle of 1.52″, find the distance of the star from the earth.

173. In a circle whose radius is 111.3 find the area inclosed between two parallel chords, on the same side of the center whose lengths are 129.3 and 97.4.

174. If $2\tan^{-1}x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}$, find x.

175. If $\tan^2(180° - x) - \sec(180° + x) = 5$, find $\cos x$.

176. In order to fix the distance between two islands C and D, a base line, AB, 900 ft. long, is measured on the shore. Also, $\angle BAC$ was found to be 110° 50′ [110.83°], $\angle DAB$, 67° 51′ [67.85°], $\angle CBA$, 49° 51′ [49.85°], $\angle ABD$, 85° 19′ [85.32°]. What was the distance between the islands?

LOGARITHMIC AND TRIGONOMETRIC TABLES

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CONTENTS

INTRODUCTION TO TABLES

1. Number of Decimal Places in Tables. All trigonometric work is based on the results of measurements. But no measurement is accurate beyond the sixth or seventh figure; this is owing to the limitations of our eyesight and sense of touch-perception, and to the ultimate imperfections in all our instruments of measurement.

Thus a mile (63,360 inches) can be measured to within $\frac{1}{10}$ inch of its true length; an inch can be measured only to within a millionth part of itself, etc. So great a degree of accuracy, however, can be obtained only by applying every possible refinement of accuracy. Ordinary measuring, such, for instance, as that done by a carpenter, is accurate only to the second or third figure, that is, to within $\frac{1}{100}$ or $\frac{1}{1000}$ part. Hence it would be absurd for a carpenter or surveyor to use a number like 7.382654 ft.; 7.38^{+} ft. is sufficient.

In 6,543,786, if the figure 6 to the right is $\frac{1}{8}$ inch long, how long would the figure 6 on the left be if its length were made proportional to its value?

Hence *four-place tables are sufficiently accurate for all ordinary work* (*such as is done by a land surveyor, or in a physical laboratory under ordinary circumstances*). *Five-place tables give all the accuracy required except in very rare cases*, when six- or seven-place tables may be used. But the latter cases are beyond the scope of this book.

TABLE I. FIVE-PLACE LOGARITHMS OF NUMBERS 1-10,000 (pp. 21-39)

2. General Description of Table I. Table I consists of two parts. Part I occupies p. 21 and gives the logarithms (both characteristic and mantissa) of numbers 1–100. Part II occupies pp. 22–39, contains mantissas only, and gives these for all numbers from 1 to 10,000.

In using Part II the characteristic of each logarithm must be determined and supplied in accordance with the methods stated in Arts. 4 and 5 of Durell's Plane Trigonometry.

Direct Use of Table I

3. To find the mantissa for a number containing four figures. In the given table the left-hand column (headed N) is a column of ordinary numbers. The first three figures of the given number whose mantissa is sought are found in this column. In the top row of each page are the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The fourth figure of the given number is found here.

Hence, to obtain the mantissa of 3647, for instance, we take 364 in the first column on page 27 and look along the row beginning with 364 till we come to the column headed 7. The mantissa thus obtained is .56194.

The first two figures of the row of mantissas, viz. 56, are supposed to be repeated in connection with each mantissa that follows till another complete mantissa is given. The use of a * indicates that the first two figures of the mantissa are to be taken from the beginning of the line of mantissas which follows.

Thus, the mantissa of 1125 is .05115, not .04115.

If the number whose mantissa is sought contains less than four figures, in using the tables we regard enough zeros as annexed to the given figures to make up four figures. In Chapter I of Durell's Plane Trigonometry it is shown that doing this does not affect the mantissa.

Thus, to find the mantissa of 271, we find the mantissa of 2710, viz. .43297.

Similarly the mantissa of 7 is the same as that of 7000, viz. .84510.

4. To find the mantissa of a number containing five or six figures. Interpolation. The method consists in finding the mantissa for the first four figures and adding a correction for

the fifth, or for the fifth and sixth figures. This correction is computed on the assumption that the differences in logarithms are proportional to the differences in the numbers to which they belong. Though this proportion is not strictly accurate, it is sufficiently accurate for practical purposes.

Ex. Find the mantissa of 1581.47.

m. for 1582 = .19921	Mantissa of 1581 = .19893
m. for 1581 = .19893	.00028 × .47 = .00013
Diff. for 1 = .00028	Mantissa of 1581.47 = .19906, *Ans.*

For since an increase of 1 in the number makes an increase of .00028 in the mantissa, an increase of .47 in the number will make an increase of .47 of .00028, that is, of .00013 in the logarithm.

As in the mantissa, so in the correction only five places of figures may be used. If the figure in the sixth place of the correction is 5 or a larger number, the figure in the fifth place of the correction is to be increased by 1; if less than 5, the figures after the fifth place are to be rejected. Thus if the above correction had been .000135 it would have been treated as .00014. If it had been .0001346 it would have been treated as 0.00013.

The difference between the mantissas of two successive numbers is called the **tabular difference**.

Hence, in general, to find a mantissa for a number containing five or six figures:

Obtain from the table the mantissa for the first four figures, and also that for the next higher number, and subtract;

Multiply the difference between the two mantissas by the fifth figure (or fifth and sixth figures) expressed as a decimal, and **add** *the result to the mantissa for the first four figures.*

5. Hence, **to find the log of a given number**:

Determine the characteristic by Art. 4 *or* 5, *Chapter I;*

Neglect the decimal point (in the given number) and obtain from the table the mantissa for the given figures.

Ex. 1. Find log 3.62057.

m. of 3.621 = .55883	log 3.620 = 0.55871
m. of 3.620 = .55871	.00012 × .57 = .00007
.00012	log 3.62057 = 0.55878, *Ans.*

Ex. 2. Find log .078546.

m. of 7855 = .89515	log .07854 = 8.89509 − 10
m. of 7854 = .89509	.00006 × .6 = .00004
.00006	log .078546 = 8.89513 − 10, *Ans.*

For examples to be worked by the pupil, see the first part of Exercise 3 of Durell's Plane Trigonometry.

Inverse Use of Table I

6. To find an antilogarithm, that is, **to find the number corresponding to a given logarithm.**

Since the characteristic depends only on the position of the decimal point and not on the figures forming the given number, the characteristic is neglected at the outset of the process of finding the antilogarithm.

(*a*) If the given mantissa can be found in the table:

Take from the table the figures corresponding to the mantissa of the given logarithm;

Use the characteristic of the given logarithm to fix the decimal point in the number obtained from the table.

Ex. 1. Find the antilogarithm of 1.44138.

The figures corresponding to the mantissa .44138 are 2763. Since the characteristic is 1, there are two figures at the left of the decimal point.

Hence the antilog 1.44138 = 27.63.

Or, if log $x = 1.44138$, $x = 27.63$.

(*b*) In case the given mantissa does not occur in the table:

Obtain from the table the next lower mantissa with the corresponding four figures of the antilogarithm;

Subtract the tabular mantissa from the given mantissa;

Divide this difference by the difference between the tabular mantissa and the next higher mantissa in the table;

Annex *the quotient to the four figures of the antilogarithm obtained from the table;*

Use the characteristic to place the decimal point in the result.

Ex. 1. Find the antilog of 2.42376.

The mantissa .42376 does not occur in the table, and the next lower mantissa is .42374. The difference between .42376 and .42374 is .00002.

If a difference of 16 in the last two figures of the mantissa makes a difference of 1 in the fourth figure of the antilog, a difference of 2 in the last figure of the mantissa will make a difference of $\frac{2}{16}$ of 1 or .125 (or ·13) with respect to the fourth figure of the antilog. Hence we have

$$\text{antilog } 2.42376 = 265.313^{-} \quad \textit{Ans.}$$

$$\begin{array}{r} \underline{374} \\ 16)\overline{2.00}(.13^{-} \\ \underline{16} \\ 40 \end{array}$$

Ex. 2. If $\log x = 7.26323 - 10$, find x.

Nearest less mantissa $= .26316$, whose number is 1833. Tab. diff. $= 24$. $7 \div 24 = .29^{+}$. Hence $x = .00183329$, *Ans.*

The first part of Exercise 4 of Durell's Plane Trigonometry should be worked at this point.

TABLE II. LOGARITHMS AND COLOGARITHMS OF MUCH-USED NUMBERS (p. 40)

This table explains itself.

TABLE III. FIVE-PLACE LOGARITHMS OF TRIGONOMETRIC FUNCTIONS FOR EVERY MINUTE OF THE QUADRANT (pp. 41-86)

7. **Description of Table III.** This table gives the logarithms of the sine, cosine, tangent, and cotangent of each minute of angle from 0° up to 90°.

Where -10 is a part of the characteristic of the log function it is omitted for the sake of economy of space. This omission occurs at the end of the log function of each angle except for log tangents from 45° to 90°, and log cotangents from 0° to 45°.

For angles between 0 *and* 45°, the required functions are printed at the *top* of the columns, the number of degrees at the top of the page, and the number of minutes in the *left-hand column.*

For angles between 45° *and* 90°, the required function is printed at the *bottom* of the columns, the number of degrees at the *bottom* of the page, and the number of minutes in the *right-hand column.*

Thus,

log sin 26° 37′ = 9.65130 − 10 (p. 68). log tan 67° 48′ = 0.38924 (p. 64).
log sin 58° 16′ = 9.92968 − 10 (p. 73). log cot 12° 23′ = 0.65845 (p. 54).

Let the pupil determine why each column of the table has the name of a trigonometric function at the top and the name of the corresponding co-function at the bottom of the column.

Let him also determine why -10 is to be annexed at the end of some log trigonometric functions as taken from the tables, and not at the end of others.

Direct Use of Table III

8. Given the degrees, minutes, and seconds of an angle, to find a logarithmic trigonometric function of the angle. After finding the log function for the given number of degrees and minutes, the log function for the given number of degrees, minutes, and seconds is found by interpolation.

Ex. 1. Find the log sin 37° 42′ 53″.

The log sin 37° 42′ is 9.78642, and the difference between this and log sin 37° 43′ is 16.

Since an increase of 1′ in the angle makes an increase of 16 in the

last two places of the log sin, an increase of $53''$ or $\frac{53}{60}$ of $1'$ will make an increase of $\frac{53}{60}$ of 16 in the log of the function.

Hence we have

$$\begin{aligned} \log \sin 37° \, 42' &= 9.78642 - 10 \\ \text{Diff. for } 53'' = \tfrac{53}{60} \text{ of } 16 &= \underline{14} \\ \log \sin 37° \, 42' \, 53'' &= 9.78656 - 10 \end{aligned}$$

Ex. 2. Find the log sin $53° \, 27' \, 18''$.

$$\begin{aligned} \log \sin 53° \, 27' &= 9.90490 - 10 \\ \text{Diff. for } 18'' = \tfrac{18}{60} \text{ of } 9 &= \underline{3} \\ \log \sin 53° \, 27' \, 18'' &= 9.90493 - 10 \end{aligned}$$

Ex. 3. Find log cos $23° \, 48' \, 12''$.

Since the cosine of an angle decreases as the angle increases, the log of $23° \, 49'$ is less than the log cos $23° \, 48'$. Hence the correction for $12''$ must be subtracted from the log cos $23° \, 48'$.

$$\begin{aligned} \text{Thus } \log \cos 23° \, 48' &= 9.96140 - 10 \\ \text{Diff. for } 12'' = \tfrac{12}{60} \text{ of } 5 &= \underline{1} \\ \log \cos 23° \, 48' \, 12'' &= 9.96139 - 10 \end{aligned}$$

Ex. 4. Find log cot $57° \, 18' \, 43''$.

$$\begin{aligned} \log \cot 57° \, 18' &= 9.80753 - 10 \\ \text{Diff. for } 43'' = 28 \times \tfrac{43}{60} &= \underline{20} \\ \log \cot 57° \, 18' \, 43'' &= 9.80733 - 10 \end{aligned}$$

Hence, in general,

Obtain from the table the log function for the given number of degrees and minutes;

Also obtain from the table the log function for the angle, 1 *minute greater; find the difference between these two log functions; multiply this difference by* $\frac{\textit{no. seconds}}{60}$*; this will give the correction for seconds;*

Add the correction for seconds in case of sine and tangent (direct functions);

Subtract the correction in case of cosine and cotangent (complementary functions).

9. Log Secants. To find the log secant of an angle, use the formula $\sec x = \dfrac{1}{\cos x}$. $\therefore \log \sec x = 0 + \operatorname{colog} \cos x$.

Thus $\log \sec 39° \, 28' \, 23'' = \operatorname{colog} \cos 39° \, 28' \, 23''$.
But $\log \cos 39° \, 28' \, 23'' = 9.88757 - 10$.
$\operatorname{colog} \cos 39° \, 28' \, 23''$ or $\log \sec 39° \, 28' \, 23'' = 0.11243$.

10. Log Functions of Angles greater than 90°. By the methods of Chapter IV, a trigonometric function of any angle greater than 90° can be reduced to a trigonometric function of an angle less than 90°.

Thus, since $\sin A = \sin (180° - A)$,
$\sin 113° \, 27' = \sin 66° \, 33'$.
$\therefore \log \sin 113° \, 27' = \log \sin 66° \, 33' = 9.96256 - 10$.

Also $\cos A = - \cos (180° - A)$.
Hence, $\log \cos A = \log \cos (180° - A)(n)$, the small n being annexed to show that the function whose log is being used is a negative quantity.

Thus $\log \cos 142° \, 18' = \log \cos 37° \, 42' \, (n) = 9.78642 - 10 \, (n)$.

At this point work the first part of Exercise 14 of Durell's Plane Trigonometry.

Inverse Use of Table III

11. Given the logarithm of a function to find the corresponding acute angle (*or find antilog sin, antilog cos, etc. or* $\angle$*log sin,* $\angle$*log cos, etc.*) Obtain from the table, if possible, the number of degrees and minutes corresponding to the given logarithmic function.

Ex. If $\log \tan A = 9.92535 - 10$, find the angle A.

By consulting the table, tangent column, we find that $A = 40° \, 6'$. Or antilog tan $9.92535 - 10 = 40° \, 6'$.

If the given logarithmic function does not occur in the table;

Obtain from the table the next less logarithm of the same function, noting the corresponding number of degrees and minutes; subtract this logarithm from the given logarithm;

Divide the difference so obtained by the tabular difference for 1′ *and multiply by* 60″*; the result will be the correction, in seconds, to be added in case of sine and tangent, and subtracted in case of cosine and cotangent, to the angle already noted.*

Ex. 1. Find antilog sin 9.78538 – 10.

$$\begin{array}{rl} \angle \log \sin 9.78538 - 10 = & 37° \; 35'^{+} \\ \underline{9.78527 - 10} & \\ 11 & \end{array}$$

Since a difference of 16 in the log makes a difference of 1′ (or of 60″) in the angle, a difference of 11 in the log makes a difference of $\frac{11}{16}$ of 60″, or 41″, in the angle.

$$\therefore \text{antilog} \sin 9.78538 - 10 = 37° \; 35' \; 41'', \textit{Ans.}$$

Ex. 2. Find antilog cos 9.96623 – 10.

$$\begin{array}{rl} \text{antilog} \cos 9.96623 - 10 = & 22° \; 19'^{-} \\ \underline{9.96619 - 10} & \\ \frac{4}{5} \text{ of } 60'' = 48'' & \end{array}$$

$$\text{antilog} \cos 9.96623 - 10 = 22° \; 18' \; 12'', \textit{Ans.}$$

Ex. 3. Find antilog cot 0.57603.

$$\begin{array}{rl} \text{antilog} \cot 0.57603 = & 14° \; 52'^{-} \\ \underline{0.57601} & \\ \frac{2}{51} \text{ of } 60'' = 2'' & \end{array}$$

$$\text{antilog} \cot 0.57603 = 14° \; 51' \; 58'', \textit{Ans.}$$

Ex. 4. Find antilog cos 9.60172 – 10.

$$\begin{array}{rl} \text{antilog} \cos 9.60172 - 10 = & 66° \; 27'^{-} \\ \underline{9.60157 - 10} & \\ \frac{15}{29} \text{ of } 60'' = 31'', & \end{array}$$

$$\text{antilog} \cos 9.60172 - 10 = 66° \; 26' \; 29'', \textit{Ans.}$$

At this point work the first part of Exercise 15 of Durell's Trigonometry.

TABLE IV. AUXILIARY FIVE—PLACE TABLE FOR SMALL ANGLES (pp. 87–89)

12. The Auxiliary Table of Logarithms of Sine and Tangent for Small Angles is needed because when an angle is smaller than 2°, the logarithms of the sine and tangent vary so rapidly that ordinary methods of interpolation are not sufficiently accurate. (The same is true for the cosine, cotangent, and tangent when the angle is between 88° and 90°, but there are other indirect methods of meeting such cases.)

Table IV is based on Art. 115 of Plane Trigonometry, where it is shown that the sine (or tangent) of a small angle is approximately the same in value as the number of radians in the angle. Hence, for example, to find sine $1° 21' 37''$, we divide the number of seconds in $1° 21' 37''$ by the number of seconds in a radian, viz. 206,265. This process is facilitated by Table IV. The column headed $''$ in this table gives the number of seconds in each angle containing an exact number of minutes, and hence is an aid in converting any given angle into seconds.

In the column headed S' is given the log of 206,265 (viz. 5.31443), modified by a slight correction owing to the change in the slight differences between the sine of a small angle and the radian measure of that angle. Similarly the column headed T'' gives log of 206,265 in use of the tangent. (The columns headed S and T give the cologs corresponding to the S' and T'' columns.) The column headed log sin gives the log sin or final answer for each even minute, these numbers being needed also in guiding the work in the inverse use of the table. Hence —

13. To find the log sin or tangent of an angle less than 2°.

Find the number of seconds in the given angle and find the log of this number in Table I;

Add to this log the corresponding log in column S or T according as the log sin or log tan is desired.

Ex. Find log sin 1° 26′ 13″.

$$\begin{aligned} 1^\circ\ 26'\ 13'' &= 5173'' \\ \log 5173 &= 3.71374 \\ S \text{ (or colog } 206265) &= \underline{4.68553 - 10} \\ \therefore \log 1^\circ\ 26'\ 13'' &= 8.39927 - 10, \textit{ Ans.} \end{aligned}$$

14. To find the angle corresponding to a given log sine or log tangent (less than 8.54282 − 10).

Look up in the L. Sin column the number nearest in size to the given log; and set down the number on the same row with this in column S′ or T′, according as the given function is a sine or tangent;

Add the given log function to the number set down from the table;

Find the antilog of the result; this will be the number of seconds in the required angle.

Ex. Find antilog tan 8.39307.

In L. Sin column, the nearest number is 8.39310.

Corresponding to this is $T' = 5.31434$

$$\begin{aligned} \text{Given tan} &= \underline{8.39307} \\ \text{antilog } 13.70741 &= 5098'' \\ &= 1^\circ\ 24'\ 58'', \textit{ Ans.} \end{aligned}$$

The reason for the above process is seen from the fact that

$$\text{sin of required } \angle = \frac{5098''}{206265''}.$$

$$\therefore 206265 \times (\text{sin of required } \angle) = 5098''.$$

$$\therefore \log 206265 + 8.39307 = \log 5098''.$$

15. Other Uses of the Auxiliary Table IV. The log cosine of an angle between 88° and 90° changes so rapidly as to make direct interpolation inaccurate. In such cases use the formula

$$\cos A = \sin (90° - A).$$

Thus, for example, log cos 88° 47′ = log sin 1° 13′, and the value of log sin 1° 13′ can be obtained by Art. 14.

The log cot A, when A is between 88° and 90°, may be obtained similarly.

Also, if A is an angle between 88° and 90°, the log tan A changes so rapidly that interpolation is inaccurate.

In this case use $\tan A = \dfrac{1}{\cot A}$.

$$\log \tan A = \text{colog} \cot A = \text{colog} \tan (90° - A).$$

Thus, for example, log tan 88° 47′ = colog tan 1° 13′, etc.

At this point work the first part of Exercise 16 of Durell's Trigonometry.

TABLE V. FOUR-PLACE TABLE OF THE NATURAL SINE, COSINE, TANGENT, AND COTANGENT FOR EVERY TEN MINUTES OF THE QUADRANT (pp. 91–96)

16. Method of using Table V.

By natural trigonometric functions are meant the actual numerical (not logarithmic) values of these functions. Thus $\frac{1}{2}$ is the natural sine of 30°. Interpolation for this table is made in the same general way as for Table V.

Ex. Find natural sine 27° 48′.

$$\begin{aligned} \text{N. Sine } 27° \, 40' &= 0.4643 \\ \tfrac{8}{10} \text{ of } 26 &= \underline{21} \\ \text{N. Sine } 27° \, 48' &= 0.4664, \textit{ Ans.} \end{aligned}$$

TABLE VI. FOUR-PLACE TABLE OF LOGARITHMS OF NUMBERS 1–2000 (pp. 97–101)

17. Method of using Table VI.

In using the four-place log of a number, when the first significant figure of the number is 1, *use pp.* 100–101; *otherwise use pp.* 98–99.

In finding the antilog of a four-place log, if the given log is less than .3010, *use pp.* 100–101; *otherwise use pp.* 98–99.

At this point work the latter part of Exercises 3 and 4 of Durell's Plane Trigonometry.

TABLE VII. FOUR-PLACE LOGARITHMIC TABLE OF THE TRIGONOMETRIC FUNCTIONS FOR ANGLES OF THE QUADRANT EXPRESSED IN DECIMALLY DIVIDED DEGREES (pp. 103–113)

18. Method of using Table VII. The explanation of the methods of using Table III given in Arts. 8–11 of this Introduction apply in general to the use of Table VII.

Hence we need only illustrate by examples the application of these methods to the table in hand.

Ex. 1. Find log sin 48.34°.

$$\begin{array}{rl} \log\sin 48.4^\circ = & 9.8738 - 10 \\ \log\sin 48.3^\circ = & \underline{9.8731 - 10} \\ & 7 \end{array} \qquad \begin{array}{rl} \log\sin 48.3^\circ = & 9.8731 - 10 \\ \frac{4}{10} \text{ of } 7 = & \underline{\quad 3 \qquad\quad} \\ \log\sin 48.34^\circ = & 9.8734 - 10, \textit{ Ans.} \end{array}$$

Ex. 2. Find the antilog tan 0.2165.

$$\begin{array}{l} \angle \log\tan 0.2165 = 58.7^{\circ+} \\ \quad\underline{2161} \\ \qquad \frac{4}{17} \text{ of } 10 = 2^{+} \\ \angle \log\tan 0.2165 = 58.72^\circ, \textit{ Ans.} \end{array}$$

At this point work the latter part of Exercises 14 and 15 of Durell's Trigonometry.

19. Four-place Log Functions of Angles near 0° or 90°. As is explained in Art. 12 of this Introduction, when an angle is less than 2°, the logarithms of the sine and tangent vary so rapidly that ordinary methods of interpolation are not sufficiently accurate. To get an accurate log function in this case we use the result obtained in Art. 106 of Plane Trigonometry, viz: sine or tangent of a very small $\angle x$ = no. radians in $\angle x$, or $= \dfrac{\angle x \text{ in degrees}}{57.296^\circ}$.

$\therefore$ log sin (or tan) of small $\angle x = \log x + \text{colog } 57.296$
$= \log x + 8.2419 - 10.$

Also when x is small $\cot x = \dfrac{1}{\tan x} = \dfrac{57.296°}{x \text{ in degrees}}.$

$\therefore$ log cot small $\angle x = 1.7581 + \text{colog } x.$

Interpolation also is not accurate for log cos, log tan, log cot, of angles between 88° and 90°.

When A is an angle between 88° and 90° proceed as follows:

$\cos A = \sin (90° - A).$

$\therefore \log \cos A = \log \sin (90° - A) = 8.2419 - 10 + \log (90° - A).$

$\cot A = \tan (90° - A).$

$\therefore \log \cot A = \log \tan (90° - A) = 8.2419 - 10 + \log (90° - A).$

$\tan A = \dfrac{1}{\cot A}. \quad \therefore \log \tan A = 1.7581 - \log (90° - A).$

Ex. 1. Find sin 0.876°.

$$\begin{aligned} \log 0.876° &= 9.9425 - 10 \\ \text{colog } 57.296° &= \underline{8.2419 - 10} \\ \therefore \log \sin 0.876° &= 8.1844 - 10, \textit{ Ans.} \end{aligned}$$

Ex. 2. Find $\angle$ log sin $7.9592 - 10$.

$$\begin{aligned} &17.9592 - 20 \\ &\underline{\ \ 8.2419 - 10} \\ \text{antilog } &9.7173 - 10 = 0.522°^{-} \\ \therefore \angle \log \sin &7.9592 - 10 = 0.522°^{-}, \textit{ Ans.} \end{aligned}$$

At this point work the latter part of Exercise 16 of Durell's Trigonometry.

TABLE VIII. TABLE FOR CONVERTING MINUTES AND SECONDS INTO THE DECIMAL PART OF A DEGREE (p. 114)

20. The method of using Table VIII is evident from the form of the table, but it should be remembered that in each

decimal equivalent ending in a significant figure the last figure is supposed to repeat indefinitely.

Hence, for example, we have

$$\begin{aligned} 36^\circ\,46' &= 36.766^{\circ+} \\ &= 36.77^\circ \end{aligned}$$

Also

$$\begin{aligned} 35^\circ\,43' \quad\;\, &= 35.716^\circ \\ 20'' &= .006^\circ \\ \hline \therefore 35^\circ\,43'\,20'' &= 35.722^\circ \\ &= \;35.72^\circ,\ \textit{Ans.} \end{aligned}$$

TABLE IX. TABLE FOR CONVERTING THE DECIMAL PARTS OF A DEGREE INTO MINUTES AND SECONDS (p. 114)

21. **The method of using Table IX** is also evident from the table itself.

TABLE I

COMMON LOGARITHMS

OF NUMBERS

PART I

LOGARITHMS (WITH CHARACTERISTICS) OF NUMBERS 1–100

N.	Log.	N.	Log.	N.	Log.	N.	Log.
0	— Infinity	**30**	1.47 712	**60**	1.77 815	**90**	1.95 424
1	0.00 000	31	1.49 136	61	1.78 533	91	1.95 904
2	0.30 103	32	1.50 51$\bar{5}$	62	1.79 239	92	1.96 379
3	0.47 712	33	1.51 851	63	1.79 934	93	1.96 848
4	0.60 206	34	1.53 148	64	1.80 618	94	1.97 313
5	0.69 897	35	1.54 407	65	1.81 291	95	1.97 772
6	0.77 815	36	1.55 630	66	1.81 954	96	1.98 227
7	0.84 510	37	1.56 820	67	1.82 607	97	1.98 677
8	0.90 309	38	1.57 978	68	1.83 251	98	1.99 123
9	0.95 424	39	1.59 106	69	1.83 88$\bar{5}$	99	1.99 564
10	1.00 000	**40**	1.60 206	**70**	1.84 510	**100**	2.00 000
11	1.04 139	41	1.61 278	71	1.85 126		
12	1.07 918	42	1.62 32$\bar{5}$	72	1.85 733		
13	1.11 394	43	1.63 347	73	1.86 332		
14	1.14 613	44	1.64 345	74	1.86 923		
15	1.17 609	45	1.65 321	75	1.87 506		
16	1.20 412	46	1.66 276	76	1.88 081		
17	1.23 04$\bar{5}$	47	1.67 210	77	1.88 649		
18	1.25 527	48	1.68 124	78	1.89 209		
19	1.27 875	49	1.69 020	79	1.89 763		
20	1.30 103	**50**	1.69 897	**80**	1.90 309		
21	1.32 222	51	1.70 757	81	1.90 849		
22	1.34 242	52	1.71 600	82	1.91 381		
23	1.36 173	53	1.72 428	83	1.91 908		
24	1.38 021	54	1.73 239	84	1.92 428		
25	1.39 794	55	1.74 036	85	1.92 942		
26	1.41 497	56	1.74 819	86	1.93 4$\bar{5}$0		
27	1.43 136	57	1.75 587	87	1.93 952		
28	1.44 716	58	1.76 343	88	1.94 448		
29	1.46 240	59	1.77 085	89	1.94 939		
30	1.47 712	**60**	1.77 815	**90**	1.95 424		

Part II

Mantissas of Numbers 1–10,000

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
01	432	475	518	561	604	647	689	732	775	817
02	860	903	945	988	*030	*072	*115	*157	*199	*242
03	01 284	326	368	410	452	494	536	578	620	662
04	703	745	787	828	870	912	953	995	*036	*078
05	02 119	160	202	243	284	325	366	407	449	490
06	531	572	612	653	694	735	776	816	857	898
07	938	979	*019	*060	*100	*141	*181	*222	*262	*302
08	03 342	383	423	463	503	543	583	623	663	703
09	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
11	532	571	610	650	689	727	766	805	844	883
12	922	961	999	*038	*077	*115	*154	*192	*231	*269
13	05 308	346	385	423	461	500	538	576	614	652
14	690	729	767	805	843	881	918	956	994	*032
15	06 070	108	145	183	221	258	296	333	371	408
16	446	483	521	558	595	633	670	707	744	781
17	819	856	893	930	967	*004	*041	*078	*115	*151
18	07 188	225	262	298	335	372	408	445	482	518
19	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
21	08 279	314	350	386	422	458	493	529	565	600
22	636	672	707	743	778	814	849	884	920	955
23	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
24	09 342	377	412	447	482	517	552	587	621	656
25	691	726	760	795	830	864	899	934	968	*003
26	10 037	072	106	140	175	209	243	278	312	346
27	380	415	449	483	517	551	585	619	653	687
28	721	755	789	823	857	890	924	958	992	*025
29	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
31	727	760	793	826	860	893	926	959	992	*024
32	12 057	090	123	156	189	222	254	287	320	352
33	385	418	450	483	516	548	581	613	646	678
34	710	743	775	808	840	872	905	937	969	*001
35	13 033	066	098	130	162	194	226	258	290	322
36	354	386	418	450	481	513	545	577	609	640
37	672	704	735	767	799	830	862	893	925	956
38	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
39	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
41	922	953	983	*014	*045	*076	*106	*137	*168	*198
42	15 229	259	290	320	351	381	412	442	473	503
43	534	564	594	625	655	685	715	746	776	806
44	836	866	897	927	957	[illegible]	*017	*047	*077	*107
45	16 137	167	197	227	256	[illegible]	[illegible]	346	376	406
46	435	465	495	524	554	[illegible]	[illegible]	643	673	702
47	732	761	791	820	850	879	909	938	967	997
48	17 026	056	085	114	143	173	202	231	260	289
49	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
51	898	926	955	984	*013	*041	*070	*099	*127	*156
52	18 184	213	241	270	298	327	355	384	412	441
53	469	498	526	554	583	611	639	667	696	724
54	752	780	808	837	865	893	921	949	977	*005
55	19 033	061	089	117	145	173	201	229	257	285
56	312	340	368	396	424	451	479	507	535	562
57	590	618	645	673	700	728	756	783	811	838
58	866	893	921	948	976	*003	*030	*058	*085	*112
59	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
61	683	710	737	763	790	817	844	871	898	925
62	952	978	*005	*032	*059	*085	*112	*139	*165	*192
63	21 219	245	272	299	325	352	378	405	431	458
64	484	511	537	564	590	617	643	669	696	722
65	748	775	801	827	854	880	906	932	958	985
66	22 011	037	063	089	115	141	167	194	220	246
67	272	298	324	350	376	401	427	453	479	505
68	531	557	583	608	634	660	686	712	737	763
69	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
71	300	325	350	376	401	426	452	477	502	528
72	553	578	603	629	654	679	704	729	754	779
73	805	830	855	880	905	930	955	980	*005	*030
74	24 055	080	105	130	155	180	204	229	254	279
75	304	329	353	378	403	428	452	477	502	527
76	551	576	601	625	650	674	699	724	748	773
77	797	822	846	871	895	920	944	969	993	*018
78	25 042	066	091	115	139	164	188	212	237	261
79	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
81	768	792	816	840	864	888	912	935	959	983
82	26 007	031	055	079	102	126	150	174	198	221
83	245	269	293	316	340	364	387	411	435	458
84	482	505	529	553	576	600	623	647	670	694
85	717	741	764	788	811	834	858	881	905	928
86	951	975	998	*021	*045	*068	*091	*114	*138	*161
87	27 184	207	231	254	277	300	323	346	370	393
88	416	439	462	485	508	531	554	577	600	623
89	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
91	28 103	126	149	171	194	217	240	262	285	307
92	330	353	375	398	421	443	466	488	511	533
93	556	578	601	623	646	668	691	713	735	758
94	780	803	825	847	870	892	914	937	959	981
95	29 003	026	048	070	092	115	137	159	181	203
96	226	248	270	292	314	336	358	380	403	425
97	447	469	491	513	535	557	579	601	623	645
98	667	688	710	732	754	776	798	820	842	863
99	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	0	1	2	3	4	5	6	7	8	9

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
01	320	341	363	384	406	428	449	471	492	514
02	535	557	578	600	621	643	664	685	707	728
03	750	771	792	814	835	856	878	899	920	942
04	963	984	*006	*027	*048	*069	*091	*112	*133	*154
05	31 175	197	218	239	260	281	302	323	345	366
06	387	408	429	450	471	492	513	534	555	576
07	597	618	639	660	681	702	723	744	765	785
08	806	827	848	869	890	911	931	952	973	994
09	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
11	428	449	469	490	510	531	552	572	593	613
12	634	654	675	695	715	736	756	777	797	818
13	838	858	879	899	919	940	960	980	*001	*021
14	33 041	062	082	102	122	143	163	183	203	224
15	244	264	284	304	325	345	365	385	405	425
16	445	465	486	506	526	546	566	586	606	626
17	646	666	686	706	726	746	766	786	806	826
18	846	866	885	905	925	945	965	985	*005	*025
19	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
21	439	459	479	498	518	537	557	577	596	616
22	635	655	674	694	713	733	753	772	792	811
23	830	850	869	889	908	928	947	967	986	*005
24	35 025	044	064	083	102	122	141	160	180	199
25	218	238	257	276	295	315	334	353	372	392
26	411	430	449	468	488	507	526	545	564	583
27	603	622	641	660	679	698	717	736	755	774
28	793	813	832	851	870	889	908	927	946	965
29	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
31	361	380	399	418	436	455	474	493	511	530
32	549	568	586	605	624	642	661	680	698	717
33	736	754	773	791	810	829	847	866	884	903
34	922	940	959	977	996	*014	*033	*051	*070	*088
35	37 107	125	144	162	181	199	218	236	254	273
36	291	310	328	346	365	383	401	420	438	457
37	475	493	511	530	548	566	585	603	621	639
38	658	676	694	712	731	749	767	785	803	822
39	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
41	202	220	238	256	274	292	310	328	346	364
42	382	399	417	435	453	471	489	507	525	543
43	561	578	596	614	632	650	668	686	703	721
44	739	757	775	792	810	828	846	863	881	899
45	917	934	952	970	987	*005	*023	*041	*058	*076
46	39 094	111	129	146	164	182	199	217	235	252
47	270	287	305	322	340	358	375	393	410	428
48	445	463	480	498	515	533	550	568	585	602
49	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N.	0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
51	967	985	*002	*019	*037	*054	*071	*088	*106	*123
52	40 140	157	175	192	209	226	243	261	278	295
53	312	329	346	364	381	398	415	432	449	466
54	483	500	518	535	552	569	586	603	620	637
55	654	671	688	705	722	739	756	773	790	807
56	824	841	858	875	892	909	926	943	960	976
57	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
58	41 162	179	196	212	229	246	263	280	296	313
59	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
61	664	681	697	714	731	747	764	780	797	814
62	830	847	863	880	896	913	929	946	963	979
63	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
64	42 160	177	193	210	226	243	259	275	292	308
65	325	341	357	374	390	406	423	439	455	472
66	488	504	521	537	553	570	586	602	619	635
67	651	667	684	700	716	732	749	765	781	797
68	813	830	846	862	878	894	911	927	943	959
69	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
71	297	313	329	345	361	377	393	409	425	441
72	457	473	489	505	521	537	553	569	584	600
73	616	632	648	664	680	696	712	727	743	759
74	775	791	807	823	838	854	870	886	902	917
75	933	949	965	981	996	*012	*028	*044	*059	*075
76	44 091	107	122	138	154	170	185	201	217	232
77	248	264	279	295	311	326	342	358	373	389
78	404	420	436	451	467	483	498	514	529	545
79	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
81	871	886	902	917	932	948	963	979	994	*010
82	45 025	040	056	071	086	102	117	133	148	163
83	179	194	209	225	240	255	271	286	301	317
84	332	347	362	378	393	408	423	439	454	469
85	484	500	515	530	545	561	576	591	606	621
86	637	652	667	682	697	712	728	743	758	773
87	788	803	818	834	849	864	879	894	909	924
88	939	954	969	984	*000	*015	*030	*045	*060	*075
89	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
91	389	404	419	434	449	464	479	494	509	523
92	538	553	568	583	598	613	627	642	657	672
93	687	702	716	731	746	761	776	790	805	820
94	835	850	864	879	894	909	923	938	953	967
95	982	997	*012	*026	*041	*056	*070	*085	*100	*114
96	47 129	144	159	173	188	202	217	232	246	261
97	276	290	305	319	334	349	363	378	392	407
98	422	436	451	465	480	494	509	524	538	553
99	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	0	1	2	3	4	5	6	7	8	9

N.	0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
01	857	871	885	900	914	929	943	958	972	986
02	48 001	015	029	044	058	073	087	101	116	130
03	144	159	173	187	202	216	230	244	259	273
04	287	302	316	330	344	359	373	387	401	416
05	430	444	458	473	487	501	515	530	544	558
06	572	586	601	615	629	643	657	671	686	700
07	714	728	742	756	770	785	799	813	827	841
08	855	869	883	897	911	926	940	954	968	982
09	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
11	276	290	304	318	332	346	360	374	388	402
12	415	429	443	457	471	485	499	513	527	541
13	554	568	582	596	610	624	638	651	665	679
14	693	707	721	734	748	762	776	790	803	817
15	831	845	859	872	886	900	914	927	941	955
16	969	982	996	*010	*024	*037	*051	*065	*079	*092
17	50 106	120	133	147	161	174	188	202	215	229
18	243	256	270	284	297	311	325	338	352	365
19	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
21	651	664	678	691	705	718	732	745	759	772
22	786	799	813	826	840	853	866	880	893	907
23	920	934	947	961	974	987	*001	*014	*028	*041
24	51 055	068	081	095	108	121	135	148	162	175
25	188	202	215	228	242	255	268	282	295	308
26	322	335	348	362	375	388	402	415	428	441
27	455	468	481	495	508	521	534	548	561	574
28	587	601	614	627	640	654	667	680	693	706
29	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
31	983	996	*009	*022	*035	*048	*061	*075	*088	*101
32	52 114	127	140	153	166	179	192	205	218	231
33	244	257	270	284	297	310	323	336	349	362
34	375	388	401	414	427	440	453	466	479	492
35	504	517	530	543	556	569	582	595	608	621
36	634	647	660	673	686	699	711	724	737	750
37	763	776	789	802	815	827	840	853	866	879
38	892	905	917	930	943	956	969	982	994	*007
39	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	250	263
41	275	288	301	314	326	339	352	364	377	390
42	403	415	428	441	453	466	479	491	504	517
43	529	542	555	567	580	593	605	618	631	643
44	656	668	681	694	706	719	732	744	757	769
45	782	794	807	820	832	845	857	870	882	895
46	908	920	933	945	958	970	983	995	*008	*020
47	54 033	045	058	070	083	095	108	120	133	145
48	158	170	183	195	208	220	233	245	258	270
49	283	295	307	320	332	345	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518
N.	0	1	2	3	4	5	6	7	8	9

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55	55 023	035	047	060	072	084	096	108	121	133
56	145̄	157	169	182	194	206	218	230	242	255̄
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58	388	400	413	425̄	437	449	461	473	485	497
59	509	522	534	546	558	570	582	594	606	618
360	630	642	654	666	678	691	703	715̄	727	739
61	751	763	775̄	787	799	811	823	835̄	847	859
62	871	883	895̄	907	919	931	943	955̄	967	979
63	991	*003	*015̄	*027	*038	*050	*062	*074	*086	*098
64	56 110	122	134	146	158	170	182	194	205	217
65	229	241	253	265̄	277	289	301	312	324	336
66	348	360	372	384	396	407	419	431	443	455̄
67	467	478	490	502	514	526	538	549	561	573
68	585̄	597	608	620	632	644	656	667	679	691
69	703	714	726	738	750̄	761	773	785̄	797	808
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71	937	949	961	972	984	996	*008	*019	*031	*043
72	57 054	066	078	089	101	113	124	136	148	159
73	171	183	194	206	217	229	241	252	264	276
74	287	299	310	322	334	345	357	368	380	392
75	403	415̄	426	438	449	461	473	484	496	507
76	519	530	542	553	565̄	576	588	600	611	623
77	634	646	657	669	680	692	703	715̄	726	738
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380	978	990	*001	*013	*024	*035	*047	*058	*070	*081
81	58 092	104	115	127	138	149	161	172	184	195̄
82	206	218	229	240	252	263	274	286	297	309
83	320	331	343	354	365	377	388	399	410	422
84	433	444	456	467	478	490	501	512	524	535̄
85	546	557	569	580	591	602	614	625̄	636	647
86	659	670	681	692	704	715̄	726	737	749	760
87	771	782	794	805̄	816	827	838	850̄	861	872
88	883	894	906	917	928	939	950	961	973	984
89	995̄	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
91	218	229	240	251	262	273	284	295	306	318
92	329	340	351	362	373	384	395	406	417	428
93	439	450	461	472	483	494	506	517	528	539
94	550̄	561	572	583	594	605̄	616	627	638	649
95	660	671	682	693	704	715̄	726	737	748	759
96	770	780	791	802	813	824	835	846	857	868
97	879	890	901	912	923	934	945̄	956	966	977
98	988	999	*010	*021	*032	*043	*054	*065̄	*076	*086
99	60 097	108	119	130	141	152	163	173	184	195
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24	737	747	757	767	778	788	798	808	818	829
25	839	849	859	870	880	890	900	910	921	931
26	941	951	961	972	982	992	*002	*012	*022	*033
27	63 043	053	063	073	083	094	104	114	124	134
28	144	155	165	175	185	195	205	215	225	236
29	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
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34	749	759	769	779	789	799	809	819	829	839
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36	949	959	969	979	988	998	*008	*018	*028	*038
37	64 048	058	068	078	088	098	108	118	128	137
38	147	157	167	177	187	197	207	217	227	237
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41	444	454	464	473	483	493	503	513	523	532
42	542	552	562	572	582	591	601	611	621	631
43	640	650	660	670	680	689	699	709	719	729
44	738	748	758	768	777	787	797	807	816	826
45	836	846	856	865	875	885	895	904	914	924
46	933	943	953	963	972	982	992	*002	*011	*021
47	65 031	040	050	060	070	079	089	099	108	118
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54	706	715	72$\bar{5}$	734	744	753	763	772	782	792
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62	464	474	483	492	502	511	521	530	539	549
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64	652	661	671	680	689	699	708	717	727	736
65	745	75$\bar{5}$	764	773	783	792	801	811	820	829
66	839	848	857	867	876	885	894	904	913	922
67	932	941	950	960	969	978	987	997	*006	*015
68	67 02$\bar{5}$	034	043	052	062	071	080	089	099	108
69	117	127	136	145	154	164	173	182	191	201
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71	302	311	321	330	339	348	357	367	376	38$\bar{5}$
72	394	403	413	422	431	440	449	459	468	477
73	486	495	504	514	523	532	541	550	560	569
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76	761	770	779	788	797	806	815	82$\bar{5}$	834	843
77	852	861	870	879	888	897	906	916	92$\bar{5}$	934
78	943	952	961	970	979	988	997	*006	*015	*024
79	68 034	043	052	061	070	079	088	097	106	115
480	124	133	142	151	160	169	178	187	196	205
81	21$\bar{5}$	224	233	242	251	260	269	278	287	296
82	30$\bar{5}$	314	323	332	341	350	359	368	377	386
83	39$\bar{5}$	404	413	422	431	440	449	458	467	476
84	48$\bar{5}$	494	502	511	520	529	538	547	556	565
85	574	583	592	601	610	619	628	637	646	65$\bar{5}$
86	664	673	681	690	699	708	717	726	735	744
87	753	762	771	780	789	797	806	815	824	833
88	842	851	860	869	878	886	895	904	913	922
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490	69 020	028	037	046	055	064	073	082	090	099
91	108	117	126	13$\bar{5}$	144	152	161	170	179	188
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93	28$\bar{5}$	294	302	311	320	329	338	346	355	364
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95	461	469	478	487	496	504	513	522	531	539
96	548	557	566	574	583	592	601	609	618	627
97	636	644	653	662	671	679	688	697	705	714
98	723	732	740	749	758	767	775	784	793	801
99	810	819	827	836	84$\bar{5}$	854	862	871	880	888
500	897	906	914	923	932	940	949	958	966	975
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01	984	992	*001	*010	*018	*027	*036	*044	*053	*062
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03	157	165	174	183	191	200	209	217	226	234
04	243	252	260	269	278	286	295̄	303	312	321
05	329	338	346	355̄	364	372	381	389	398	406
06	415	424	432	441	449	458	467	475	484	492
07	501	509	518	526	535	544	552	561	569	578
08	586	595̄	603	612	621	629	638	646	655̄	663
09	672	680	689	697	706	714	723	731	740	749
510	757	766	774	783	791	800	808	817	825	834
11	842	851	859	868	876	885̄	893	902	910	919
12	927	935	944	952	961	969	978	986	995̄	*003
13	71 012	020	029	037	046	054	063	071	079	088
14	096	105̄	113	122	130	139	147	155	164	172
15	181	189	198	206	214	223	231	240	248	257
16	265̄	273	282	290	299	307	315	324	332	341
17	349	357	366	374	383	391	399	408	416	425̄
18	433	441	450̄	458	466	475̄	483	492	500̄	508
19	517	525	533	542	550	559	567	575	584	592
520	600	609	617	625	634	642	650	659	667	675
21	684	692	700	709	717	725	734	742	750	759
22	767	775	784	792	800	809	817	825	834	842
23	850	858	867	875	883	892	900	908	917	925̄
24	933	941	950̄	958	966	975̄	983	991	999	*008
25	72 016	024	032	041	049	057	066	074	082	090
26	099	107	115	123	132	140	148	156	165̄	173
27	181	189	198	206	214	222	230	239	247	255
28	263	272	280	288	296	304	313	321	329	337
29	346	354	362	370	378	387	395̄	403	411	419
530	428	436	444	452	460	469	477	485̄	493	501
31	509	518	526	534	542	550	558	567	575̄	583
32	591	599	607	616	624	632	640	648	656	665̄
33	673	681	689	697	705	713	722	730	738	746
34	754	762	770	779	787	795̄	803	811	819	827
35	835	843	852	860	868	876	884	892	900	908
36	916	925̄	933	941	949	957	965	973	981	989
37	997	*006	*014	*022	*030	*038	*046	*054	*062	*070
38	73 078	086	094	102	111	119	127	135̄	143	151
39	159	167	175̄	183	191	199	207	215	223	231
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41	320	328	336	344	352	360	368	376	384	392
42	400	408	416	424	432	440	448	456	464	472
43	480	488	496	504	512	520	528	536	544	552
44	560	568	576	484	592	600	608	616	624	632
45	640	648	656	664	672	679	687	695	703	711
46	719	727	735	743	751	759	767	775̄	783	791
47	799	807	815̄	823	830	838	846	854	862	870
48	878	886	894	902	910	918	926	933	941	949
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52	194	202	210	218	225	233	241	249	257	265̅
53	273	280	288	296	304	312	320	327	335	343
54	351	359	367	374	382	390	398	406	414	421
55	429	437	445̅	453	461	468	476	484	492	5̅00
56	507	515	523	531	539	547	554	562	570	578
57	586	593	601	609	617	624	632	640	648	656
58	663	671	679	687	695̅	702	710	718	726	733
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61	896	904	912	920	927	935̅	943	950	958	966
62	974	981	989	997	*005̅	*012	*020	*028	*035	*043
63	75 051	059	066	074	082	089	097	105̅	113	120
64	128	136	143	151	159	166	174	182	189	197
65	205̅	213	220	228	236	243	251	259	266	274
66	282	289	297	305̅	312	320	328	335	343	351
67	358	366	374	381	389	397	404	412	420	427
68	435̅	442	450	458	465	473	481	488	496	504
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71	664	671	679	686	694	702	709	717	724	732
72	740	747	755̅	762	770	778	785	793	800	808
73	815	823	831	838	846	853	861	868	876	884
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75	967	974	982	989	997	*005̅	*012	*020	*027	*035̅
76	76 042	05̅0	057	065̅	072	080	087	095̅	103	110
77	118	125	133	140	148	155	163	170	178	185
78	193	200	208	215	223	230	238	245	253	260
79	268	275	283	290	298	305	313	320	328	335
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84	641	649	656	664	671	678	686	693	701	708
85	716	723	730	738	745	753	760	768	775̅	782
86	790	797	805̅	812	819	827	834	842	849	856
87	864	871	879	886	893	901	908	916	923	930
88	938	945	953	960	967	975̅	982	989	997	*004
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590	085	093	100	107	115̅	122	129	137	144	151
91	159	166	173	181	188	195	203	210	217	225̅
92	232	240	247	254	262	269	276	283	291	298
93	305	313	320	327	335̅	342	349	357	364	371
94	379	386	393	401	408	415	422	430	437	444
95	452	459	466	474	481	488	495	503	510	517
96	525̅	532	539	546	554	561	568	576	583	590
97	597	605̅	612	619	627	634	641	648	656	663
98	670	677	685̅	692	699	706	714	721	728	735
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02	960	967	974	981	988	996	*003	*010	*017	*02$\bar{5}$
03	78 032	039	046	053	061	068	07$\bar{5}$	082	089	097
04	104	111	118	125	132	140	147	154	161	168
05	176	183	190	197	204	211	219	226	233	240
06	247	254	262	269	276	283	290	297	30$\bar{5}$	312
07	319	326	333	340	347	35$\bar{5}$	362	369	376	383
08	390	398	40$\bar{5}$	412	419	426	433	440	447	45$\bar{5}$
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17	79 029	036	043	0$\bar{5}$0	057	064	071	078	08$\bar{5}$	092
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22	379	386	393	400	407	414	421	428	43$\bar{5}$	442
23	449	456	463	470	477	484	491	498	50$\bar{5}$	511
24	518	525	532	539	546	553	560	567	574	581
25	588	59$\bar{5}$	602	609	616	623	630	637	644	650
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27	727	734	741	748	754	761	768	775	782	789
28	796	803	810	817	824	831	837	844	851	858
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630	934	941	948	95$\bar{5}$	962	969	975	982	989	996
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32	072	079	085	092	099	106	113	120	127	134
33	140	147	154	161	168	17$\bar{5}$	182	188	195	202
34	209	216	223	229	236	243	250	257	264	271
35	277	284	291	298	30$\bar{5}$	312	318	325	332	339
36	346	353	359	366	373	380	387	393	400	407
37	414	421	428	434	441	448	45$\bar{5}$	462	468	475
38	482	489	496	502	509	516	523	530	536	543
39	550	557	564	570	577	584	591	598	604	611
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41	686	693	699	706	713	720	726	733	740	747
42	754	760	767	774	781	787	794	801	808	814
43	821	828	83$\bar{5}$	841	848	85$\bar{5}$	862	868	875	882
44	889	895	902	909	916	922	929	936	943	949
45	956	963	969	976	983	990	996	*003	*010	*017
46	81 023	030	037	043	050	057	064	070	077	084
47	090	097	104	111	117	124	131	137	144	151
48	158	164	171	178	184	191	198	204	211	218
49	224	231	238	24$\bar{5}$	251	258	26$\bar{5}$	271	278	28$\bar{5}$
650	291	298	30$\bar{5}$	311	318	32$\bar{5}$	331	338	34$\bar{5}$	351
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54	558	564	571	578	584	591	598	604	611	617
55	624	631	637	644	651	657	664	671	677	684
56	690	697	704	710	717	723	730	737	743	750
57	757	763	770	776	783	790	796	803	809	816
58	823	829	836	842	849	856	862	869	875	882
59	889	895	902	908	915	921	928	935	941	948
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61	82 020	027	033	040	046	053	060	066	073	079
62	086	092	099	105	112	119	125	132	138	145
63	151	158	164	171	178	184	191	197	204	210
64	217	223	230	236	243	249	256	263	269	276
65	282	289	295	302	308	315	321	328	334	341
66	347	354	360	367	373	380	387	393	400	406
67	413	419	426	432	439	445	452	458	465	471
68	478	484	491	497	504	510	517	523	530	536
69	543	549	556	562	569	575	582	588	595	601
670	607	614	620	627	633	640	646	653	659	666
71	672	679	685	692	698	705	711	718	724	730
72	737	743	750	756	763	769	776	782	789	795
73	802	808	814	821	827	834	840	847	853	860
74	866	872	879	885	892	898	905	911	918	924
75	930	937	943	950	956	963	969	975	982	988
76	995	*001	*008	*014	*020	*027	*033	*040	*046	*052
77	83 059	065	072	078	085	091	097	104	110	117
78	123	129	136	142	149	155	161	168	174	181
79	187	193	200	206	213	219	225	232	238	245
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85	569	575	582	588	594	601	607	613	620	626
86	632	639	645	651	658	664	670	677	683	689
87	696	702	708	715	721	727	734	740	746	753
88	759	765	771	778	784	790	797	803	809	816
89	822	828	835	841	847	853	860	866	872	879
690	885	891	897	904	910	916	923	929	935	942
91	948	954	960	967	973	979	985	992	998	*004
92	84 011	017	023	029	036	042	048	055	061	067
93	073	080	086	092	098	105	111	117	123	130
94	136	142	148	155	161	167	173	180	186	192
95	198	205	211	217	223	230	236	242	248	255
96	261	267	273	280	286	292	298	305	311	317
97	323	330	336	342	348	354	361	367	373	379
98	386	392	398	404	410	417	423	429	435	442
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01	572	578	584	590	597	603	609	615	621	628
02	634	640	646	652	658	66$\bar{5}$	671	677	683	689
03	696	702	708	714	720	726	733	739	74$\bar{5}$	751
04	757	763	770	776	782	788	794	800	807	813
05	819	825	831	837	844	8$\bar{5}$0	856	862	868	874
06	880	887	893	899	905	911	917	924	930	936
07	942	948	954	960	967	973	979	98$\bar{5}$	991	997
08	85 003	009	016	022	028	034	040	046	052	058
09	06$\bar{5}$	071	077	083	089	095	101	107	114	120
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11	187	193	199	205	211	217	224	230	236	242
12	248	254	260	266	272	278	28$\bar{5}$	291	297	303
13	309	315	321	327	333	339	345	352	358	364
14	370	376	382	388	394	400	406	412	418	42$\bar{5}$
15	431	437	443	449	45$\bar{5}$	461	467	473	479	485
16	491	497	503	509	516	522	528	534	540	546
17	552	558	564	570	576	582	588	594	600	606
18	612	618	62$\bar{5}$	631	637	643	649	65$\bar{5}$	661	667
19	673	679	68$\bar{5}$	691	697	703	709	715	721	727
720	733	739	745	751	757	763	769	775	781	788
21	794	800	806	812	818	824	830	836	842	848
22	854	860	866	872	878	884	890	896	902	908
23	914	920	926	932	938	944	9$\bar{5}$0	956	962	968
24	974	980	986	992	998	*004	*010	*016	*022	*028
25	86 034	040	046	052	058	064	070	076	082	088
26	094	100	106	112	118	124	130	136	141	147
27	153	159	165	171	177	183	189	195	201	207
28	213	219	225	231	237	243	249	25$\bar{5}$	261	267
29	273	279	28$\bar{5}$	291	297	303	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
31	392	390	404	410	415	421	427	433	439	445
32	451	457	463	469	47$\bar{5}$	481	487	493	499	504
33	510	516	522	528	534	540	546	552	558	564
34	570	576	581	587	593	599	605	611	617	623
35	629	63$\bar{5}$	641	646	652	658	664	670	676	682
36	688	694	700	705	711	717	723	729	73$\bar{5}$	741
37	747	753	759	764	770	776	782	788	794	800
38	806	812	817	823	829	835	841	847	853	859
39	864	870	876	882	888	894	900	906	911	917
740	923	929	93$\bar{5}$	941	947	953	958	964	970	976
41	982	988	994	999	*005	*011	*017	*023	*029	*03$\bar{5}$
42	87 040	046	052	058	064	070	075	081	087	093
43	099	10$\bar{5}$	111	116	122	128	134	140	146	151
44	157	163	169	17$\bar{5}$	181	186	192	198	204	210
45	216	221	227	233	239	24$\bar{5}$	251	256	262	268
46	274	280	286	291	297	303	309	31$\bar{5}$	320	326
47	332	338	344	349	355	361	367	373	379	384
48	390	396	402	408	413	419	42$\bar{5}$	431	437	442
49	448	45$\bar{4}$	460	466	471	477	483	489	49$\bar{5}$	500
750	506	512	518	523	529	535	541	547	552	558
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52	622	628	633	639	$64\overline{5}$	651	656	662	668	674
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54	737	743	749	754	760	766	772	777	783	789
55	$79\overline{5}$	800	806	812	818	823	829	$83\overline{5}$	841	846
56	852	858	864	869	875	881	887	892	898	904
57	910	915	921	927	933	938	944	950	955	961
58	967	973	978	984	990	996	*001	*007	*013	*018
59	88 024	030	036	041	047	053	058	064	070	076
760	081	087	093	098	104	110	116	121	127	133
61	138	144	150	156	161	167	173	178	184	190
62	195	201	207	213	218	224	230	235	241	247
63	252	258	264	270	275	281	287	292	298	304
64	309	315	321	326	332	338	343	349	$35\overline{5}$	360
65	366	372	377	383	389	$39\overline{5}$	400	406	412	417
66	423	429	434	440	446	451	457	463	468	474
67	480	485	491	497	502	508	513	519	$52\overline{5}$	530
68	536	542	547	553	559	564	570	576	581	587
69	593	598	604	610	615	621	627	632	638	643
770	649	$65\overline{5}$	660	666	672	677	683	689	694	700
71	705	711	717	722	728	734	739	$74\overline{5}$	750	756
72	762	767	773	779	784	790	795	801	807	812
73	818	824	829	$83\overline{5}$	840	846	852	857	863	868
74	874	880	885	891	897	902	908	913	919	$92\overline{5}$
75	930	936	941	947	953	958	964	969	$97\overline{5}$	981
76	986	992	997	*003	*009	*014	*020	*025	*031	*037
77	89 042	048	053	059	064	070	076	081	087	092
78	098	104	109	$11\overline{5}$	120	126	131	137	143	148
79	154	159	$16\overline{5}$	170	176	182	187	193	198	204
780	209	215	221	226	232	237	243	248	254	260
81	265	271	276	282	287	293	298	304	310	315
82	321	326	332	337	343	348	354	360	365	371
83	376	382	387	393	398	404	409	$41\overline{5}$	421	426
84	432	437	443	448	454	459	$46\overline{5}$	470	476	481
85	487	492	498	504	509	$51\overline{5}$	520	526	531	537
86	542	548	553	559	564	570	575	581	586	592
87	597	603	609	614	620	625	631	636	642	647
88	653	658	664	669	$67\overline{5}$	680	686	691	697	702
89	708	713	719	724	730	735	741	746	752	757
790	763	768	774	779	$78\overline{5}$	790	796	801	807	812
91	818	823	829	834	840	845	851	856	862	867
92	873	878	883	889	894	900	905	911	916	922
93	927	933	938	944	949	$95\overline{5}$	960	966	971	977
94	982	988	993	998	*004	*009	$*01\overline{5}$	*020	*026	*031
95	90 037	042	048	053	059	064	069	$07\overline{5}$	080	086
96	091	097	102	108	113	119	124	129	$13\overline{5}$	140
97	146	151	157	162	168	173	179	184	189	$19\overline{5}$
98	200	206	211	217	222	227	233	238	244	249
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800	309	314	320	325	331	336	342	347	352	358
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02	417	423	428	434	439	44$\overline{5}$	4$\overline{5}$0	455	461	466
03	472	477	482	488	493	499	504	509	51$\overline{5}$	520
04	526	531	536	542	547	553	558	563	569	574
05	580	58$\overline{5}$	590	596	601	607	612	617	623	628
06	634	639	644	6$\overline{5}$0	655	660	666	671	677	682
07	687	693	698	703	709	714	720	725	730	736
08	741	747	752	757	763	768	773	779	784	789
09	79$\overline{5}$	800	806	811	816	822	827	832	838	843
810	849	854	859	86$\overline{5}$	870	875	881	886	891	897
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12	956	961	966	972	977	982	988	993	998	*004
13	91 009	014	020	025	030	036	041	046	052	057
14	062	068	073	078	084	089	094	100	105	110
15	116	121	126	132	137	142	148	153	158	164
16	169	174	180	18$\overline{5}$	190	196	201	206	212	217
17	222	228	233	238	243	249	254	259	26$\overline{5}$	270
18	275	281	286	291	297	302	307	312	318	323
19	328	334	339	344	3$\overline{5}$0	35$\overline{5}$	360	365	371	376
820	381	387	392	397	403	408	413	418	424	429
21	434	440	44$\overline{5}$	450	455	461	466	471	477	482
22	487	492	498	503	508	514	519	524	529	53$\overline{5}$
23	540	545	551	556	561	566	572	577	582	587
24	593	598	603	609	614	619	624	630	63$\overline{5}$	640
25	645	651	656	661	666	672	677	682	687	693
26	698	703	709	714	719	724	730	73$\overline{5}$	740	745
27	751	756	761	766	772	777	782	787	793	798
28	803	808	814	819	824	829	834	840	84$\overline{5}$	850
29	855	861	866	871	876	882	887	892	897	903
830	908	913	918	924	929	934	939	944	9$\overline{5}$0	95$\overline{5}$
31	960	965	971	976	981	986	991	997	*002	*007
32	92 012	018	023	028	033	038	044	049	054	059
33	06$\overline{5}$	070	07$\overline{5}$	080	085	091	096	101	106	111
34	117	122	127	132	137	143	148	153	158	163
35	169	174	179	184	189	19$\overline{5}$	200	205	210	215
36	221	226	231	236	241	247	252	257	262	267
37	273	278	283	288	293	298	304	309	314	319
38	324	330	33$\overline{5}$	340	345	350	355	361	366	371
39	376	381	387	392	397	402	407	412	418	423
840	428	433	438	443	449	454	459	464	469	474
41	480	48$\overline{5}$	490	495	500	505	511	516	521	526
42	531	536	542	547	552	557	562	567	572	578
43	583	588	593	598	603	609	614	619	624	629
44	634	639	64$\overline{5}$	6$\overline{5}$0	65$\overline{5}$	660	665	670	675	681
45	686	691	696	701	706	711	716	722	727	732
46	737	742	747	752	758	763	768	773	778	783
47	788	793	799	804	809	814	819	824	829	834
48	840	84$\overline{5}$	8$\overline{5}$0	85$\overline{5}$	860	865	870	875	881	886
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51	993	998	*003	*008	*013	*018	*024	*029	*034	*039
52	93 044	049	054	059	064	069	07$\bar{5}$	080	08$\bar{5}$	090
53	09$\bar{5}$	100	105	110	115	120	125	131	136	141
54	146	151	156	161	166	171	176	181	186	192
55	197	202	207	212	217	222	227	232	237	242
56	247	252	258	263	268	273	278	283	288	293
57	298	303	308	313	318	323	328	334	339	344
58	349	354	359	364	369	374	379	384	389	394
59	399	404	409	414	420	42$\bar{5}$	430	43$\bar{5}$	440	44$\bar{5}$
860	4$\bar{5}$0	45$\bar{5}$	460	46$\bar{5}$	470	475	480	485	490	495
61	500	505	510	515	520	526	531	536	541	546
62	551	556	561	566	571	576	581	586	591	596
63	601	606	611	616	621	626	631	636	641	646
64	651	656	661	666	671	676	682	687	692	697
65	702	707	712	717	722	727	732	737	742	747
66	752	757	762	767	772	777	782	787	792	797
67	802	807	812	817	822	827	832	837	842	847
68	852	857	862	867	872	877	882	887	892	897
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870	952	957	962	967	972	977	982	987	992	997
71	94 002	007	012	017	022	027	032	037	042	047
72	052	057	062	067	072	077	082	086	091	096
73	101	106	111	116	121	126	131	136	141	146
74	151	156	161	166	171	176	181	186	191	196
75	201	206	211	216	221	226	231	236	240	245
76	250	255	260	265	270	275	280	285	290	295
77	300	30$\bar{5}$	310	31$\bar{5}$	320	32$\bar{5}$	330	33$\bar{5}$	340	34$\bar{5}$
78	349	354	359	364	369	374	379	384	389	394
79	399	404	409	414	419	424	429	433	438	443
880	448	453	458	463	468	473	478	483	488	493
81	498	503	507	512	517	522	527	532	537	542
82	547	552	557	562	567	571	576	581	586	591
83	596	601	606	611	616	621	626	630	635	640
84	645	650	655	660	66$\bar{5}$	670	67$\bar{5}$	680	68$\bar{5}$	689
85	694	699	704	709	714	719	724	729	734	738
86	743	748	753	758	763	768	773	778	783	787
87	792	797	802	807	812	817	822	827	832	836
88	841	846	851	856	861	866	871	876	880	885
89	890	895	900	90$\bar{5}$	910	91$\bar{5}$	919	924	929	934
890	939	944	949	954	959	963	968	973	978	983
91	988	993	998	*002	*007	*012	*017	*022	*027	*032
92	95 036	041	046	051	056	061	066	071	075	080
93	085	090	09$\bar{5}$	100	10$\bar{5}$	109	114	119	124	129
94	134	139	143	148	153	158	163	168	173	177
95	182	187	192	197	202	207	211	216	221	226
96	231	236	240	245	250	255	260	26$\bar{5}$	270	274
97	279	284	289	294	299	303	308	313	318	323
98	328	332	337	342	347	352	357	361	366	371
99	376	381	386	390	395	400	40$\bar{5}$	410	41$\bar{5}$	419
900	424	429	434	439	444	448	453	458	463	468
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01	472	477	482	487	492	497	501	506	511	516
02	521	525	530	535	540	545	550	554	559	564
03	569	574	578	583	588	593	598	602	607	612
04	617	622	626	631	636	641	646	650	655	660
05	665̄	670	674	679	684	689	694	698	703	708
06	713	718	722	727	732	737	742	746	751	756
07	761	766	770	775	780	785̄	789	794	799	804
08	809	813	818	823	828	832	837	842	847	852
09	856	861	866	871	875	880	885	890	895̄	899
910	904	909	914	918	923	928	933	938	942	947
11	952	957	961	966	971	976	980	985	990	995̄
12	999	*004	*009	*014	*019	*023	*028	*033	*038	*042
13	96 047	052	057	061	066	071	076	080	085	090
14	095̄	099	104	109	114	118	123	128	133	137
15	142	147	152	156	161	166	171	175	180	185̄
16	190	194	199	204	209	213	218	223	227	232
17	237	242	246	251	256	261	265	270	275̄	280
18	284	289	294	298	303	308	313	317	322	327
19	332	336	341	346	350	355	360	365̄	369	374
920	379	384	388	393	398	402	407	412	417	421
21	426	431	435	440	445̄	450	454	459	464	468
22	473	478	483	487	492	497	501	506	511	515
23	520	525̄	530	534	539	544	548	553	558	562
24	567	572	577	581	586	591	595	600	605̄	609
25	614	619	624	628	633	638	642	647	652	656
26	661	666	670	675	680	685̄	689	694	699	703
27	708	713	717	722	727	731	736	741	745	750
28	755̄	759	764	769	774	778	783	788	792	797
29	802	806	811	816	820	825̄	830	834	839	844
930	848	853	858	862	867	872	876	881	886	890
31	895̄	900	904	909	914	918	923	928	932	937
32	942	946	951	956	960	965̄	970	974	979	984
33	988	993	997	*002	*007	*011	*016	*021	*025	*030
34	97 035̄	039	044	049	053	058	063	067	072	077
35	081	086	090	095	100	104	109	114	118	123
36	128	132	137	142	146	151	155	160	165̄	169
37	174	179	183	188	192	197	202	206	211	216
38	220	225̄	230	234	239	243	248	253	257	262
39	267	271	276	280	285	290	294	299	304	308
940	313	317	322	327	331	336	340	345	350	354
41	359	364	368	373	377	382	387	391	396	400
42	405	410	414	419	424	428	433	437	442	447
43	451	456	460	465̄	470	474	479	483	488	493
44	497	502	506	511	516	520	525̄	529	534	539
45	543	548	552	557	562	566	571	575	580	585̄
46	589	594	598	603	607	612	617	621	626	630
47	635̄	640	644	649	653	658	663	667	672	676
48	681	685	690	695̄	699	704	708	713	717	722
49	727	731	736	740	745̄	749	754	759	763	768
950	772	777	782	786	791	795	800	804	809	813
N.	0	1	2	3	4	5	6	7	8	9

N.	0	1	2	3	4	5	6	7	8	9
950	97 772	777	782	786	791	795	800	804	809	813
51	818	823	827	832	836	841	845	850	855̄	859
52	864	868	873	877	882	886	891	896	900	905̄
53	909	914	918	923	928	932	937	941	946	950
54	955̄	959	964	968	973	978	982	987	991	996
55	98 000	005̄	009	014	019	023	028	032	037	041
56	046	050	055̄	059	064	068	073	078	082	087
57	091	096	100	105̄	109	114	118	123	127	132
58	137	141	146	150	155̄	159	164	168	173	177
59	182	186	191	195	200	204	209	214	218	223
960	227	232	236	241	245	25̄0	254	259	263	268
61	272	277	281	286	290	295̄	299	304	308	313
62	318	322	327	331	336	340	345̄	349	354	358
63	363	367	372	376	381	385	390	394	399	403
64	408	412	417	421	426	430	435̄	439	444	448
65	453	457	462	466	471	475	480	484	489	493
66	498	502	507	511	516	520	525̄	529	534	538
67	543	547	552	556	561	565	570	574	579	583
68	588	592	597	601	605	610	614	619	623	628
69	632	637	641	646	650	655̄	659	664	668	673
970	677	682	686	691	695	700	704	709	713	717
71	722	726	731	735	740	744	749	753	758	762
72	767	771	776	780	784	789	793	798	802	807
73	811	816	820	825̄	829	834	838	843	847	851
74	856	860	865̄	869	874	878	883	887	892	896
75	900	905̄	909	914	918	923	927	932	936	941
76	945̄	949	954	958	963	967	972	976	981	985
77	989	994	998	*003	*007	*012	*016	*021	*025	*029
78	99 034	038	043	047	052	056	061	065̄	069	074
79	078	083	087	092	096	100	105̄	109	114	118
980	123	127	131	136	140	145̄	149	154	158	162
81	167	171	176	180	185̄	189	193	198	202	207
82	211	216	220	224	229	233	238	242	247	251
83	255	260	264	269	273	277	282	286	291	295
84	300	304	308	313	317	322	326	330	335̄	339
85	344	348	352	357	361	366	370	374	379	383
86	388	392	396	401	405	410	414	419	423	427
87	432	436	441	445̄	449	454	458	463	467	471
88	476	480	484	489	493	498	502	506	511	515
89	520	524	528	533	537	542	546	550	555̄	559
990	564	568	572	577	581	585	590	594	599	603
91	607	612	616	621	625̄	629	634	638	642	647
92	651	656	660	664	669	673	677	682	686	691
93	695̄	699	704	708	712	717	721	726	730	734
94	739	743	747	752	756	760	765̄	769	774	778
95	782	787	791	795	800	804	808	813	817	822
96	826	830	835̄	839	843	848	852	856	861	865
97	870	874	878	883	887	891	896	900	904	909
98	913	917	922	926	930	935̄	939	944	948	952
99	957	961	965	970	974	978	983	987	991	996
1000	00 000	004	009	013	017	022	026	030	035̄	039
N.	0	1	2	3	4	5	6	7	8	9

TABLE II

LOGS AND COLOGS OF CERTAIN MUCH-USED NUMBERS

Number	Logarithm	Cologarithm
2	0.3010300	9.6989700–10
3	0.4771213	9.5228787–10
$\sqrt{2}$	0.1505150	9.8494850–10
$\sqrt{3}$	0.2385607	9.7614393–10
π	0.4971499	9.5028501–10
π^2	0.9942997	9.0057003–10
2π	0.7981799	9.2018201–10
$\sqrt{\pi}$	0.2485749	9.7514251–10
57.2957795	1.7581226	8.2418774–10
206264.806	5.3144251	4.6855749–10

Five Place

2	0.30103	9.69897–10
3	0.47712	9.52288–10
$\sqrt{2}$	0.15052	9.84948–10
$\sqrt{3}$	0.23856	9.76144–10
π	0.49715	9.50285–10
π^2	0.99430	9.00570–10
2π	0.79818	9.20182–10
$\sqrt{\pi}$	0.24857	9.75143–10
57.2957795	1.75812	8.24188–10
206264.806	5.31443	4.68557–10

Four Place

2	0.3010	9.6990–10
3	0.4771	9.5229–10
$\sqrt{2}$	0.1505	9.8495–10
$\sqrt{3}$	0.2386	9.7614–10
π	0.4971	9.5029–10
π^2	0.9943	9.0057–10
2π	0.7982	9.2018–10
$\sqrt{\pi}$	0.2486	9.7514–10
57.2956695	1.7581	8.2419–10
206264.806	5.3144	4.6858–10

TABLE III

FIVE-PLACE LOGARITHMS

OF THE

SINE, COSINE, TANGENT, AND COTANGENT

FOR

EACH MINUTE OF THE QUADRANT

0° | 89°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	∞	∞	∞	0.00 000	**60**
1	6.46 373	6.46 373	3.53 627	0.00 000	59
2	6.76 476	6.76 476	3.23 524	0.00 000	58
3	6.94 08$\bar{5}$	6.94 08$\bar{5}$	3.05 915	0.00 000	57
4	7.06 579	7.06 579	2.93 421	0.00 000	56
5	7.16 270	7.16 270	2.83 730	0.00 000	55
6	7.24 188	7.24 188	2.75 812	0.00 000	54
7	7.30 882	7.30 882	2.69 118	0.00 000	53
8	7.36 682	7.36 682	2.63 318	0.00 000	52
9	7.41 797	7.41 797	2.58 203	0.00 000	51
10	7.46 373	7.46 373	2.53 627	0.00 000	**50**
11	7.50 512	7.50 512	2.49 488	0.00 000	49
12	7.54 291	7.54 291	2.45 709	0.00 000	48
13	7.57 767	7.57 767	2.42 233	0.00 000	47
14	7.60 985	7.60 986	2.39 014	0.00 000	46
15	7.63 982	7.63 982	2.36 018	0.00 000	45
16	7.66 784	7.66 78$\bar{5}$	2.33 215	0.00 000	44
17	7.69 417	7.69 418	2.30 582	9.99 999	43
18	7.71 900	7.71 900	2.28 100	9.99 999	42
19	7.74 248	7.74 248	2.25 752	9.99 999	41
20	7.76 475	7.76 476	2.23 524	9.99 999	**40**
21	7.78 594	7.78 595	2.21 40$\bar{5}$	9.99 999	39
22	7.80 61$\bar{5}$	7.80 615	2.19 38$\bar{5}$	9.99 999	38
23	7.82 545	7.82 546	2.17 454	9.99 999	37
24	7.84 393	7.84 394	2.15 606	9.99 999	36
25	7.86 166	7.86 167	2.13 833	9.99 999	35
26	7.87 870	7.87 871	2.12 129	9.99 999	34
27	7.89 509	7.89 510	2.10 490	9.99 999	33
28	7.91 088	7.91 089	2.08 911	9.99 999	32
29	7.92 612	7.92 613	2.07 387	9.99 998	31
30	7.94 084	7.94 086	2.05 914	9.99 998	**30**
31	7.95 508	7.95 510	2.04 490	9.99 998	29
32	7.96 887	7.96 889	2.03 111	9.99 998	28
33	7.98 223	7.98 225	2.01 77$\bar{5}$	9.99 998	27
34	7.99 520	7.99 522	2.00 478	9.99 998	26
35	8.00 779	8.00 781	1.99 219	9.99 998	25
36	8.02 002	8.02 004	1.97 996	9.99 998	24
37	8.03 192	8.03 194	1.96 806	9.99 997	23
38	8.04 350	8.04 353	1.95 647	9.99 997	22
39	8.05 478	8.05 481	1.94 519	9.99 997	21
40	8.06 578	8.06 581	1.93 419	9.99 997	**20**
41	8.07 6$\bar{5}$0	8.07 653	1.92 347	9.99 997	19
42	8.08 696	8.08 700	1.91 300	9.99 997	18
43	8.09 718	8.09 722	1.90 278	9.99 997	17
44	8.10 717	8.10 720	1.89 280	9.99 996	16
45	8.11 693	8.11 696	1.88 304	9.99 996	15
46	8.12 647	8.12 651	1.87 349	9.99 996	14
47	8.13 581	8.13 585	1.86 41$\bar{5}$	9.99 996	13
48	8.14 495	8.14 $\bar{5}$00	1.85 500	9.99 996	12
49	8.15 391	8.15 395	1.84 60$\bar{5}$	9.99 996	11
50	8.16 268	8.16 273	1.83 727	9.99 995	**10**
51	8.17 128	8.17 133	1.82 867	9.99 995	9
52	8.17 971	8.17 976	1.82 024	9.99 995	8
53	8.18 798	8.18 804	1.81 196	9.99 99$\bar{5}$	7
54	8.19 610	8.19 616	1.80 384	9.99 99$\bar{5}$	6
55	8.20 407	8.20 413	1.79 587	9.99 994	5
56	8.21 189	8.21 195	1.78 80$\bar{5}$	9.99 994	4
57	8.21 958	8.21 964	1.78 036	9.99 994	3
58	8.22 713	8.22 720	1.77 280	9.99 994	2
59	8.23 456	8.23 462	1.76 538	9.99 994	1
60	8.24 186	8.24 192	1.75 808	9.99 993	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

1°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	8.24 186	8.24 192	1.75 808	9.99 993	**60**
1	8.24 903	8.24 910	1.75 090	9.99 993	59
2	8.25 609	8.25 616	1.74 384	9.99 993	58
3	8.26 304	8.26 312	1.73 688	9.99 993	57
4	8.26 988	8.26 996	1.73 004	9.99 992	56
5	8.27 661	8.27 669	1.72 331	9.99 992	55
6	8.28 324	8.28 332	1.71 668	9.99 992	54
7	8.28 977	8.28 986	1.71 014	9.99 992	53
8	8.29 621	8.29 629	1.70 371	9.99 992	52
9	8.30 255	8.30 263	1.69 737	9.99 991	51
10	8.30 879	8.30 888	1.69 112	9.99 991	**50**
11	8.31 495	8.31 505	1.68 495	9.99 991	49
12	8.32 103	8.32 112	1.67 888	9.99 990	48
13	8.32 702	8.32 711	1.67 289	9.99 990	47
14	8.33 292	8.33 302	1.66 698	9.99 990	46
15	8.33 875	8.33 886	1.66 114	9.99 990	45
16	8.34 450	8.34 461	1.65 539	9.99 989	44
17	8.35 018	8.35 029	1.64 971	9.99 989	43
18	8.35 578	8.35 590	1.64 410	9.99 989	42
19	8.36 131	8.36 143	1.63 857	9.99 989	41
20	8.36 678	8.36 689	1.63 311	9.99 988	**40**
21	8.37 217	8.37 229	1.62 771	9.99 988	39
22	8.37 750	8.37 762	1.62 238	9.99 988	38
23	8.38 276	8.38 289	1.61 711	9.99 987	37
24	8.38 796	8.38 809	1.61 191	9.99 987	36
25	8.39 310	8.39 323	1.60 677	9.99 987	35
26	8.39 818	8.39 832	1.60 168	9.99 986	34
27	8.40 320	8.40 334	1.59 666	9.99 986	33
28	8.40 816	8.40 830	1.59 170	9.99 986	32
29	8.41 307	8.41 321	1.58 679	9.99 985	31
30	8.41 792	8.41 807	1.58 193	9.99 985	**30**
31	8.42 272	8.42 287	1.57 713	9.99 985	29
32	8.42 746	8.42 762	1.57 238	9.99 984	28
33	8.43 216	8.43 232	1.56 768	9.99 984	27
34	8.43 680	8.43 696	1.56 304	2.99 984	26
35	8.44 139	8.44 156	1.55 844	9.99 983	25
36	8.44 594	8.44 611	1.55 389	9.99 983	24
37	8.45 044	8.45 061	1.54 939	9.99 983	23
38	8.45 589	8.45 507	1.54 493	9.99 982	22
39	8.45 930	8.45 948	1.54 052	9.99 982	21
40	8.46 366	8.46 385	1.53 615	9.99 982	**20**
41	8.46 799	8.46 817	1.53 183	9.99 981	19
42	8.47 226	8.47 245	1.52 755	9.99 981	18
43	8.47 650	8.47 669	1.52 331	9.99 981	17
44	8.48 069	8.48 089	1.51 911	9.99 980	16
45	8.48 485	8.48 505	1.51 495	9.99 980	15
46	8.48 896	8.48 917	1.51 083	9.99 979	14
47	8.49 304	8.49 325	1.50 675	9.99 979	13
48	8.49 708	8.49 729	1.50 271	9.99 979	12
49	8.50 108	8.50 130	1.49 870	9.99 978	11
50	8.50 504	8.50 527	1.49 473	9.99 978	**10**
51	8.50 897	8.50 920	1.49 080	9.99 977	9
52	8.51 287	8.51 310	1.48 690	9.99 977	8
53	8.51 673	8.51 696	1.48 304	9.99 977	7
54	8.52 055	8.52 079	1.47 921	9.99 976	6
55	8.52 434	8.52 459	1.47 541	9.99 976	5
56	8.52 810	8.52 835	1.47 165	9.99 975	4
57	8.53 183	8.53 208	1.46 792	9.99 975	3
58	8.53 552	8.53 578	1.46 422	9.99 974	2
59	8.53 919	8.53 945	1.46 055	9.99 974	1
60	8.54 282	8.54 308	1.45 692	9.99 974	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

88°

2°	'	L. Sin.	L. Tang.	L. Cotg.	L. Cos.		87°
	0	8.54 282	8.54 308	1.45 692	9.99 974	60	
	1	8.54 642	8.54 669	1.45 331	9.99 973	59	
	2	8.54 999	8.55 027	1.44 973	9.99 973	58	
	3	8.55 354	8.55 382	1.44 618	9.99 972	57	
	4	8.55 705	8.55 734	1.44 266	9.99 972	56	
	5	8.56 054	8.56 083	1.43 917	9.99 971	55	
	6	8.56 400	8.56 429	1.43 571	9.99 971	54	
	7	8.56 743	8.56 773	1.43 227	9.99 970	53	
	8	8.57 084	8.57 114	1.42 886	9.99 970	52	
	9	8.57 421	8.57 452	1.42 548	9.99 969	51	
	10	8.57 757	8.57 788	1.42 212	9.99 969	50	
	11	8.58 089	8.58 121	1.41 879	9.99 968	49	
	12	8.58 419	8.58 451	1.41 549	9.99 968	48	
	13	8.58 747	8.58 779	1.41 221	9.99 967	47	
	14	8.59 072	8.59 105	1.40 89$\bar{5}$	9.99 967	46	
	15	8.59 39$\bar{5}$	8.59 428	1.40 572	9.99 967	45	
	16	8.59 715	8.59 749	1.40 251	9.99 966	44	
	17	8.60 033	8.60 068	1.39 932	9.99 966	43	
	18	8.60 349	8.60 384	1.39 616	9.99 96$\bar{5}$	42	
	19	8.60 662	8.60 698	1.39 302	9.99 964	41	
	20	8.60 973	8.61 009	1.38 991	9.99 964	40	
	21	8.61 282	8.61 319	1.38 681	9.99 963	39	
	22	8.61 589	8.61 626	1.38 374	9.99 963	38	
	23	8.61 894	8.61 931	1.38 069	9.99 962	37	
	24	8.62 196	8.62 234	1.37 766	9.99 962	36	
	25	8.62 497	8.62 535	1.37 46$\bar{5}$	9.99 961	35	
	26	8.62 79$\bar{5}$	8.62 834	1.37 166	9.99 961	34	
	27	8.63 091	8.63 131	1.36 869	9.99 960	33	
	28	8.63 385	8.63 426	1.36 574	9.99 960	32	
	29	8.63 678	8.63 718	1.36 282	9.99 959	31	
2°	30	8.63 968	8.64 009	1.35 991	9.99 959	30	87°
	31	8.64 256	8.64 298	1.35 702	9.99 958	29	
	32	8.64 543	8.64 585	1.35 41$\bar{5}$	9.99 958	28	
	33	8.64 827	8.64 870	1.35 130	9.99 957	27	
	34	8.65 110	8.65 154	1.34 846	9.99 956	26	
	35	8.65 391	8 65 435	1.34 56$\bar{5}$	9.99 956	25	
	36	8.65 670	8.65 71$\bar{5}$	1.34 285	9.99 955	24	
	37	8.65 947	8.65 993	1.34 007	9.99 95$\bar{5}$	23	
	38	8.66 223	8.66 269	1.33 731	9.99 954	22	
	39	8.66 497	8.66 543	1.33 457	9.99 954	21	
	40	8.66 769	8.66 816	1.33 184	9.99 953	20	
	41	8.67 039	8.67 087	1.32 913	9.99 952	19	
	42	8.67 308	8.67 356	1.32 644	9.99 952	18	
	43	8.67 575	8.67 624	1.32 376	9.99 951	17	
	44	8.67 841	8.67 890	1.32 110	9.99 951	16	
	45	8.68 104	8.68 154	1.31 846	9.99 9$\bar{5}$0	15	
	46	8.68 367	8.68 417	1.31 583	9.99 949	14	
	47	8.68 627	8.68 678	1.31 322	9.99 949	13	
	48	8.68 886	8.68 938	1.31 062	9.99 948	12	
	49	8.69 144	8.69 196	1.30 804	9.99 948	11	
	50	8.69 400	8.69 453	1.30 547	9.99 947	10	
	51	8.69 654	8.69 708	1.30 292	9.99 946	9	
	52	8.69 907	8.69 962	1.30 038	9.99 946	8	
	53	8.70 159	8.70 214	1.29 786	9.99 94$\bar{5}$	7	
	54	8.70 409	8.70 46$\bar{5}$	1.29 535	9.99 944	6	
	55	8.70 658	8.70 714	1.29 286	9.99 944	5	
	56	8.70 90$\bar{5}$	8.70 962	1.29 038	9.99 943	4	
	57	8.71 151	8.71 208	1.28 792	9.99 942	3	
	58	8.71 395	8.71 453	1.28 547	9.99 942	2	
	59	8.71 638	8.71 697	1.28 303	9.99 941	1	
	60	8.71 880	8.71 940	1.28 060	9.99 940	0	
		L. Cos.	L. Cotg.	L. Tang.	L. Sin.	'	

3°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	8.71 880	8.71 940	1.28 060	9.99 940	**60**
1	8.72 120	8.72 181	1.27 819	9.99 940	59
2	8.72 359	8.72 420	1.27 580	9.99 939	58
3	8.72 597	8.72 659	1.27 341	9.99 938	57
4	8.72 834	8.72 896	1.27 104	9.99 938	56
5	8.73 069	8.73 132	1.26 868	9.99 937	55
6	8.73 303	8.73 366	1.26 634	9.99 936	54
7	8.73 535	8.73 600	1.26 400	9.99 936	53
8	8.73 767	8.73 832	1.26 168	9.99 935	52
9	8.73 997	8.74 063	1.25 937	9.99 934	51
10	8.74 226	8.74 292	1.25 708	9.99 934	**50**
11	8.74 454	8.74 521	1.25 479	9.99 933	49
12	8.74 680	8.74 748	1.25 252	9.99 932	48
13	8.74 906	8.74 974	1.25 026	9.99 932	47
14	8.75 130	8.75 199	1.24 801	9.99 931	46
15	8.75 353	8.75 423	1.24 577	9.99 930	45
16	8.75 575	8.75 645	1.24 355	9.99 929	44
17	8.75 795	8.75 867	1.24 133	9.99 929	43
18	8.76 015	8.76 087	1.23 913	9.99 928	42
19	8.76 234	8.76 306	1.23 694	9.99 927	41
20	8.76 451	8.76 525	1.23 475	9.99 926	**40**
21	8.76 667	8.76 742	1.23 258	9.99 926	39
22	8.76 883	8.76 958	1.23 042	9.99 925	38
23	8.77 097	8.77 173	1.22 827	9.99 924	37
24	8.77 310	8.77 387	1.22 613	9.99 923	36
25	8.77 522	8.77 600	1.22 400	9.99 923	35
26	8.77 733	8.77 811	1.22 189	9.99 922	34
27	8.77 943	8.78 022	1.21 978	9.99 921	33
28	8.78 152	8.78 232	1.21 768	9.99 920	32
29	8.78 360	8.78 441	1.21 559	9.99 920	31
30	8.78 568	8.78 649	1.21 351	9.99 919	**30**
31	8.78 774	8.78 855	1.21 145	9.99 918	29
32	8.78 979	8.79 061	1.20 939	9.99 917	28
33	8.79 183	8.79 266	1.20 734	9.99 917	27
34	8.79 386	8.79 470	1.20 530	9.99 916	26
35	8.79 588	8.79 673	1.20 327	9.99 915	25
36	8.79 789	8.79 875	1.20 125	9.99 914	24
37	8.79 990	8.80 076	1.19 924	9.99 913	23
38	8.80 189	8.80 277	1.19 723	9.99 913	22
39	8.80 388	8.80 476	1.19 524	9.99 912	21
40	8.80 585	8.80 674	1.19 326	9.99 911	**20**
41	8.80 782	8.80 872	1.19 128	9.99 910	19
42	8.80 978	8.81 068	1.18 932	9.99 909	18
43	8.81 173	8.81 264	1.18 736	9.99 909	17
44	8.81 367	8.81 459	1.18 541	9.99 908	16
45	8.81 560	8.81 653	1.18 347	9.99 907	15
46	8.81 752	8.81 846	1.18 154	9.99 906	14
47	8.81 944	8.82 038	1.17 962	9.99 905	13
48	8.82 134	8.82 230	1.17 770	9.99 904	12
49	8.82 324	8.82 420	1.17 580	9.99 904	11
50	8.82 513	8.82 610	1.17 390	9.99 903	**10**
51	8.82 701	8.82 799	1.17 201	9.99 902	9
52	8.82 888	8.82 987	1.17 013	9.99 901	8
53	8.83 075	8.83 175	1.16 825	9.99 900	7
54	8.83 261	8.83 361	1.16 639	9.99 899	6
55	8.83 446	8.83 547	1.16 453	9.99 898	5
56	8.83 630	8.83 732	1.16 268	9.99 898	4
57	8.83 813	8.83 916	1.16 084	9.99 897	3
58	8.83 996	8.84 100	1.15 900	9.99 896	2
59	8.84 177	8.84 282	1.15 718	9.99 895	1
60	8.84 358	8.84 464	1.15 536	9.99 894	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

86°

4° | 85°

'	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	8.84 358	8.84 464	1.15 536	9.99 894	60
1	8.84 539	8.84 646	1.15 354	9.99 893	59
2	8.84 718	8.84 826	1.15 174	9.99 892	58
3	8.84 897	8.85 006	1.14 994	9.99 891	57
4	8.85 075	8.85 185	1.14 815	9.99 891	56
5	8.85 252	8.85 363	1.14 637	9.99 890	55
6	8.85 429	8.85 540	1.14 460	9.99 889	54
7	8.85 605	8.85 717	1.14 283	9.99 888	53
8	8.85 780	8.85 893	1.14 107	9.99 887	52
9	8.85 955	8.86 069	1.13 931	9.99 886	51
10	8.86 128	8.86 243	1.13 757	9.99 885	50
11	8.86 301	8.86 417	1.13 583	9.99 884	49
12	8.86 474	8.86 591	1.13 409	9.99 883	48
13	8.86 645	8.86 763	1.13 237	9.99 882	47
14	8.86 816	8.86 935	1.13 065	9.99 881	46
15	8.86 987	8.87 106	1.12 894	9.99 880	45
16	8.87 156	8.87 277	1.12 723	9.99 879	44
17	8.87 325	8.87 447	1.12 553	9.99 879	43
18	8.87 494	8.87 616	1.12 384	9.99 878	42
19	8.87 661	8.87 785	1.12 215	9.99 877	41
20	8.87 829	8.87 953	1.12 047	9.99 876	40
21	8.87 995	8.88 120	1.11 880	9.99 875	39
22	8.88 161	8.88 287	1.11 713	9.99 874	38
23	8.88 326	8.88 453	1.11 547	9.99 873	37
24	8.88 490	8.88 618	1.11 382	9.99 872	36
25	8.88 654	8.88 783	1.11 217	9.99 871	35
26	8.88 817	8.88 948	1.11 052	9.99 870	34
27	8.88 980	8.89 111	1.10 889	9.99 869	33
28	8.89 142	8.89 274	1.10 726	9.99 868	32
29	8.89 304	8.89 437	1.10 563	9.99 867	31
30	8.89 464	8.89 598	1.10 402	9.99 866	30
31	8.89 625	8.89 760	1.10 240	9.99 865	29
32	8.89 784	8.89 920	1.10 080	9.99 864	28
33	8.89 943	8.90 080	1.09 920	9.99 863	27
34	8.90 102	8.90 240	1.09 760	9.99 862	26
35	8.90 260	8.90 399	1.09 601	9.99 861	25
36	8.90 417	8.90 557	1.09 443	9.99 860	24
37	8.90 574	8.90 715	1.09 285	9.99 859	23
38	8.90 730	8.90 872	1.09 128	9.99 858	22
39	8.90 885	8.91 029	1.08 971	9.99 857	21
40	8.91 040	8.91 185	1.08 815	9.99 856	20
41	8.91 195	8.91 340	1.08 660	9.99 855	19
42	8.91 349	8.91 495	1.08 505	9.99 854	18
43	8.91 502	8.91 650	1.08 350	9.99 853	17
44	8.91 655	8.91 803	1.08 197	9.99 852	16
45	8.91 807	8.91 957	1.08 043	9.99 851	15
46	8.91 959	8.92 110	1.07 890	9.99 850	14
47	8.92 110	8.92 262	1.07 738	9.99 848	13
48	8.92 261	8.92 414	1.07 586	9.99 847	12
49	8.92 411	8.92 565	1.07 435	9.99 846	11
50	8.92 561	8.92 716	1.07 284	9.99 845	10
51	8.92 710	8.92 866	1.07 134	9.99 844	9
52	8.92 859	8.93 016	1.06 984	9.99 843	8
53	8.93 007	8.93 165	1.06 835	9.99 842	7
54	8.93 154	8.93 313	1.06 687	9.99 841	6
55	8.93 301	8.93 462	1.06 538	9.99 840	5
56	8.93 448	8.93 609	1.06 391	9.99 839	4
57	8.93 594	8.93 756	1.06 244	9.99 838	3
58	8.93 740	8.93 903	1.06 097	9.99 837	2
59	8.93 885	8.94 049	1.05 951	9.99 836	1
60	8.94 030	8.94 195	1.05 805	9.99 834	0
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	'

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′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	8.94 030	8.94 195	1.05 805̄	9.99 834	**60**
1	8.94 174	8.94 340	1.05 660	9.99 833	59
2	8.94 317	8.94 485	1.05 515̄	9.99 832	58
3	8.94 461	8.94 630	1.05 370	9.99 831	57
4	8.94 603	8.94 773	1.05 227	9.99 830	56
5	8.94 746	8.94 917	1.05 083	9.99 829	55
6	8.94 887	8.95 060	1.04 940	9.99 828	54
7	8.95 029	8.95 202	1.04 798	9.99 827	53
8	8.95 170	8.95 344	1.04 656	9.99 825	52
9	8.95 310	8.95 486	1.04 514	9.99 824	51
10	8.95 45̄0	8.95 627	1.04 373	9.99 823	**50**
11	8.95 589	8.95 767	1.04 233	9.99 822	49
12	8.95 728	8.95 908	1.04 092	9.99 821	48
13	8.95 867	8.96 047	1.03 953	9.99 820	47
14	8.96 005	8.96 187	1.03 813	9.99 819	46
15	8.96 143	8.96 325	1.03 675̄	9.99 817	45
16	8.96 280	8.96 464	1.03 536	9.99 816	44
17	8.96 417	8.96 602	1.03 398	9.99 815	43
18	8.96 553	8.96 739	1.03 261	9.99 814	42
19	8.96 689	8.96 877	1.03 123	9.99 813	41
20	8.96 825̄	8.97 013	1.02 987	9.99 812	**40**
21	8.96 960	8.97 15̄0	1.02 850	9.99 810	39
22	8.97 095̄	8.97 285	1.02 715̄	9.99 809	38
23	8.97 229	8.97 421	1.02 579	9.99 808	37
24	8.97 363	8.97 556	1.02 444	9.99 807	36
25	8.97 496	8.97 691	1.02 309	9.99 806	35
26	8.97 629	8.97 825̄	1.02 175	9.99 804	34
27	8.97 762	8.97 959	1.02 041	9.99 803	33
28	8.97 894	8.98 092	1.01 908	9.99 802	32
29	8.98 026	8.98 225	1.01 775̄	9.99 801	31
30	8.98 157	8.98 358	1.01 642	9.99 800	**30**
31	8.98 288	8.98 490	1.01 510	9.99 798	29
32	8.98 419	8.98 622	1.01 378	9.99 797	28
33	8.98 549	8.98 753	1.01 247	9.99 796	27
34	8.98 679	8.98 884	1.01 116	9.99 795̄	26
35	8.98 808	8.99 015̄	1.00 985	9.99 793	25
36	8.98 937	8.99 145	1.00 855̄	9.99 792	24
37	8.99 066	8.99 275	1.00 725̄	9.99 791	23
38	8.99 194	8.99 405̄	1.00 595	9.99 790	22
39	8.99 322	8.99 534	1.00 466	9.99 788	21
40	8.99 45̄0	8.99 662	1.00 338	9.99 787	**20**
41	8.99 577	8.99 791	1.00 209	9.99 786	19
42	8.99 704	8.99 919	1.00 081	9.99 785̄	18
43	8.99 830	9.00 046	0.99 954	9.99 783	17
44	8.99 956	9.00 174	0.99 826	9.99 782	16
45	9.00 082	9.00 301	0.99 699	9.99 781	15
46	9.00 207	9.00 427	0.99 573	9.99 780	14
47	9.00 332	9.00 553	0.99 447	9.99 778	13
48	9.00 456	9.00 679	0.99 321	9.99 777	12
49	9.00 581	9.00 805̄	0.99 195	9.99 776	11
50	9.00 704	9.00 930	0.99 070	9.99 775̄	**10**
51	9.00 828	9.01 055̄	0.98 945	9.99 773	9
52	9.00 951	9.01 179	0.98 821	9.99 772	8
53	9.01 074	9.01 303	0.98 697	9.99 771	7
54	9.01 196	9.01 427	0.98 573	9.99 769	6
55	9.01 318	9.01 550	0.98 45̄0	9.99 768	5
56	9.01 440	9.01 673	0.98 327	9.99 767	4
57	9.01 561	9.01 796	0.98 204	9.99 765	3
58	9.01 682	9.01 918	0.98 082	9.99 764	2
59	9.01 803	9.02 040	0.97 960	9.99 763	1
60	9.01 923	9.02 162	0.97 838	9.99 761	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

6°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.01 923	9.02 162	0.97 838	9.99 761	**60**
1	9.02 043	9.02 283	0.97 717	9.99 760	59
2	9.02 163	9.02 404	0.97 596	9.99 759	58
3	9.02 283	9.02 525	0.97 475̄	9.99 757	57
4	9.02 402	9.02 645	0.97 355̄	9.99 756	56
5	9.02 520	9.02 766	0.97 234	9.99 755̄	55
6	9.02 639	9.02 885	0.97 115̄	9.99 753	54
7	9.02 757	9.03 005̄	0.96 995	9.99 752	53
8	9.02 874	9.03 124	0.96 876	9.99 751	52
9	9.02 992	9.03 242	0.96 758	9.99 749	51
10	9.03 109	9.03 361	0.96 639	9.99 748	**50**
11	9.03 226	9.03 479	0.96 521	9.99 747	49
12	9.03 342	9.03 597	0.96 403	9.99 745	48
13	9.03 458	9.03 714	0.96 286	9.99 744	47
14	9.03 574	9.03 832	0.96 168	9.99 742	46
15	9.03 690	9.03 948	0.96 052	9.99 741	45
16	9.03 805̄	9.04 065	0.95 935̄	9.99 740	44
17	9.03 920	9.04 181	0.95 819	9.99 738	43
18	9.04 034	9.04 297	0.95 703	9.99 737	42
19	9.04 149	9.04 413	0.95 587	9.99 736	41
20	9.04 262	9.04 528	0.95 472	9.99 734	**40**
21	9.04 376	9.04 643	0.95 357	9.99 733	39
22	9.04 490	9.04 758	0.95 242	9.99 731	38
23	9.04 603	9.04 873	0.95 127	9.99 730	37
24	9.04 715	9.04 987	0.95 013	9.99 728	36
25	9.04 828	9.05 101	0.94 899	9.99 727	35
26	9.04 940	9.05 214	0.94 786	9.99 726	34
27	9.05 052	9.05 328	0.94 672	9.99 724	33
28	9.05 164	9.05 441	0.94 559	9.99 723	32
29	9.05 275̄	9.05 553	0.94 447	9.99 721	31
30	9.05 386	9.05 666	0.94 334	9.99 720	**30**
31	9.05 497	9.05 778	0.94 222	9.99 718	29
32	9.05 607	9.05 890	0.94 110	9.99 717	28
33	9.05 717	9.06 002	0.93 998	9.99 716	27
34	9.05 827	9.06 113	0.93 887	9.99 714	26
35	9.05 937	9.06 224	0.93 776	9.99 713	25
36	9.06 046	9.06 335̄	0.93 665	9.99 711	24
37	9.06 155	9.06 445	0.93 555̄	9.99 710	23
38	9.06 264	9.06 556	0.93 444	9.99 708	22
39	9.06 372	9.06 666	0.93 334	9.99 707	21
40	9.06 481	9.06 775	0.93 225̄	9.99 705	**20**
41	9.06 589	9.06 885̄	0.93 115	9.99 704	19
42	9.06 696	9.06 994	0.93 006	9.99 702	18
43	9.06 804	9.07 103	0.92 897	9.99 701	17
44	9.06 911	9.07 211	0.92 789	9.99 699	16
45	9.07 018	9.07 320	0.92 680	9.99 698	15
46	9.07 124	9.07 428	0.92 572	9.99 696	14
47	9.07 231	9.07 536	0.92 464	9.99 695̄	13
48	9.07 337	9.07 643	0.92 357	9.99 993	12
49	9.07 442	9.07 751	0.92 249	9.99 692	11
50	9.07 548	9.07 858	0.92 142	9.99 690	**10**
51	9.07 653	9.07 964	0.92 036	9.99 689	9
52	9.07 758	9.08 071	0.91 929	9.99 687	8
53	9.07 863	9.08 177	0.91 823	9.99 686	7
54	9.07 968	9.08 283	0.91 717	9.99 684	6
55	9.08 072	9.08 389	0.91 611	9.99 683	5
56	9.08 176	9.08 495̄	0.91 505	9.99 681	4
57	9.08 280	9.08 600	0.91 400	9.99 680	3
58	9.08 383	9.08 705	0.91 295̄	9.99 678	2
59	9.08 486	9.08 810	0.91 190	9.99 677	1
60	9.08 589	9.08 914	0.91 086	9.99 675	0
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

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7°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.08 589	9.08 914	0.91 086	9.99 675	**60**
1	9.08 692	9.09 019	0.90 981	9.99 674	59
2	9.08 795	9.09 123	0.90 877	9.99 672	58
3	9.08 897	9.09 227	0.90 773	9.99 670	57
4	9.08 999	9.09 330	0.90 670	9.99 669	56
5	9.09 101	9.09 434	0.90 566	9.99 667	55
6	9.09 202	9.09 537	0.90 463	9.99 666	54
7	9.09 304	9.09 640	0.90 360	9.99 664	53
8	9.09 405	9.09 742	0.90 258	9.99 663	52
9	9.09 506	9.09 845	0.90 155	9.99 661	51
10	9.09 606	9.09 947	0.90 053	9.99 659	**50**
11	9.09 707	9.10 049	0.89 951	9.99 658	49
12	9.09 807	9.10 150	0.89 850	9.99 656	48
13	9.09 907	9.10 252	0.89 748	9.99 655	47
14	9.10 006	9.10 353	0.89 647	9.99 653	46
15	9.10 106	9.10 454	0.89 546	9.99 651	45
16	9.10 205	9.10 555	0.89 445	9.99 650	44
17	9.10 304	9.10 656	0.89 344	9.99 648	43
18	9.10 402	9.10 756	0.89 244	9.99 647	42
19	9.10 501	9.10 856	0.89 144	9.99 645	41
20	9.10 599	9.10 956	0.89 044	9.99 643	**40**
21	9.10 697	9.11 056	0.88 944	9.99 642	39
22	9.10 795	9.11 155	0.88 845	9.99 640	38
23	9.10 893	9.11 254	0.88 746	9.99 638	37
24	9.10 990	9.11 353	0.88 647	9.99 637	36
25	9.11 087	9.11 452	0.88 548	9.99 635	35
26	9.11 184	9.11 551	0.88 449	9.99 633	34
27	9.11 281	9.11 649	0.88 351	9.99 632	33
28	9.11 377	9.11 747	0.88 253	9.99 630	32
29	9.11 474	9.11 845	0.88 155	9.99 629	31
30	9.11 570	9.11 943	0.88 057	9.99 627	**30**
31	9.11 666	9.12 040	0.87 960	9.99 625	29
32	9.11 761	9.12 138	0.87 862	9.99 624	28
33	9.11 857	9.12 235	0.87 765	9.99 622	27
34	9.11 952	9.12 332	0.87 668	9.99 620	26
35	9.12 047	9.12 428	0.87 572	9.99 618	25
36	9.12 142	9.12 525	0.87 475	9.99 617	24
37	9.12 236	9.12 621	0.87 379	9.99 615	23
38	9.12 331	9.12 717	0.87 283	9.99 613	22
39	9.12 425	9.12 813	0.87 187	9.99 612	21
40	9.12 519	9.12 909	0.87 091	9.99 610	**20**
41	9.12 612	9.13 004	0.86 996	9.99 608	19
42	9.12 706	9.13 099	0.86 901	9.99 607	18
43	9.12 799	9.13 194	0.86 806	9.99 605	17
44	9.12 892	9.13 289	0.86 711	9.99 603	16
45	9.12 985	9.13 384	0.86 616	9.99 601	15
46	9.13 078	9.13 478	0.86 522	9.99 600	14
47	9.13 171	9.13 573	0.86 427	9.99 598	13
48	9.13 263	9.13 667	0.86 333	9.99 596	12
49	9.13 355	9.13 761	0.86 239	9.99 595	11
50	9.13 447	9.13 854	0.86 146	9.99 593	**10**
51	9.13 539	9.13 948	0.86 052	9.99 591	9
52	9.13 630	9.14 041	0.85 959	9.99 589	8
53	9.13 722	9.14 134	0.85 866	9.99 588	7
54	9.13 813	9.14 227	0.85 773	9.99 586	6
55	9.13 904	9.14 320	0.85 680	9.99 584	5
56	9.13 994	9.14 412	0.85 588	9.99 582	4
57	9.14 085	9.14 504	0.85 496	9.99 581	3
58	9.14 175	9.14 597	0.85 403	9.99 579	2
59	9.14 266	9.14 688	0.85 312	9.99 577	1
60	9.14 356	9.14 780	0.85 220	9.99 575	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

82°

8°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.14 356	9.14 780	0.85 220	9.99 575	60
1	9.14 445	9.14 872	0.85 128	9.99 574	59
2	9.14 53$\bar{5}$	9.14 963	0.85 037	9.99 572	58
3	9.14 624	9.15 054	0.84 946	9.99 570	57
4	9.14 714	9.15 145	0.84 85$\bar{5}$	9.99 568	56
5	9.14 803	9.15 236	0.84 764	9.99 566	55
6	9.14 891	9.15 327	0.84 673	9.99 56$\bar{5}$	54
7	9·14 980	9.15 417	0.84 583	9.99 563	53
8	9.15 069	9.15 508	0.84 492	9.99 561	52
9	9.15 157	9.15 598	0.84 402	9.99 559	51
10	9.15 245	9.15 688	0.84 312	9.99 557	**50**
11	9.15 333	9.15 777	0.84 223	9.99 556	49
12	9.15 421	9.15 867	0.84 133	9.99 554	48
13	9.15 508	9.15 956	0.84 044	9.99 552	47
14	9.15 596	9.16 046	0.83 954	9.99 550	46
15	9.15 683	9.16 135	0.83 865	9.99 548	45
16	9.15 770	9.16 224	0 83 776	9.99 546	44
17	9.15 857	9.16 312	0.83 688	9.99 54$\bar{5}$	43
18	9.15 944	9.16 401	0.83 599	9.99 543	42
19	9.16 030	9.16 489	0.83 511	9.99 541	41
20	9.16 116	9.16 577	0.83 423	9.99 539	**40**
21	9.16 203	9.16 665	0.83 33$\bar{5}$	9.99 537	39
22	9.16 289	9.16 753	0.83 247	9.99 535	38
23	9.16 374	9.16 841	0.83 159	9.99 533	37
24	9.16 460	9.16 928	0.83 072	9.99 532	36
25	9.16 545	9.17 016	0.82 984	9.99 530	35
26	9.16 631	9.17 103	0.82 897	9.99 528	34
27	9.16 716	9.17 190	0.82 810	9.99 526	33
28	9.16 801	9.17 277	0.82 723	9.99 524	32
29	9.16 886	9.17 363	0.82 637	9.99 522	31
30	9.16 970	9.17 450	0.82 550	9.99 520	**30**
31	9.17 05$\bar{5}$	9.17 536	0.82 464	9.99 518	29
32	9.17 139	9.17 622	0.82 378	9.99 517	28
33	9.17 223	9.17 708	0.82 292	9.99 51$\bar{5}$	27
34	9.17 307	9.17 794	0.82 206	9.99 513	26
35	9.17 391	9.17 880	0.82 120	9.99 511	25
36	9.17 474	9.17 965	0.82 03$\bar{5}$	9.99 509	24
37	9.17 558	9.18 051	0.81 949	9.99 507	23
38	9.17 641	9.18 136	0.81 864	9.99 505	22
39	9.17 724	9.18 221	0.81 779	9.99 503	21
40	9.17 807	9.18 306	0.81 694	9.99 501	**20**
41	9.17 890	9.18 391	0.81 609	9.99 499	19
42	9.17 973	9.18 475	0.81 52$\bar{5}$	9.99 497	18
43	9.18 055	9.18 560	0.81 440	9.99 495	17
44	9.18 137	9.18 644	0.81 356	9.99 494	16
45	9.18 220	9.18 728	0.81 272	9.99 492	15
46	9.18 302	9.18 812	0.81 188	9.99 490	14
47	9.18 383	9.18 896	0.81 104	9.99 488	13
48	9.18 465	9.18 979	0.81 021	9.99 486	12
49	9.18 547	9.19 063	0.80 937	9.99 484	11
50	9.18 628	9.19 146	0.80 854	9.99 482	**10**
51	9.18 709	9.19 229	0.80 771	9.99 480	9
52	9.18 790	9.19 312	0.80 688	9.99 478	8
53	9.18 871	9.19 395	0.80 60$\bar{5}$	9.99 476	7
54	9.18 952	9.19 478	0.80 522	9.99 474	6
55	9.19 033	9.19 561	0.80 439	9.99 472	5
56	9.19 113	9.19 643	0.80 357	9.99 470	4
57	9.19 193	9.19 725	0.80 275	9.99 468	3
58	9.19 273	9.19 807	0.80 193	9.99 466	2
59	9.19 353	9.19 889	0.80 111	9.99 464	1
60	9.19 433	9.19 971	0.80 029	9.99 462	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

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9°

′	L. Sin.	L. Tan.	L. Cotg.	L. Cos.	
0	9.19 433	9.19 971	0.80 029	9.99 462	**60**
1	9.19 513	9.20 053	0.79 947	9.99 460	59
2	9.19 592	9.20 134	0.79 866	9.99 458	58
3	9.19 672	9.20 216	0.79 784	9.99 456	57
4	9.19 751	9.20 297	0.79 703	9.99 454	56
5	9.19 830	9.20 378	0.79 622	9.99 452	55
6	9.19 909	9.20 459	0.79 541	9.99 450	54
7	9.19 988	9.20 540	0.79 460	9.99 448	53
8	9.20 067	9.20 621	0.79 379	9.99 446	52
9	9.20 145	9.20 701	0.79 299	9.99 444	51
10	9.20 223	9.20 782	0.79 218	9.99 442	**50**
11	9.20 302	9.20 862	0.79 138	9.99 440	49
12	9.20 380	9.20 942	0.79 058	9.99 438	48
13	9.20 458	9.21 022	0.78 978	9.99 436	47
14	9.20 535	9.21 102	0.78 898	9.99 434	46
15	9.20 613	9.21 182	0.78 818	9.99 432	45
16	9.20 691	9.21 261	0.78 739	9.99 429	44
17	9.20 768	9.21 341	0.78 659	9.99 427	43
18	9.20 845	9.21 420	0.78 580	9.99 425	42
19	9.20 922	9.21 499	0.78 501	9.99 423	41
20	9.20 999	9.21 578	0.78 422	9.99 421	**40**
21	9.21 076	9.21 657	0.78 343	9.99 419	39
22	9.21 153	9.21 736	0.78 264	9.99 417	38
23	9.21 229	9.21 814	0.78 186	9.99 415	37
24	9.21 306	9.21 893	0.78 107	9.99 413	36
25	9.21 382	9.21 971	0.78 029	9.99 411	35
26	9.21 458	9.22 049	0.77 951	9.99 409	34
27	9.21 534	9.22 127	0.77 873	9.99 407	33
28	9.21 610	9.22 205	0.77 795	9.99 404	32
29	9.21 685	9.22 283	0.77 717	9.99 402	31
30	9.21 761	9.22 361	0.77 639	9.99 400	**30**
31	9.21 836	9.22 438	0.77 562	9.99 398	29
32	9.21 912	9.22 516	0.77 484	9.99 396	28
33	9.21 987	9.22 593	0.77 407	9.99 394	27
34	9.22 062	9.22 670	0.77 330	9.99 392	26
35	9.22 137	9.22 747	0.77 253	9.99 390	25
36	9.22 211	9.22 824	0.77 176	9.99 388	24
37	9.22 286	9.22 901	0.77 099	9.99 385	23
38	9.22 361	9.22 977	0.77 023	9.99 383	22
39	9.22 435	9.23 054	0.76 946	9.99 381	21
40	9.22 509	9.23 130	0.76 870	9.99 379	**20**
41	9.22 583	9.23 206	0.76 794	9.99 377	19
42	9.22 657	9.23 283	0.76 717	9.99 375	18
43	9.22 731	9.23 359	0.76 641	9.99 372	17
44	9.22 805	9.23 435	0.76 565	9.99 370	16
45	9.22 878	9.23 510	0.76 490	9.99 368	15
46	9.22 952	9.23 586	0.76 414	9.99 366	14
47	9.23 025	9.23 661	0.76 339	9.99 364	13
48	9.23 098	9.23 737	0.76 263	9.99 362	12
49	9.23 171	9 23 812	0.76 188	9.99 359	11
50	9.23 244	9.23 887	0.76 113	9.99 357	**10**
51	9.23 317	9.23 962	0.76 038	9.99 355	9
52	9.23 390	9.24 037	0.75 963	9.99 353	8
53	9.23 462	9.24 112	0.75 888	9.99 351	7
54	9.23 535	9.24 186	0.75 814	9.99 348	6
55	9.23 607	9.24 261	0.75 739	9.99 346	5
56	9.23 679	9.24 335	0.75 665	9.99 344	4
57	9.23 752	9.24 410	0.75 590	9.99 342	3
58	9.23 823	9.24 484	0.75 516	9.99 340	2
59	9.23 895	9.24 558	0.75 442	9.99 337	1
60	9.23 967	9.24 632	0.75 368	9.99 335	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

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10°

'	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.23 967	9.24 632	0.75 368	9.99 335	**60**
1	9.24 039	9.24 706	0.75 294	9.99 333	59
2	9.24 110	9.24 779	0.75 221	9.99 331	58
3	9.24 181	9.24 853	0.75 147	9.99 328	57
4	9.24 253	9.24 926	0.75 074	9.99 326	56
5	9.24 324	9.2$\bar{5}$ 000	0.75 000	9.99 324	55
6	9.24 39$\bar{5}$	9.25 073	0.74 927	9.99 322	54
7	9.24 466	9.25 146	0.74 854	9.99 319	53
8	9.24 536	9.25 219	0.74 781	9.99 317	52
9	9.24 607	9.25 292	0.74 708	9.99 31$\bar{5}$	51
10	9.24 677	9.25 36$\bar{5}$	0.74 635	9.99 313	**50**
11	9.24 748	9.25 437	0 74 563	9.99 310	49
12	9.24 818	9.25 510	0.74 490	9.99 308	48
13	9.24 888	9.25 582	0.74 418	9.99 306	47
14	9.24 958	9.25 65$\bar{5}$	0.74 345	9.99 304	46
15	9.25 028	9.25 727	0.74 273	9.99 301	45
16	9.25 098	9.25 799	0.74 201	9.99 299	44
17	9.25 168	9.25 871	0.74 129	9.99 297	43
18	9.25 237	9.25 943	0.74 057	9.99 294	42
19	9.25 307	9.26 01$\bar{5}$	0.73 985	9.99 292	41
20	9.25 376	9.26 086	0.73 914	9.99 290	**40**
21	9.25 445	9.26 158	0.73 842	9.99 288	39
22	9.25 514	9.26 229	0.73 771	9.99 285	38
23	9.25 583	9.26 301	0.73 699	9.99 283	37
24	9.25 652	9.26 372	0.73 628	9.99 281	36
25	9.25 721	9.26 443	0.73 557	9.99 278	35
26	9.25 790	9.26 514	0.73 486	9.99 276	34
27	9.25 858	9.26 58$\bar{5}$	0.73 415	9.99 274	33
28	9.25 927	9.26 655	0.73 34$\bar{5}$	9.99 271	32
29	9.25 995	9.26 726	0.73 274	9.99 269	31
30	9.26 063	9.26 797	0.73 203	9.99 267	**30**
31	9.26 131	9.26 867	0.73 133	9.99 264	29
32	9.26 199	9.26 937	0.73 063	9.99 262	28
33	9.26 267	9.27 008	0.72 992	9.99 260	27
34	9.26 335	9.27 078	0.72 922	9.99 257	26
35	9.26 403	9.27 148	0.72 852	9.99 25$\bar{5}$	25
36	9.26 470	9.27 218	0.72 782	9.99 252	24
37	9.26 538	9.27 288	0.72 712	9.99 250	23
38	9.26 605	9.27 357	0.72 643	9.99 248	22
39	9.26 672	9.27 427	0.72 573	9.99 245	21
40	9.26 739	9.27 496	0.72 504	9.99 243	**20**
41	9.26 806	9.27 566	0.72 434	9.99 241	19
42	9.26 873	9.27 635	0.72 36$\bar{5}$	9.99 238	18
43	9.26 940	9.27 704	0.72 296	9.99 236	17
44	9.27 007	9.27 773	0.72 227	9.99 233	16
45	9.27 073	9.27 842	0.72 158	9.99 231	15
46	9.27 140	9.27 911	0.72 089	9.99 229	14
47	9.27 206	9.27 980	0.72 020	9.99 226	13
48	9.27 273	9.28 049	0.71 951	9.99 224	12
49	9.27 339	9.28 117	0.71 883	9.99 221	11
50	9.27 40$\bar{5}$	9.28 186	0.71 814	9.99 219	**10**
51	9.27 471	9.28 254	0.71 746	9.99 217	9
52	9.27 537	9.28 323	0.71 677	9.99 214	8
53	9.27 602	9.28 391	0.71 609	9.99 212	7
54	9.27 668	9.28 459	0.71 541	9.99 209	6
55	9.27 734	9.28 527	0.71 473	9.99 207	5
56	9.27 799	9.28 59$\bar{5}$	0.71 405	9.99 204	4
57	9.27 864	9.28 662	0.71 338	9.99 202	3
58	9.27 930	9.28 730	0.71 270	9.99 200	2
59	9.27 99$\bar{5}$	9.28 798	0.71 202	9.99 197	1
60	9.28 060	9.28 865	0.71 13$\bar{5}$	9.99 19$\bar{5}$	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Cos.	'

79°

	'	L. Sin.	L. Tang.	L. Cotg.	L. Cos.		
	0	9.28 060	9.28 865	0.71 13$\bar{5}$	9.99 19$\bar{5}$	**60**	
	1	9.28 12$\bar{5}$	9.28 933	0.71 067	9.99 192	59	
	2	9.28 190	9.29 000	0.71 000	9.99 190	58	
	3	9.28 254	9.29 067	0.70 933	9.99 187	57	
	4	9.28 319	9.29 134	0.70 866	9.99 18$\bar{5}$	56	
	5	9.28 384	9.29 201	0.70 799	9.99 182	55	
	6	9.28 448	9.29 268	0.70 732	9.99 180	54	
	7	9.28 512	9.29 33$\bar{5}$	0.70 665	9.99 177	53	
	8	9.28 577	9.29 402	0.70 598	9.99 17$\bar{5}$	52	
	9	9.28 641	9.29 468	0.70 532	9.99 172	51	
	10	9.28 70$\bar{5}$	9.29 53$\bar{5}$	0.70 465	9.99 170	**50**	
	11	9.28 769	9.29 601	0.70 399	9.99 167	49	
	12	9.28 833	9.29 668	0.70 332	9.99 16$\bar{5}$	48	
	13	9.28 896	9.29 734	0.70 266	9.99 162	47	
	14	9.28 960	9.29 800	0.70 200	9.99 160	46	
	15	9.29 024	9.29 866	0.70 134	9.99 157	45	
	16	9.29 087	9.29 932	0.70 068	9.99 15$\bar{5}$	44	
	17	9.29 150	9.29 998	0.70 002	9.99 152	43	
	18	9.29 214	9.30 064	0.69 936	9.99 1$\bar{5}$0	42	
	19	9.29 277	9.30 130	0.69 870	9.99 147	41	
	20	9.29 340	9.30 195	0.69 80$\bar{5}$	9.99 14$\bar{5}$	**40**	
	21	9.29 403	9.30 261	0.69 739	9.99 142	39	
	22	9.29 466	9.30 326	0.69 674	9.99 140	38	
	23	9.29 529	9.30 391	0.69 609	9.99 137	37	
	24	9.29 591	9.30 457	0.69 543	9.99 13$\bar{5}$	36	
	25	9.29 654	9.30 522	0.69 478	9.99 132	35	
	26	9.29 716	9.30 587	0.69 413	9.99 130	34	
	27	9.29 779	9.30 652	0.69 348	9.99 127	33	
	28	9.29 841	9.30 717	0.69 283	9.99 124	32	
11°	29	9.29 903	9.30 782	0.69 218	9.99 122	31	**78°**
	30	9.29 966	9.30 846	0.69 154	9.99 119	**30**	
	31	9.30 028	9.30 911	0.69 089	9.99 117	29	
	32	9.30 090	9.30 975	0.69 02$\bar{5}$	9.99 114	28	
	33	9.30 151	9.31 040	0.68 960	9.99 112	27	
	34	9.30 213	9.31 104	0.68 896	9.99 109	26	
	35	9.30 27$\bar{5}$	9.31 168	0.68 832	9.99 106	25	
	36	9.30 336	9.31 233	0.68 767	9.99 104	24	
	37	9.30 398	9.31 297	0.68 703	9.99 101	23	
	38	9.30 459	9.31 361	0.68 639	9.99 099	22	
	39	9.30 521	9.31 42$\bar{5}$	0.68 575	9.99 096	21	
	40	9.30 582	9.31 489	0.68 511	9.99 093	**20**	
	41	9.30 643	9.31 552	0.68 448	9.99 091	19	
	42	9.30 704	9.31 616	0.68 384	9.99 088	18	
	43	9.30 765	9.31 679	0.68 321	9.99 086	17	
	44	9.30 826	9.31 743	0.68 257	9.99 083	16	
	45	9.30 887	9.31 806	0.68 194	9.99 080	15	
	46	9.30 947	9.31 870	0.68 130	9.99 078	14	
	47	9.31 008	9.31 933	0.68 067	9.99 075	13	
	48	9.31 068	9.31 996	0.68 004	9.99 072	12	
	49	9.31 129	9.32 059	0.67 941	9.99 070	11	
	50	9.31 189	9.32 122	0.67 878	9.99 067	**10**	
	51	9.31 2$\bar{5}$0	9.32 185	0.67 81$\bar{5}$	9.99 064	9	
	52	9.31 310	9.32 248	0.67 752	9.99 062	8	
	53	9.31 370	9.32 311	0.67 689	9.99 059	7	
	54	9.31 430	9.32 373	0.67 627	9.99 056	6	
	55	9.31 490	9.32 436	0.67 564	9.99 054	5	
	56	9.31 549	9.32 498	0.67 502	9.99 051	4	
	57	9.31 609	9.32 561	0.67 439	9.99 048	3	
	58	9.31 669	9.32 623	0.67 377	9.99 046	2	
	59	9.31 728	9.32 685	0.67 31$\bar{5}$	9.99 043	1	
	60	9.31 788	9.32 747	0.67 253	9.99 040	**0**	
		L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	'	

	′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.		
	0	9.31 788	9.32 747	0.67 253	9.99 040	60	
	1	9.31 847	9.32 810	0.67 190	9.99 038	59	
	2	9.31 907	9.32 872	0.67 128	9.99 035	58	
	3	9.31 966	9.32 933	0.67 067	9.99 032	57	
	4	9.32 02$\bar{5}$	9.32 995	0.67 00$\bar{5}$	9.99 030	56	
	5	9.32 084	9.33 057	0.66 943	9.99 027	55	
	6	9.32 143	9.33 119	0.66 881	9.99 024	54	
	7	9.32 202	9.33 180	0.66 820	9.99 022	53	
	8	9.32 261	9.33 242	0.66 758	9.99 019	52	
	9	9.32 319	9.33 303	0.66 697	9.99 016	51	
	10	9.32 378	9.33 36$\bar{5}$	0.66 635	9.99 013	50	
	11	9.32 437	9.33 426	0.66 574	9.99 011	49	
	12	9.32 495	9.33 487	0.66 513	9.99 008	48	
	13	9.32 553	9.33 548	0.66 452	9.99 005	47	
	14	9.32 612	9.33 609	0.66 391	9.99 002	46	
	15	9.32 670	9.33 670	0.66 330	9.99 000	45	
	16	9.32 728	9.33 731	0.66 269	9.98 997	44	
	17	9.32 786	9.33 792	0.66 208	9.98 994	43	
	18	9.32 844	9.33 853	0.66 147	9.98 991	42	
	19	9.32 902	9.33 913	0.66 087	9.98 989	41	
	20	9.32 960	9.33 974	0.66 026	9.98 986	40	
	21	9.33 018	9.34 034	0.65 966	9.98 983	39	
	22	9.33 075	9.34 09$\bar{5}$	0.65 905	9.98 980	38	
	23	9.33 133	9.34 155	0.65 84$\bar{5}$	9.98 978	37	
	24	9.33 190	9.34 215	0.65 78$\bar{5}$	9.98 97$\bar{5}$	36	
	25	9.33 248	9.34 276	0.65 724	9.98 972	35	
	26	9.33 305	9.34 336	0.65 664	9.98 969	34	
	27	9.33 362	9.34 396	0.65 604	9.98 967	33	
	28	9.33 420	9.34 456	0.65 544	9.98 964	32	
12°	29	9.33 477	9.34 516	0.65 484	9.98 961	31	77°
	30	9.33 534	9.34 576	0.65 424	9.98 958	30	
	31	9.33 591	9.34 635	0.65 36$\bar{5}$	9.98 955	29	
	32	9.33 647	9.34 69$\bar{5}$	0.65 305	9.98 953	28	
	33	9.33 704	9.34 75$\bar{5}$	0.65 245	9.98 9$\bar{5}$0	27	
	34	9.33 761	9.34 814	0.65 186	9.98 947	26	
	35	9.33 818	9.34 874	0.65 126	9.98 944	25	
	36	9.33 874	9.34 933	0.65 067	9.98 941	24	
	37	9.33 931	9.34 992	0.65 008	9.98 938	23	
	38	9.33 987	9.35 051	0.64 949	9.98 936	22	
	39	9.34 043	9.35 111	0.64 889	9.98 933	21	
	40	9.34 100	9.35 170	0.64 830	9.98 930	20	
	41	9.34 156	9.35 229	0.64 771	9.98 927	19	
	42	9.34 212	9.35 288	0.64 712	9.98 924	18	
	43	9.34 268	9.35 347	0.64 653	9.98 921	17	
	44	9.34 324	9.35 405	0.64 59$\bar{5}$	9.98 919	16	
	45	9.34 380	9.35 464	0.64 536	9.98 916	15	
	46	9.34 436	9.35 523	0.64 477	9.98 913	14	
	47	9.34 491	9.35 581	0.64 419	9.98 910	13	
	48	9.34 547	9.35 640	0.64 360	9.98 907	12	
	49	9.34 602	9.35 698	0.64 302	9.98 904	11	
	50	9.34 658	9.35 757	0.64 243	9.98 901	10	
	51	9.34 713	9.35 81$\bar{5}$	0.64 185	9.98 898	9	
	52	9.34 769	9.35 873	0.64 127	9.98 896	8	
	53	9.34 824	9.35 931	0.64 069	9.98 893	7	
	54	9.34 879	9.35 989	0.64 011	9.98 890	6	
	55	9.34 934	9.36 047	0.63 953	9.98 887	5	
	56	9.34 989	9.36 105	0.63 89$\bar{5}$	9.98 884	4	
	57	9.35 044	9.36 163	0.63 837	9.98 881	3	
	58	9.35 099	9.36 221	0.63 779	9.98 878	2	
	59	9.35 154	9.36 279	0.63 721	9.98 875	1	
	60	9.35 209	9.36 336	0.63 664	9.98 872	0	
		L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′	

13°	′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.		76°
	0	9.35 209	9.36 336	0.63 664	9.98 872	**60**	
	1	9.35 263	9.36 394	0.63 606	9.98 869	59	
	2	9.35 318	9.36 452	0.63 548	9.98 867	58	
	3	9.35 373	9.36 509	0.63 491	9.98 864	57	
	4	9.35 427	9.36 566	0.63 434	9.98 861	56	
	5	9.35 481	9.36 624	0.63 376	9.98 858	55	
	6	9.35 536	9.36 681	0.63 319	9.98 85$\bar{5}$	54	
	7	9.35 590	9.36 738	0.63 262	9.98 852	53	
	8	9.35 644	9.36 795	0.63 20$\bar{5}$	9.98 849	52	
	9	9.35 698	9.36 852	0.63 148	9.98 846	51	
	10	9.35 752	9.36 909	0.63 091	9.98 843	**50**	
	11	9.35 806	9.36 966	0.63 034	9.98 840	49	
	12	9.35 860	9.37 023	0.62 977	9.98 837	48	
	13	9.35 914	9.37 080	0.62 920	9.98 834	47	
	14	9.35 968	9.37 137	0.62 863	9.98 831	46	
	15	9.36 022	9.37 193	0.62 807	9.98 828	45	
	16	9.36 075	9.37 2$\bar{5}$0	0.62 750	9.98 825	44	
	17	9.36 129	9.37 306	0.62 694	9.98 822	43	
	18	9.36 182	9.37 363	0.62 637	9.98 819	42	
	19	9.36 236	9.37 419	0.62 581	9.98 816	41	
	20	9.36 289	9.37 476	0.62 524	9.98 813	**40**	
	21	9.36 342	9.37 532	0.62 468	9.98 810	39	
	22	9.36 395	9.37 588	0.62 412	9.98 807	38	
	23	9.36 449	9.37 644	0.62 356	9.98 804	37	
	24	9.36 502	9.37 700	0.62 300	9.98 801	36	
	25	9.36 55$\bar{5}$	9.37 756	0.62 244	9.98 798	35	
	26	9.36 608	9.37 812	0.62 188	9.98 795	34	
	27	9.36 660	9.37 868	0.62 132	9.98 792	33	
	28	9.36 713	9.37 924	0.62 076	9.98 789	32	
	29	9.36 766	9.37 980	0.62 020	9.98 786	31	
	30	9.36 819	9.38 035	0.61 96$\bar{5}$	9.98 783	**30**	
	31	9.36 871	9.38 091	0.61 909	9.98 780	29	
	32	9.36 924	9.38 147	0.61 853	9.98 777	28	
	33	9.36 976	9.38 202	0.61 798	9.98 774	27	
	34	9.37 028	9.38 257	0.61 743	9.98 771	26	
	35	9.37 081	9.38 313	0.61 687	9.98 768	25	
	36	9.37 133	9.38 368	0.61 632	9.98 76$\bar{5}$	24	
	37	9.37 185	9.38 423	0.61 577	9.98 762	23	
	38	9.37 237	9.38 479	0.61 521	9.98 759	22	
	39	9.37 289	9.38 534	0.61 466	9.98 756	21	
	40	9.37 341	9.38 589	0.61 411	9.98 753	**20**	
	41	9.37 393	9.38 644	0.61 356	9.98 7$\bar{5}$0	19	
	42	9.37 445	9.38 699	0.61 301	9.98 746	18	
	43	9.37 497	9.38 754	0.61 246	9.98 743	17	
	44	9.37 549	9.38 808	0.61 192	9.98 740	16	
	45	9.37 600	9.38 863	0.61 137	9.98 737	15	
	46	9.37 652	9.38 918	0.61 082	9.98 734	14	
	47	9.37 703	9.38 972	0.61 028	9.98 731	13	
	48	9.37 75$\bar{5}$	9.39 027	0.60 973	9.98 728	12	
	49	9.37 806	9.39 082	0.60 918	9.98 72$\bar{5}$	11	
	50	9.37 858	9.39 136	0.60 864	9.98 722	**10**	
	51	9.37 909	9.39 190	0.60 810	9.98 719	9	
	52	9.37 960	9.39 24$\bar{5}$	0.60 755	9.98 715	8	
	53	9.38 011	9.39 299	0.60 701	9.98 712	7	
	54	9.38 062	9.39 353	0.60 647	9.98 709	6	
	55	9.38 113	9.39 407	0.60 593	9.98 706	5	
	56	9.38 164	9.39 461	0.60 539	9.98 703	4	
	57	9.38 215	9.39 515	0.60 48$\bar{5}$	9.98 700	3	
	58	9.38 266	9.39 569	0.60 431	9.98 697	2	
	59	9.38 317	9.39 623	0.60 377	9.98 694	1	
	60	9.38 368	9.39 677	0.60 323	9.98 690	**0**	
		L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′	

14°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.38 368	9.39 677	0.60 323	9.98 690	**60**
1	9.38 418	9.39 731	0.60 269	9.98 687	59
2	9.38 469	9.39 78$\bar{5}$	0.60 215	9.98 684	58
3	9.38 519	9.39 838	0.60 162	9.98 681	57
4	9.38 570	9.39 892	0.60 108	9.98 678	56
5	9.38 620	9.39 945	0.60 05$\bar{5}$	9.98 67$\bar{5}$	55
6	9.38 670	9.39 999	0.60 001	9.98 671	54
7	9.38 721	9.40 052	0.59 948	9.98 668	53
8	9.38 771	9.40 106	0.59 894	9.98 665	52
9	9.38 821	9.40 159	0.59 841	9.98 662	51
10	9.38 871	9.40 212	0.59 788	9.98 659	**50**
11	9.38 921	9.40 266	0.59 734	9.98 656	49
12	9.38 971	9.40 319	0.59 681	9.98 652	48
13	9.39 021	9.40 372	0.59 628	9.98 649	47
14	9.39 071	9.40 42$\bar{5}$	0.59 575	9.98 646	46
15	9.39 121	9.40 478	0.59 522	9.98 643	45
16	9.39 170	9.40 531	0.59 469	9.98 640	44
17	9.39 220	9.40 584	0.59 416	9.98 636	43
18	9.39 270	9.40 636	0.59 364	9.98 633	42
19	9.39 319	9.40 689	0.59 311	9.98 630	41
20	9.39 369	9.40 742	0.59 258	9.98 627	**40**
21	9.39 418	9.40 79$\bar{5}$	0.59 205	9.98 623	39
22	9.39 467	9.40 847	0.59 153	9.98 620	38
23	9.39 517	9.40 900	0.59 100	9.98 617	37
24	9.39 566	9.40 952	0.59 048	9.98 614	36
25	9.39 61$\bar{5}$	9.41 00$\bar{5}$	0.58 995	9.98 610	35
26	9.39 664	9.41 057	0.58 943	9.98 607	34
27	9.39 713	9.41 109	0.58 891	9.98 604	33
28	9.39 762	9.41 161	0.58 839	9.98 601	32
29	9.39 811	9.41 214	0.58 786	9.98 597	31
30	9.39 860	9.41 266	0.58 734	9.98 594	**30**
31	9.39 909	9.41 318	0.58 682	9.98 591	29
32	9.39 958	9.41 370	0.58 630	9.98 588	28
33	9.40 006	9.41 422	0.58 578	9.98 584	27
34	9.40 05$\bar{5}$	9.41 474	0.58 526	9.98 581	26
35	9.40 103	9.41 526	0.58 474	9.98 578	25
36	9.40 152	9.41 578	0.58 422	9.98 574	24
37	9.40 200	9.41 629	0.58 371	9.98 571	23
38	9.40 249	9.41 681	0.58 319	9.98 568	22
39	9.40 297	9.41 733	0.58 267	9.98 56$\bar{5}$	21
40	9.40 346	9.41 784	0.58 216	9.98 561	**20**
41	9.40 394	9.41 836	0.58 164	9.98 558	19
42	9.40 442	9.41 887	0.58 113	9.98 55$\bar{5}$	18
43	9.40 490	9.41 939	0.58 061	9.98 551	17
44	9.40 538	9.41 990	0.58 010	9.98 548	16
45	9.40 586	9.42 041	0.57 959	9.98 54$\bar{5}$	15
46	9.40 634	9.42 093	0.57 907	9.98 541	14
47	9.40 682	9.42 144	0.57 856	9.98 538	13
48	9.40 730	9.42 195	0.57 80$\bar{5}$	9.98 53$\bar{5}$	12
49	9.40 778	9.42 246	0.57 754	9.98 531	11
50	9.40 825	9.42 297	0.57 703	9.98 528	**10**
51	9.40 873	9.42 348	0.57 652	9.98 52$\bar{5}$	9
52	9.40 921	9.42 399	0.57 601	9.98 521	8
53	9.40 968	9.42 450	0.57 550	9.98 518	7
54	9.41 016	9.42 501	0.57 499	9.98 51$\bar{5}$	6
55	9.41 063	9.42 552	0.57 448	9.98 511	5
56	9.41 111	9.42 603	0.57 397	9.98 508	4
57	9.41 158	9.42 653	0.57 347	9.98 50$\bar{5}$	3
58	9.41 205	9.42 704	0.57 296	9.98 501	2
59	9.41 252	9.42 75$\bar{5}$	0.57 245	9.98 498	1
60	9.41 300	9.42 805	0.57 19$\bar{5}$	9.98 494	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

75°

15°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.41 300	9.42 805	0.57 195	9.98 494	**60**
1	9.41 347	9.42 856	0.57 144	9.98 491	59
2	9.41 394	9.42 906	0.57 094	9.98 488	58
3	9.41 441	9.42 957	0.57 043	9.98 484	57
4	9.41 488	9.43 007	0.56 993	9.98 481	56
5	9.41 53$\bar{5}$	9.43 057	0.56 943	9.98 477	55
6	9.41 582	9.43 108	0.56 892	9.98 474	54
7	9.41 628	9.43 158	0.56 842	9.98 471	53
8	9.41 675	9.43 208	0.56 792	9.98 467	52
9	9.41 722	9.43 258	0.56 742	9.98 464	51
10	9.41 768	9.43 308	0.56 692	9.98 460	**50**
11	9.41 81$\bar{5}$	9.43 358	0.56 642	9.98 457	49
12	9.41 861	9.43 408	0.56 592	9.98 453	48
13	9.41 908	9.43 458	0.56 542	9.98 450	47
14	9.41 954	9.43 508	0.56 492	9.98 447	46
15	9.42 001	9.43 558	0.56 442	9.98 443	45
16	9.42 047	9.43 607	0.56 393	9.98 440	44
17	9.42 093	9.43 657	0.56 343	9.98 436	43
18	9.42 140	9.43 707	0.56 293	9.98 433	42
19	9.42 186	9.43 756	0.56 244	9.98 429	41
20	9.42 232	9.43 806	0.56 194	9.98 426	**40**
21	9.42 278	9.43 855	0.56 14$\bar{5}$	9.98 422	39
22	9.42 324	9.43 90$\bar{5}$	0.56 095	9.98 419	38
23	9.42 370	9.43 954	0.56 046	9.98 415	37
24	9.42 416	9.44 004	0.55 996	9.98 412	36
25	9.42 461	9.44 053	0.55 947	9.98 409	35
26	9.42 507	9.44 102	0.55 898	9.98 405	34
27	9.42 553	9.44 151	0.55 849	9.98 402	33
28	9.42 599	9.44 201	0.55 799	9.98 398	32
29	9.42 644	9.44 2$\bar{5}$0	0.55 750	9.98 39$\bar{5}$	31
30	9.42 690	9.44 299	0.55 701	9.98 391	**30**
31	9.42 735	9.44 348	0.55 652	9.98 388	29
32	9.42 781	9.44 397	0.55 603	9.98 384	28
33	9.42 826	9.44 446	0.55 554	9.98 381	27
34	9.42 872	9.44 49$\bar{5}$	0.55 505	9.98 377	26
35	9.42 917	9.44 544	0.55 456	9.98 373	25
36	9.42 962	9.44 592	0.55 408	9.98 370	24
37	9.43 008	9.44 641	0.55 359	9.98 366	23
38	9.43 053	9.44 690	0.55 310	9.98 363	22
39	9.43 098	9.44 738	0.55 262	9.98 359	21
40	9.43 143	9.44 787	0.55 213	9.98 356	**20**
41	9.43 188	9.44 836	0.55 164	9.98 352	19
42	9.43 233	9.44 884	0.55 116	9.98 349	18
43	9.43 278	9.44 933	0.55 067	9.98 345	17
44	9.43 323	9.44 981	0.55 019	9.98 342	16
45	9.43 367	9.45 029	0.54 971	9.98 338	15
46	9.43 412	9.45 078	0.54 922	9.98 334	14
47	9.43 457	9.45 126	0.54 874	9.98 331	13
48	9.43 502	9.45 174	0.54 826	9.98 327	12
49	9.43 546	9.45 222	0.54 778	9.98 324	11
50	9.43 591	9.45 271	0.54 729	9.98 320	**10**
51	9.43 635	9 45 319	0.54 681	9.98 317	9
52	9.43 680	9.45 367	0.54 633	9.98 313	8
53	9.43 724	9.45 414	0.54 585	9.98 309	7
54	9.43 769	9.45 463	0.54 537	9.98 306	6
55	9.43 813	9.45 511	0.54 489	9.98 302	5
56	9.43 857	9.45 559	0.54 441	9.98 299	4
57	9.43 901	9·45 606	0.54 394	9.98 295	3
58	9.43 946	9.45 654	0.54 346	9.98 291	2
59	9.43 990	9.45 702	0.54 298	9.98 288	1
60	9.44 034	9.45 7$\bar{5}$0	0.54 250	9.98 284	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

74°

16°

'	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.44 034	9.45 750	0.54 250	9.98 284	**60**
1	9.44 078	9.45 797	0.54 203	9.98 281	59
2	9.44 122	9.45 845	0.54 155	9.98 277	58
3	9.44 166	9.45 892	0.54 108	9.98 273	57
4	9.44 210	9.45 940	0.54 060	9.98 270	56
5	9.44 253	9.45 987	0.54 013	9.98 266	55
6	9.44 297	9.46 035	0.53 965	9.98 262	54
7	9.44 341	9.46 082	0.53 918	9.98 259	53
8	9.44 385	9.46 130	0.53 870	9.98 255	52
9	9.44 428	9.46 177	0.53 823	9.98 251	51
10	9.44 472	9.46 224	0.53 776	9.98 248	**50**
11	9.44 516	9.46 271	0.53 729	9.98 244	49
12	9.44 559	9.46 319	0.53 681	9.98 240	48
13	9.44 602	9.46 366	0.53 634	9.98 237	47
14	9.44 646	9.46 413	0.53 587	9.98 233	46
15	9.44 689	9.46 460	0.53 540	9.98 229	45
16	9.44 733	9.46 507	0.53 493	9.98 226	44
17	9.44 776	9.46 554	0.53 446	9.98 222	43
18	9.44 819	9.46 601	0.53 399	9.98 218	42
19	9.44 862	9.46 648	0.53 352	9.98 215	41
20	9.44 905	9.46 694	0.53 306	9.98 211	**40**
21	9.44 948	9.46 741	0.53 259	9.98 207	39
22	9.44 992	9.46 788	0.53 212	9.98 204	38
23	9.45 035	9.46 835	0.53 165	9.98 200	37
24	9.45 077	9.46 881	0.53 119	9.98 196	36
25	9.45 120	9.46 928	0.53 072	9.98 192	35
26	9.45 163	9.46 975	0.53 025	9.98 189	34
27	9.45 206	9.47 021	0.52 979	9.98 185	33
28	9.45 249	9.47 068	0.52 932	9.98 181	32
29	9.45 292	9.47 114	0.52 886	9.98 177	31
30	9.45 334	9.47 160	0.52 840	9.98 174	**30**
31	9.45 377	9.47 207	0.52 793	9.98 170	29
32	9.45 419	9.47 253	0.52 747	9.98 166	28
33	9.45 462	9.47 299	0.52 701	9.98 162	27
34	9.45 504	9.47 346	0.52 654	9.98 159	26
35	9.45 547	9.47 392	0.52 608	9.98 155	25
36	9.45 589	9.47 438	0.52 562	9.98 151	24
37	9.45 632	9.47 484	0.52 516	9.98 147	23
38	9.45 674	9.47 530	0.52 470	9.98 144	22
39	9.45 716	9.47 576	0.52 424	9.98 140	21
40	9.45 758	9.47 622	0.52 378	9.98 136	**20**
41	9.45 801	9.47 668	0.52 332	9.98 132	19
42	9.45 843	9.47 714	0.52 286	9.98 129	18
43	9.45 885	9.47 760	0.52 240	9.98 125	17
44	9.45 927	9.47 806	0.52 194	9.98 121	16
45	9.45 969	9.47 852	0.52 148	9.98 117	15
46	9.46 011	9.47 897	0.52 103	9.98 113	14
47	9.46 053	9.47 943	0.52 057	9.98 110	13
48	9.46 095	9.47 989	0.52 011	9.98 106	12
49	9.46 136	9.48 035	0.51 965	9.98 102	11
50	9.46 178	9.48 080	0.51 920	9.98 098	**10**
51	9.46 220	9.48 126	0.51 874	9.98 094	9
52	9.46 262	9.48 171	0.51 829	9.98 090	8
53	9.46 303	9.48 217	0.51 783	9.98 087	7
54	9.46 345	9.48 262	0.51 738	9.98 083	6
55	9.46 386	9.48 307	0.51 693	9.98 079	5
56	9.46 428	9.48 353	0.51 647	9.98 075	4
57	9.46 469	9.48 398	0.51 602	9.98 071	3
58	9.46 511	9.48 443	0.51 557	9.98 067	2
59	9.46 552	9.48 489	0.51 511	9.98 063	1
60	9.46 594	9.48 534	0.51 466	9.98 060	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	'

73°

17°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.46 594	9.48 534	0.51 466	9.98 060	**60**
1	9.46 63$\bar{5}$	9.48 579	0.51 421	9.98 056	59
2	9.46 676	9.48 624	0.51 376	9.98 052	58
3	9.46 717	9.48 669	0.51 331	9.98 048	57
4	9.46 758	9.48 714	0.51 286	9.98 044	56
5	9.46 800	9.48 759	0.51 241	9.98 040	55
6	9.46 841	9.48 804	0·51 196	9.98 036	54
7	9.46 882	9.48 849	0.51 151	9.98 032	53
8	9.46 923	9.48 894	0.51 106	9.98 029	52
9	9.46 964	9.48 939	0.51 061	9.98 02$\bar{5}$	51
10	9.47 00$\bar{5}$	9.48 984	0.51 016	9.98 021	**50**
11	9.47 045	9.49 029	0.50 971	9.98 017	49
12	9.47 086	9.49 073	0.50 927	9.98 013	48
13	9.47 127	9.49 118	0.50 882	9.98 009	47
14	9.47 168	9.49 163	0.50 837	9.98 005	46
15	9.47 209	9.49 207	0.50 793	9.98 001	45
16	9.47 249	9.49 252	0.50 748	9.97 997	44
17	9.47 290	9.49 296	0.50 704	9.97 993	43
18	9·47 330	9.49 341	0.50 659	9.97 989	42
19	9.47 371	9.49 385	0.50 61$\bar{5}$	9.97 986	41
20	9.47 411	9.49 430	0.50 570	9.97 982	**40**
21	9.47 452	9.49 474	0.50 526	9.97 978	39
22	9.47 492	9.49 519	0.50 481	9.97 974	38
23	9.47 533	9.49 563	0.50 437	9.97 970	37
24	9.47 573	9.49 607	0.50 393	9.97 966	36
25	9.47 613	9.49 652	0.50 348	9.97 962	35
26	9.47 654	9.49 696	0.50 304	9.97 958	34
27	9.47 694	9.49 740	0.50 260	9.97 954	33
28	9.47 734	9.49 784	0.50 216	9.97 9$\bar{5}$0	32
29	9.47 774	9.49 828	0.50 172	9.97 946	31
30	9.47 814	9.49 872	0.50 128	9.97 942	**30**
31	9.47 854	9.49 916	0.50 084	9.97 938	29
32	9.47 894	9.49 960	0.50 040	9.97 934	28
33	9.47 934	9.50 004	0.49 996	9.97 930	27
34	9.47 974	9.50 048	0.49 952	9.97 926	26
35	9.48 014	9.50 092	0.49 908	9.97 922	25
36	9.48 054	9.50 136	0.49 864	9.97 918	24
37	9.48 094	9.50 180	0.49 820	9.97 914	23
38	9.48 133	9.50 223	0.49 777	9.97 910	22
39	9.48 173	9.50 267	0.49 733	9.97 906	21
40	9.48 213	9.50 311	0.49 689	9.97 902	**20**
41	9.48 252	9.50 35$\bar{5}$	0.49 645	9.97 898	19
42	9.48 292	9.50 398	0.49 602	9.97 894	18
43	9.48 332	9.50 442	0.49 558	9.97 890	17
44	9.48 371	9.50 485	0.49 51$\bar{5}$	9.97 886	16
45	9.48 411	9.50 529	0.49 471	9.97 882	15
46	9.48 450	9.50 572	0.49 428	9.97 878	14
47	9.48 490	9.50 616	0.49 384	9.97 874	13
48	9.48 529	9.50 659	0.49 341	9.97 870	12
49	9.48 568	9.50 703	0.49 297	9.97 866	11
50	9.48 607	9.50 746	0.49 254	9.97 861	**10**
51	9.48 647	9.50 789	0.49 211	9.97 857	9
52	9.48 686	9.50 833	0.49 167	9.97 853	8
53	9.48 725	9.50 876	0.49 124	9.97 849	7
54	9.48 764	9.50 919	0.49 081	9.97 845	6
55	9.48 803	9.50 962	0.49 038	9.97 841	5
56	9.48 842	9.51 005	0.48 99$\bar{5}$	9.97 837	4
57	9.48 881	9.51 048	0.48 952	9.97 833	3
58	9.48 920	9.51 092	0.48 908	9.97 829	2
59	9.48 959	9.51 13$\bar{5}$	0.48 865	9.97 82$\bar{5}$	1
60	9.48 998	9.51 178	0.48 822	9.97 821	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L.Sin.**	′

72°

18°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.48 998	9.51 178	0.48 822	9.97 821	**60**
1	9.49 037	9.51 221	0.48 779	9.97 817	59
2	9.49 076	9.51 264	0.48 736	9.97 812	58
3	9.49 115	9.51 306	0.48 694	9.97 808	57
4	9.49 153	9.51 349	0.48 651	9.97 804	56
5	9.49 192	9.51 392	0.48 608	9.97 800	55
6	9.49 231	9.51 435	0.48 565	9.97 796	54
7	9.49 269	9.51 478	0.48 522	9.97 792	53
8	9.49 308	9.51 520	0.48 480	9.97 788	52
9	9.49 347	9.51 563	0.48 437	9.97 784	51
10	9.49 385	9.51 606	0.48 394	9.97 779	**50**
11	9.49 424	9.51 648	0.48 352	9.97 775	49
12	9.49 462	9.51 691	0.48 309	9.97 771	48
13	9.49 500	9.51 734	0.48 266	9.97 767	47
14	9.49 539	9.51 776	0.48 224	9.97 763	46
15	9.49 577	9.51 819	0.48 181	9.97 759	45
16	9.49 615	9.51 861	0.48 139	9.97 754	44
17	9.49 654	9.51 903	0.48 097	9.97 750	43
18	9.49 692	9.51 946	0.48 054	9.97 746	42
19	9.49 730	9.51 988	0.48 012	9.97 742	41
20	9.49 768	9.52 031	0.47 969	9.97 738	**40**
21	9.49 806	9.52 073	0.47 927	9.97 734	39
22	9.49 844	9.52 115	0.47 885	9.97 729	38
23	9.49 882	9.52 157	0.47 843	9.97 725	37
24	9.49 920	9.52 200	0.47 800	9.97 721	36
25	9.49 958	9.52 242	0.47 758	9.97 717	35
26	9.49 996	9.52 284	0.47 716	9.97 713	34
27	9.50 034	9.52 326	0.47 674	9.97 708	33
28	9.50 072	9.52 368	0.47 632	9.97 704	32
29	9.50 110	9.52 410	0.47 590	9.97 700	31
30	9.50 148	9.52 452	0.47 548	9.97 696	**30**
31	9.50 185	9.52 494	0.47 506	9.97 691	29
32	9.50 223	9.52 536	0.47 464	9.97 687	28
33	9.50 261	9.52 578	0.47 422	9.97 683	27
34	9.50 298	9.52 620	0.47 380	9.97 679	26
35	9.50 336	9.52 661	0.47 339	9.97 674	25
36	9.50 374	9.52 703	0.47 297	9.97 670	24
37	9.50 411	9.52 745	0.47 255	9.97 666	23
38	9.50 449	9.52 787	0.47 213	9.97 662	22
39	9.50 486	9.52 829	0.47 171	9.97 657	21
40	9.50 523	9.52 870	0.47 130	9.97 653	**20**
41	9.50 561	9.52 912	0.47 088	9.97 649	19
42	9.50 598	9.52 953	0.47 047	9.97 645	18
43	9.50 635	9.52 995	0.47 005	9.97 640	17
44	9.50 673	9.53 037	0.46 963	9.97 636	16
45	9.50 710	9.53 078	0.46 922	9.97 632	15
46	9.50 747	9.53 120	0.46 880	9.97 628	14
47	9.50 784	9.53 161	0.46 839	9.97 623	13
48	9.50 821	9.53 202	0.46 798	9.97 619	12
49	9.50 858	9.53 244	0.46 756	9.97 615	11
50	9.50 896	9.53 285	0.46 715	9.97 610	**10**
51	9.50 933	9.53 327	0.46 673	9.97 606	9
52	9.50 970	9.53 368	0.46 632	9.97 602	8
53	9.51 007	9.53 409	0.46 591	9.97 597	7
54	9.51 043	9.53 450	0.46 550	9.97 593	6
55	9.51 080	9.53 492	0.46 508	9.97 589	5
56	9.51 117	9.53 533	0.46 467	9.97 584	4
57	9.51 154	9.53 574	0.46 426	9.97 580	3
58	9.51 191	9.53 615	0.46 385	9.97 576	2
59	9.51 227	9.53 656	0.46 344	9.97 571	1
60	9.51 264	9.53 697	0.46 303	9.97 567	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

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19°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.51 264	9.53 697	0.46 303	9.97 567	60
1	9.51 301	9.53 738	0.46 262	9.97 563	59
2	9.51 338	9.53 779	0.46 221	9.97 558	58
3	9.51 374	9.53 820	0.46 180	9.97 554	57
4	9.51 411	9.53 861	0.46 139	9.97 550	56
5	9.51 447	9.53 902	0.46 098	9.97 545	55
6	9.51 484	9.53 943	0.46 057	9.97 541	54
7	9.51 520	9.53 984	0.46 016	9.97 536	53
8	9.51 557	9.54 025	0.45 975	9.97 532	52
9	9.51 593	9.54 065	0.45 935	9.97 528	51
10	9.51 629	9.54 106	0.45 894	9.97 523	50
11	9.51 666	9.54 147	0.45 853	9.97 519	49
12	9.51 702	9.54 187	0.45 813	9.97 515	48
13	9.51 738	9.54 228	0.45 772	9.97 510	47
14	9.51 774	9.54 269	0.45 731	9.97 506	46
15	9.51 811	9.54 309	0.45 691	9.97 501	45
16	9.51 847	9.54 350	0.45 650	9.97 497	44
17	9.51 883	9.54 390	0.45 610	9.97 492	43
18	9.51 919	9.54 431	0.45 569	9.97 488	42
19	9.51 955	9.54 471	0.45 529	9.97 484	41
20	9.51 991	9.54 512	0.45 488	9.97 479	40
21	9.52 027	9.54 552	0.45 448	9.97 475	39
22	9.52 063	9.54 593	0.45 407	9.97 470	38
23	9.52 099	9.54 633	0.45 367	9.97 466	37
24	9.52 135	9.54 673	0.45 327	9.97 461	36
25	9.52 171	9.54 714	0.45 286	9.97 457	35
26	9.52 207	9.54 754	0.45 246	9.97 453	34
27	9.52 242	9.54 794	0.45 206	9.97 448	33
28	9.52 278	9.54 835	0.45 165	9.97 444	32
29	9.52 314	9.54 875	0.45 125	9.97 439	31
30	9.52 350	9.54 915	0.45 085	9.97 435	30
31	9.52 385	9.54 955	0.45 045	9.97 430	29
32	9.52 421	9.54 995	0.45 005	9.97 426	28
33	9.52 456	9.55 035	0.44 965	9.97 421	27
34	9.52 492	9.55 075	0.44 925	9.97 417	26
35	9.52 527	9.55 115	0.44 885	9.97 412	25
36	9.52 563	9.55 155	0.44 845	9.97 408	24
37	9.52 598	9.55 195	0.44 805	9.97 403	23
38	9.52 634	9.55 235	0.44 765	9.97 399	22
39	9.52 669	9.55 275	0.44 725	9.97 394	21
40	9.52 705	9.55 315	0.44 685	9.97 390	20
41	9.52 740	9.55 355	0.44 645	9.97 385	19
42	9.52 775	9.55 395	0.44 605	9.97 381	18
43	9.52 811	9.55 434	0.44 566	9.97 376	17
44	9.52 846	9.55 474	0.44 526	9.97 372	16
45	9.52 881	9.55 514	0.44 486	9.97 367	15
46	9.52 916	9.55 554	0.44 446	9.97 363	14
47	9.52 951	9.55 593	0.44 407	9.97 358	13
48	9.52 986	9.55 633	0.44 367	9.97 353	12
49	9.53 021	9.55 673	0.44 327	9.97 349	11
50	9.53 056	9.55 712	0.44 288	9.97 344	10
51	9.53 092	9.55 752	0.44 248	9.97 340	9
52	9.53 126	9.55 791	0.44 209	9.97 335	8
53	9.53 161	9.55 831	0.44 169	9.97 331	7
54	9.53 196	9.55 870	0.44 130	9.97 326	6
55	9.53 231	9.55 910	0.44 090	9.97 322	5
56	9.53 266	9.55 949	0.44 051	9.97 317	4
57	9.53 301	9.55 989	0.44 011	9.97 312	3
58	9.53 336	9.56 028	0.43 972	9.97 308	2
59	9.53 370	9.56 067	0.43 933	9.97 303	1
60	9.53 405	9.56 107	0.43 893	9.97 299	0
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

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20°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.53 405	9.56 107	0.43 893	9.97 299	**60**
1	9.53 440	9.56 146	0.43 854	9.97 294	59
2	9.53 475̅	9.56 185	0.43 815̅	9.97 289	58
3	9.53 509	9.56 224	0.43 776	9.97 285̅	57
4	9.53 544	9.56 264	0.43 736	9.97 280	56
5	9.53 578	9.56 303	0.43 697	9.97 276	55
6	9.53 613	9.56 342	0.43 658	9.97 271	54
7	9.53 647	9.56 381	0.43 619	9.97 266	53
8	9.53 682	9.56 420	0.43 580	9.97 262	52
9	9.53 716	9.56 459	0.43 541	9.97 257	51
10	9.53 751	9.56 498	0.43 502	9.97 252	**50**
11	9.53 785	9.56 537	0.43 463	9.97 248	49
12	9.53 819	9.56 576	0.43 424	9.97 243	48
13	9.53 854	9.56 615	0.43 385̅	9.97 238	47
14	9.53 888	9.56 654	0.43 346	9.97 234	46
15	9.53 922	9.56 693	0.43 307	9.97 229	45
16	9.53 957	9.56 732	0.43 268	9.97 224	44
17	9.53 991	9.56 771	0.43 229	9.97 220	43
18	9.54 025̅	9.56 810	0.43 190	9.97 215	42
19	9.54 059	9.56 849	0.43 151	9.97 210	41
20	9.54 093	9.56 887	0.43 113	9.97 206	**40**
21	9.54 127	9.56 926	0.43 074	9.97 201	39
22	9.54 161	9.56 965̅	0.43 035	9.97 196	38
23	9.54 195	9.57 004	0.42 996	9.97 192	37
24	9.54 229	9.57 042	0.42 958	9.97 187	36
25	9.54 263	9.57 081	0.42 919	9.97 182	35
26	9.54 297	9.57 120	0.42 880	9.97 178	34
27	9.54 331	9.57 158	0.42 842	9.97 173	33
28	9.54 365̅	9.57 197	0.42 803	9.97 168	32
29	9.54 399	9.57 235	0.42 765̅	9.97 163	31
30	9.54 433	9.57 274	0.42 726	9.97 159	**30**
31	9.54 466	9.57 312	0.42 688	9.97 154	29
32	9.54 500	9.57 351	0.42 649	9.97 149	28
33	9.54 534	9.57 389	0.42 611	9.97 145̅	27
34	9.54 567	9.57 428	0.42 572	9.97 140	26
35	9.54 601	9.57 466	0.42 534	9.97 135	25
36	9.54 635̅	9.57 504	0.42 496	9.97 130	24
37	9.54 668	9.57 543	0.42 457	9.97 126	23
38	9.54 702	9.57 581	0.42 419	9.97 121	22
39	9.54 735	9.57 619	0.42 381	9.97 116	21
40	9.54 769	9.57 658	0.42 342	9.97 111	**20**
41	9.54 802	9.57 696	0.42 304	9.97 107	19
42	9.54 836	9.57 734	0.42 266	9.97 102	18
43	9.54 869	9.57 772	0.42 228	9.97 097	17
44	9.54 903	9.57 810	0.42 190	9.97 092	16
45	9.54 936	9.57 849	0.42 151	9.97 087	15
46	9.54 969	9.57 887	0.42 113	9.97 083	14
47	9.55 003	9.57 925̅	0.42 075	9.97 078	13
48	9.55 036	9.57 963	0.42 037	9.97 073	12
49	9.55 069	9.58 001	0.41 999	9.97 068	11
50	9.55 102	9.58 039	0.41 961	9.97 063	**10**
51	9.55 136	9.58 077	0.41 923	9.97 059	9
52	9.55 169	9.58 115̅	0.41 885	9.97 054	8
53	9.55 202	9.58 153	0.41 847	9.97 049	7
54	9.55 235̅	9.58 191	0.41 809	9.97 044	6
55	9.55 268	9.58 229	0.41 771	9.97 039	5
56	9.55 301	9.58 267	0.41 733	9.97 035̅	4
57	9.55 334	9.58 304	0.41 696	9.97 030	3
58	9.55 367	9.58 342	0.41 658	9.97 025̅	2
59	9.55 400	9.58 380	0.41 620	9.97 020	1
60	9.55 433	9.58 418	0.41 582	9.97 015	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

69°

21° | 68°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.55 433	9.58 418	0.41 582	9.97 015	**60**
1	9.55 466	9.58 455	0.41 545	9.97 010	59
2	9.55 499	9.58 493	0.41 507	9.97 005	58
3	9.55 532	9.58 531	0.41 469	9.97 001	57
4	9.55 564	9.58 569	0.41 431	9.96 996	56
5	9.55 597	9.58 606	0.41 394	9.96 991	55
6	9.55 630	9.58 644	0.41 356	9.96 986	54
7	9.55 663	9.58 681	0.41 319	9.96 981	53
8	9.55 695	9.58 719	0.41 281	9.96 976	52
9	9.55 728	9.58 757	0.41 243	9.96 971	51
10	9.55 761	9.58 794	0.41 206	9.96 966	**50**
11	9.55 793	9.58 832	0.41 168	9.96 962	49
12	9.55 826	9.58 869	0.41 131	9.96 957	48
13	9.55 858	9.58 907	0.41 093	9.96 952	47
14	9.55 891	9.58 944	0.41 056	9.96 947	46
15	9.55 923	9.58 981	0.41 019	9.96 942	45
16	9.55 956	9.59 019	0.40 981	9.96 937	44
17	9.55 988	9.59 056	0.40 944	9.96 932	43
18	9.56 021	9.59 094	0.40 906	9.96 927	42
19	9.56 053	9.59 131	0.40 869	9.96 922	41
20	9.56 085	9.59 168	0.40 832	9.96 917	**40**
21	9.56 118	9.59 205	0.40 795	9.96 912	39
22	9.56 150	9.59 243	0.40 757	9.96 907	38
23	9.56 182	9.59 280	0.40 720	9.96 903	37
24	9.56 215	9.59 317	0.40 683	9.96 898	36
25	9.56 247	9.59 354	0.40 646	9.96 893	35
26	9.56 279	9.59 391	0.40 609	9.96 888	34
27	9.56 311	9.59 429	0.40 571	9.96 883	33
28	9.56 343	9.59 466	0.40 534	9.96 878	32
29	9.56 375	9.59 503	0.40 497	9.96 873	31
30	9.56 408	9.59 540	0.40 460	9.96 868	**30**
31	9.56 440	9.59 577	0.40 423	9.96 863	29
32	9.56 472	9.59 614	0.40 386	9.96 858	28
33	9.56 504	9.59 651	0.40 349	9.96 853	27
34	9.56 536	9.59 688	0.40 312	9.96 848	26
35	9.56 568	9.59 725	0.40 275	9.96 843	25
36	9.56 599	9.59 762	0.40 238	9.96 838	24
37	9.56 631	9.59 799	0.40 201	9.96 833	23
38	9.56 663	9.59 835	0.40 165	9.96 828	22
39	9.56 695	9.59 872	0.40 128	9.96 823	21
40	9.56 727	9.59 909	0.40 091	9.96 818	**20**
41	9.56 759	9.59 946	0.40 054	9.96 813	19
42	9.56 790	9.59 983	0.40 017	9.96 808	18
43	9.56 822	9.60 019	0.39 981	9.96 803	17
44	9.56 854	9.60 056	0.39 944	9.96 798	16
45	9.56 886	9.60 093	0.39 907	9.96 793	15
46	9.56 917	9.60 130	0.39 870	9.96 788	14
47	9.56 949	9.60 166	0.39 834	9.96 783	13
48	9.56 980	9.60 203	0.39 797	9.96 778	12
49	9.57 012	9.60 240	0.39 760	9.96 772	11
50	9.57 044	9.60 276	0.39 724	9.96 767	**10**
51	9.57 075	9.60 313	0.39 687	9.96 762	9
52	9.57 107	9.60 349	0.39 651	9.96 757	8
53	9.57 138	9.60 386	0.39 614	9.96 752	7
54	9.57 169	9.60 422	0.39 578	9.96 747	6
55	9.57 201	9.60 459	0.39 541	9.96 742	5
56	9.57 232	9.60 495	0.39 505	9.96 737	4
57	9.57 264	9.60 532	0.39 468	9.96 732	3
58	9.57 295	9.60 568	0.39 432	9.96 727	2
59	9.57 326	9.60 605	0.39 395	9.96 722	1
60	9.57 358	9.60 641	0.39 359	9.96 717	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

	'	L. Sin.	L. Tang.	L. Cotg.	L. Cos.		
	0	9.57 358	9.60 641	0.39 359	9.96 717	**60**	
	1	9.57 389	9.60 677	0.39 323	9.96 711	59	
	2	9.57 420	9.60 714	0.39 286	9.96 706	58	
	3	9.57 451	9.60 7$\bar{5}$0	0.39 250	9.96 701	57	
	4	9.57 482	9.60 786	0.39 214	9.96 696	56	
	5	9.57 514	9.60 823	0.39 177	9.96 691	55	
	6	9.57 54$\bar{5}$	9.60 859	0.39 141	9.96 686	54	
	7	9.57 576	9.60 895	0.39 10$\bar{5}$	9.96 681	53	
	8	9.57 607	9.60 931	0.39 069	9.96 676	52	
	9	9.57 638	9.60 967	0.39 033	9.96 670	51	
	10	9.57 669	9.61 004	0.38 996	9.96 665	**50**	
	11	9.57 700	9.61 040	0.38 960	9.96 660	49	
	12	9.57 731	9.61 076	0.38 924	9.96 655	48	
	13	9.57 762	9.61 112	0.38 888	9.96 6$\bar{5}$0	47	
	14	9.57 793	9.61 148	0.38 852	9.96 64$\bar{5}$	46	
	15	9.57 824	9.61 184	0.38 816	9.96 640	45	
	16	9.57 85$\bar{5}$	9.61 220	0.38 780	9.96 634	44	
	17	9.57 885	9.61 256	0.38 744	9.96 629	43	
	18	9.57 916	9.61 292	0.38 708	9.96 624	42	
	19	9.57 947	9.61 328	0.38 672	9.96 619	41	
	20	9.57 978	9.61 364	0.38 636	9.96 614	**40**	
	21	9.58 008	9.61 400	0.38 600	9.96 608	39	
	22	9.58 039	9.61 436	0.38 564	9.96 603	38	
	23	9.58 070	9.61 472	0.38 528	9.96 598	37	
	24	9.58 101	9.61 508	0.38 492	9.96 593	36	
	25	9.58 131	9.61 544	0.38 456	9.96 588	35	
	26	9.58 162	9.61 579	0.38 421	9.96 582	34	
	27	9.58 192	9.61 615	0.38 38$\bar{5}$	9.96 577	33	
	28	9.58 223	9.61 651	0.38 349	9.96 572	32	
22°	29	9.58 253	9.61 687	0.38 313	9.96 567	31	**67°**
	30	9.58 284	9.61 722	0.38 278	9.96 562	**30**	
	31	9.58 314	9.61 758	0.38 242	9.96 556	29	
	32	9.58 34$\bar{5}$	9.61 794	0.38 206	9.96 551	28	
	33	9.58 375	9.61 830	0.38 170	9.96 546	27	
	34	9.58 406	9.61 865	0.38 13$\bar{5}$	9.96 541	26	
	35	9.58 436	9.61 901	0.38 099	9.96 535	25	
	36	9.58 467	9.61 936	0.38 064	9.96 730	24	
	37	9.58 497	9.61 972	0.38 028	9.96 52$\bar{5}$	23	
	38	9.58 527	9.62 008	0.37 992	9.96 520	22	
	39	9.58 557	9.62 043	0.37 957	9.96 514	21	
	40	9.58 588	9.62 079	0.37 921	9.96 509	**20**	
	41	9.58 618	9.62 114	0.37 886	9.96 504	19	
	42	9.58 648	9.62 1$\bar{5}$0	0.37 850	9.96 498	18	
	43	9.58 678	9.62 185	0.37 81$\bar{5}$	9.96 493	17	
	44	9.58 709	9.62 221	0.37 779	9.96 488	16	
	45	9.58 739	9.62 256	0.37 744	9.96 483	15	
	46	9.58 769	9.62 292	0.37 708	9.96 477	14	
	47	9.58 799	9.62 327	0.37 673	9.96 472	13	
	48	9.58 829	9.62 362	0.37 638	9.96 467	12	
	49	9.58 859	9.62 398	0.37 602	9.96 461	11	
	50	9.58 889	9.62 433	0.37 567	9.96 456	**10**	
	51	9.58 919	9.62 468	0.37 532	9.96 451	9	
	52	9.58 949	9.62 504	0.37 496	9.96 445	8	
	53	9.58 979	9.62 539	0.37 461	9.96 440	7	
	54	9.59 009	9.62 574	0.37 426	9.96 43$\bar{5}$	6	
	55	9.59 039	9.62 609	0.37 391	9.96 429	5	
	56	9.59 069	9.62 64$\bar{5}$	0.37 355	9.96 424	4	
	57	9.59 098	9.62 680	0.37 320	9.96 419	3	
	58	9.59 128	9.62 71$\bar{5}$	0.37 285	9.96 413	2	
	59	9.59 158	9.62 750	0.37 2$\bar{5}$0	9.96 408	1	
	60	9.59 188	9.62 785	0.37 21$\bar{5}$	9.96 403	**0**	
		L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	'	

23°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.59 188	9.62 785	0.37 21$\bar{5}$	9.96 403	**60**
1	9.59 218	9.62 820	0.37 180	9.96 397	59
2	9.59 247	9.62 855	0.37 14$\bar{5}$	9.96 392	58
3	9.59 277	9.62 890	0.37 110	9.96 387	57
4	9.59 307	9.62 926	0.37 074	9.96 381	56
5	9.59 336	9.62 961	0.37 039	9.96 376	55
6	9.59 366	9.62 996	0.37 004	9.96 370	54
7	9.59 396	9.63 031	0.36 969	9.96 36$\bar{5}$	53
8	9.59 425	9.63 066	0.36 934	9.96 360	52
9	9.59 45$\bar{5}$	9.63 101	0.36 899	9.96 354	51
10	9.59 484	9.63 135	0.36 86$\bar{5}$	9.96 349	**50**
11	9.59 514	9.63 170	0.36 830	9.96 343	49
12	9.59 543	9.63 205	0.36 79$\bar{5}$	9.96 338	48
13	9.59 573	9.63 240	0.36 760	9.96 333	47
14	9.59 602	9.63 275	0.36 72$\bar{5}$	9.96 327	46
15	9.59 632	9.63 310	0.36 690	9.96 322	45
16	9.59 661	9.63 34$\bar{5}$	0.36 655	9.96 316	44
17	9.59 690	9.63 379	0.36 621	9.96 311	43
18	9.59 720	9.63 414	0.36 586	9.96 305	42
19	9.59 749	9.63 449	0.36 551	9.96 300	41
20	9.59 778	9.63 484	0.36 516	9.96 294	**40**
21	9.59 808	9.63 519	0.36 481	9.96 289	39
22	9.59 837	9.63 553	0.36 447	9.96 284	38
23	9.59 866	9.63 588	0.36 412	9.96 278	37
24	9.59 895	9.63 623	0.36 377	9.96 273	36
25	9.59 924	9.63 657	0.36 343	9.96 267	35
26	9.59 954	9.63 692	0.36 308	9.96 262	34
27	9.59 983	9.63 726	0.36 274	9.96 256	33
28	9.60 012	9.63 761	0.36 239	9.96 251	32
29	9.60 041	9.63 796	0.36 204	9.96 245	31
30	9.60 070	9.63 830	0.36 170	9.96 240	**30**
31	9.60 099	9.63 86$\bar{5}$	0.36 135	9.96 234	29
32	9.60 128	9.63 899	0.36 101	9.96 229	28
33	9.60 157	9.63 934	0.36 066	9.96 223	27
34	9.60 186	9.63 968	0.36 032	9.96 218	26
35	9.60 21$\bar{5}$	9.64 003	0.35 997	9.96 212	25
36	9.60 244	9.64 037	0.35 963	9.96 207	24
37	9.60 273	9.64 072	0.35 928	9.96 201	23
38	9.60 302	9.64 106	0.35 894	9.96 196	22
39	9.60 331	9.64 140	0.35 860	9.96 190	21
40	9.60 359	9.64 17$\bar{5}$	0.35 825	9.96 18$\bar{5}$	**20**
41	9.60 388	9.64 209	0.35 791	9.96 179	19
42	9.60 417	9.64 243	0.35 757	9.96 174	18
43	9.60 446	9.64 278	0.35 722	9.96 168	17
44	9.60 474	9.64 312	0.35 688	9.96 162	16
45	9.60 503	9.64 346	0.35 654	9.96 157	15
46	9.60 532	9.64 381	0.35 619	9.96 151	14
47	9.60 561	9.64 41$\bar{5}$	0.35 585	9.96 146	13
48	9.60 589	9.64 449	0.35 551	9.96 140	12
49	9.60 618	9.64 483	0.35 517	9.96 13$\bar{5}$	11
50	9.60 646	9.64 517	0.35 483	9.96 129	**10**
51	9.60 675	9.64 552	0.35 448	9.96 123	9
52	9.60 704	9.64 586	0.35 414	9.96 118	8
53	9.60 732	9.64 620	0.35 380	9.96 112	7
54	9.60 761	9.64 654	0.35 346	9.96 107	6
55	9.60 789	9.64 688	0.35 312	9.96 101	5
56	9.60 818	9.64 722	0.35 278	9.96 095	4
57	9.60 846	9.64 756	0.35 244	9.96 090	3
58	9.60 87$\bar{5}$	9.64 790	0.35 210	9.96 084	2
59	9.60 903	9.64 824	0.35 176	9.96 079	1
60	9.60 931	9.64 858	0.35 142	9.96 073	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

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24°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.60 931	9.64 858	0.35 142	9.96 073	**60**
1	9.60 960	9.64 892	0.35 108	9.96 067	59
2	9.60 988	9.64 926	0.35 074	9.96 062	58
3	9.61 016	9.64 960	0.35 040	9.96 056	57
4	9.61 04$\bar{5}$	9.64 994	0.35 006	9.96 050	56
5	9.61 073	9.65 028	0.34 972	9.96 04$\bar{5}$	55
6	9.61 101	9.65 062	0.34 938	9.96 039	54
7	9.61 129	9.65 096	0.34 904	9.96 034	53
8	9.61 158	9.65 130	0.34 870	9.96 028	52
9	9.61 186	9.65 164	0.34 836	9.96 022	51
10	9.61 214	9.65 197	0.34 803	9.96 017	**50**
11	9.61 242	9.65 231	0.34 769	9.96 011	49
12	9.61 270	9.65 265	0.34 73$\bar{5}$	9.96 005	48
13	9.61 298	9.65 299	0.34 701	9.96 000	47
14	9.61 326	9.65 333	0.34 667	9.95 994	46
15	9.61 354	9.65 366	0.34 634	9.95 988	45
16	9.61 382	9.65 400	0.34 600	9.95 982	44
17	9.61 411	9.65 434	0.34 566	9.95 977	43
18	9.61 438	9.65 467	0.34 533	9.95 971	42
19	9.61 466	9.65 501	0.34 499	9.95 965	41
20	9.61 494	9.65 53$\bar{5}$	0.34 465	9.95 960	**40**
21	9.61 522	9.65 568	0.34 432	9.95 954	39
22	9.61 550	9.65 602	0.34 398	9.95 948	38
23	9.61 578	9.65 636	0.34 364	9.95 942	37
24	9.61 606	9.65 669	0.34 331	9.95 937	36
25	9.61 634	9.65 703	0.34 297	9.95 931	35
26	9.61 662	9.65 736	0.34 264	9.95 925	34
27	9.61 689	9.65 770	0.34 230	9.95 920	33
28	9.61 717	9.65 803	0.34 197	9.95 914	32
29	9.61 74$\bar{5}$	9.65 837	0.34 163	9.95 908	31
30	9.61 773	9.65 870	0.34 130	9.95 902	**30**
31	9.61 800	9.65 904	0.34 096	9.95 897	29
32	9.61 828	9.65 937	0.34 063	9.95 891	28
33	9.61 856	9.65 971	0.34 029	9.95 88$\bar{5}$	27
34	9.61 883	9.66 004	0.33 996	9.95 879	26
35	9.61 911	9.66 038	0.33 962	9.95 873	25
36	9.61 939	9.66 071	0.33 929	9.95 868	24
37	9.61 966	9.66 104	0.33 896	9.95 862	23
38	9.61 994	9.66 138	0.33 862	9.95 856	22
39	9.62 021	9.66 171	0.33 829	9.95 850	21
40	9.62 049	9.66 204	0.33 796	9.95 844	**20**
41	9.62 076	9.66 238	0.33 762	9.95 839	19
42	9.62 104	9.66 271	0.33 729	9.95 833	18
43	9.62 131	9.66 304	0.33 696	9.95 827	17
44	9.62 159	9.66 337	0.33 663	9.95 821	16
45	9.62 186	9.66 371	0.33 629	9.95 815	15
46	9.62 214	9.66 404	0.33 596	9.95 810	14
47	9.62 241	9.66 437	0.33 563	9.95 804	13
48	9.62 268	9.66 470	0.33 530	9.95 798	12
49	9.62 296	9.66 503	0.33 497	9.95 792	11
50	9.62 323	9.66 537	0.33 463	9.95 786	**10**
51	9.62 350	9.66 570	0.33 430	9.95 780	9
52	9.62 377	9.66 603	0.33 397	9.95 77$\bar{5}$	8
53	9.62 40$\bar{5}$	9.66 636	0.33 364	9.95 769	7
54	9.62 432	9.66 669	0.33 331	9.95 763	6
55	9.62 459	9.66 702	0.33 298	9.95 757	5
56	9.62 486	9.66 735	0.33 26$\bar{5}$	9.95 751	4
57	9.62 513	9.66 768	0.33 232	9.95 745	3
58	9.62 541	9.66 801	0.33 199	9.95 739	2
59	9.62 568	9.66 834	0.33 166	9.95 733	1
60	9.62 59$\bar{5}$	9.66 867	0.33 133	9.95 728	0
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

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	'	L. Sin.	L. Tang.	L. Cotg.	L. Cos.		
	0	9.62 59$\bar{5}$	9.66 867	0.33 133	9.95 728	60	
	1	9.62 622	9.66 900	0.33 100	9.95 722	59	
	2	9.62 649	9.66 933	0.33 067	9.95 716	58	
	3	9.62 676	9.66 966	0.33 034	9.95 710	57	
	4	9.62 703	9.66 999	0.33 001	9.95 704	56	
	5	9.62 730	9.67 032	0.32 968	9.95 698	55	
	6	9.62 757	9.67 06$\bar{5}$	0.32 935	9.95 692	54	
	7	9.62 784	9.67 098	0.32 902	9.95 686	53	
	8	9.62 811	9.67 131	0.32 869	9.95 680	52	
	9	9.62 838	9.67 163	0.32 837	9.95 674	51	
	10	9.62 86$\bar{5}$	9.67 196	0.32 804	9.95 668	50	
	11	9.62 892	9.67 229	0.32 771	9.95 663	49	
	12	9.62 918	9.67 262	0.32 738	9.95 657	48	
	13	9.62 945	9.67 29$\bar{5}$	0.32 705	9.95 651	47	
	14	9.62 972	9.67 327	0.32 673	9 95 64$\bar{5}$	46	
	15	9.62 999	9.67 360	0.32 640	9.95 639	45	
	16	9.63 026	9.67 393	0.32 607	9.95 633	44	
	17	9.63 052	9.67 426	0.32 574	9.95 627	43	
	18	9.63 079	9.67 458	0.32 542	9.95 621	42	
	19	9.63 106	9.67 491	0.32 509	9.95 61$\bar{5}$	41	
	20	9.63 133	9.67 524	0.32 476	9.95 609	40	
	21	9.63 159	9.67 556	0.32 444	9.95 603	39	
	22	9.63 186	9.67 589	0.32 411	9.95 597	38	
	23	9.63 213	9.67 622	0.32 378	9.95 591	37	
	24	9.63 239	9.67 654	0.32 346	9.95 58$\bar{5}$	36	
	25	9.63 266	9.67 687	0.32 313	9.95 579	35	
	26	9.63 292	9.67 719	0.32 281	9.95 573	34	
	27	9.63 319	9.67 752	0.32 248	9.95 567	33	
	28	9.63 345	9.67 78$\bar{5}$	0.32 215	9.95 561	32	
25°	29	9.63 372	9.67 817	0.32 183	9.95 55$\bar{5}$	31	64°
	30	9.63 398	9.67 8$\bar{5}$0	0.32 150	9.95 549	30	
	31	9.63 42$\bar{5}$	9.67 882	0.32 118	9.95 543	29	
	32	9.63 451	9.67 91$\bar{5}$	0.32 085	9.95 537	28	
	33	9.63 478	9.67 947	0.32 053	9.95 531	27	
	34	9.63 504	9.67 980	0.32 020	9.95 52$\bar{5}$	26	
	35	9.63 531	9.68 012	0.31 988	9.95 519	25	
	36	9.63 557	9.68 044	0.31 956	9.95 513	24	
	37	9.63 583	9.68 077	0.31 923	9.95 507	23	
	38	9.63 610	9.68 109	0.31 891	9.95 500	22	
	39	9.63 636	9.68 142	0.31 858	9.95 494	21	
	40	9.63 662	9.68 174	0.31 826	9.95 488	20	
	41	9.63 689	9.68 206	0.31 794	9.95 482	19	
	42	9.63 71$\bar{5}$	9.68 239	0.31 761	9.95 476	18	
	43	9.63 741	9.68 271	0.31 729	9.95 470	17	
	44	9.63 767	9.68 303	0.31 697	9.95 464	16	
	45	9.63 794	9.68 336	0.31 664	9.95 458	15	
	46	9.63 820	9.68 368	0.31 632	9.95 452	14	
	47	9.63 846	9.68 400	0.31 600	9.95 446	13	
	48	9.63 872	9.68 432	0.31 568	9.95 440	12	
	49	9.63 898	9.68 46$\bar{5}$	0.31 535	9.95 434	11	
	50	9.63 924	9.68 497	0.31 503	9.95 427	10	
	51	9.63 950	9.68 529	0.31 471	9.95 421	9	
	52	9.63 976	9.68 561	0.31 439	9.95 415	8	
	53	9.64 002	9.68 593	0.31 407	9.95 409	7	
	54	9.64 028	9.68 626	0.31 374	9.95 403	6	
	55	9.64 054	9.68 658	0.31 342	9.95 397	5	
	56	9.64 080	9.68 690	0.31 310	9.95 391	4	
	57	9.64 106	9.68 722	0.31 278	9.95 384	3	
	58	9.64 132	9.68 754	0.31 246	9.95 378	2	
	59	9.64 158	9.68 786	0.31 214	9.95 372	1	
	60	9.64 184	9.68 818	0.31 182	9.95 366	0	
		L. Cos.	L. Cotg.	L. Tang.	L. Sin.	'	

26°	′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.		63°
	0	9.64 184	9.68 818	0.31 182	9.95 366	**60**	
	1	9.64 210	9.68 850	0.31 150	9.95 360	59	
	2	9.64 236	9.68 882	0.31 118	9.95 354	58	
	3	9.64 262	9.68 914	0.31 086	9.95 348	57	
	4	9.64 288	9.68 946	0.31 054	9.95 341	56	
	5	9.64 313	9.68 978	0.31 022	9.95 335	55	
	6	9.64 339	9.69 010	0.30 990	9.95 329	54	
	7	9.64 365	9.69 042	0.30 958	9.95 323	53	
	8	9.64 391	9.69 074	0.30 926	9.95 317	52	
	9	9.64 417	9.69 106	0.30 894	9.95 310	51	
	10	9.64 442	9.69 138	0.30 862	9.95 304	**50**	
	11	9.64 468	9.69 170	0.30 830	9.95 298	49	
	12	9.64 494	9.69 202	0.30 798	9.95 292	48	
	13	9.64 519	9.69 234	0.30 766	9.95 286	47	
	14	9.64 545	9.69 266	0.30 734	9.95 279	46	
	15	9.64 571	9.69 298	0.30 702	9.95 273	45	
	16	9.64 596	9.69 329	0.30 671	9.95 267	44	
	17	9.64 622	9.69 361	0.30 639	9.95 261	43	
	18	9.64 647	9.69 393	0.30 607	9.95 254	42	
	19	9.64 673	9.69 425	0.30 575	9.95 248	41	
	20	9.64 698	9.69 457	0.30 543	9.95 242	**40**	
	21	9.64 724	9.69 488	0.30 512	9.95 236	39	
	22	9.64 749	9.69 520	0.30 480	9.95 229	38	
	23	9.64 775	9.69 552	0.30 448	9.95 223	37	
	24	9.64 800	9.69 584	0.30 416	9.95 217	36	
	25	9.64 826	9.69 615	0.30 385	9.95 211	35	
	26	9.64 851	9.69 647	0.30 353	9.95 204	34	
	27	9.64 877	9.69 679	0.30 321	9.95 198	33	
	28	9.64 902	9.69 710	0.30 290	9.95 192	32	
	29	9.64 927	9.69 742	0.30 258	9.95 185	31	
	30	9.64 953	9.69 774	0.30 226	9.95 179	**30**	
	31	9.64 978	9.69 805	0.30 195	9.95 173	29	
	32	9.65 003	9.69 837	0.30 163	9.95 167	28	
	33	9.65 029	9.69 868	0.30 132	9.95 160	27	
	34	9.65 054	9.69 900	0.30 100	9.95 154	26	
	35	9.65 079	9.69 932	0.30 068	9.95 148	25	
	36	9.65 104	9.69 963	0.30 037	9.95 141	24	
	37	9.65 130	9.69 995	0.30 005	9.95 135	23	
	38	9.65 155	9.70 026	0.29 974	9.95 129	22	
	39	9.65 180	9.70 058	0.29 942	9.95 122	21	
	40	9.65 205	9.70 089	0.29 911	9.95 116	**20**	
	41	9.65 230	9.70 121	0.29 879	9.95 110	19	
	42	9.65 255	9.70 152	0.29 848	9.95 103	18	
	43	9.65 281	9.70 184	0.29 816	9.95 097	17	
	44	9.65 306	9.70 215	0.29 785	9.95 090	16	
	45	9.65 331	9.70 247	0.29 753	9.95 084	15	
	46	9.65 356	9.70 278	0.29 722	9.95 078	14	
	47	9.65 381	9.70 309	0.29 691	9.95 071	13	
	48	9.65 406	9.70 341	0.29 659	9.95 065	12	
	49	9.65 431	9.70 372	0.29 628	9.95 059	11	
	50	9.65 456	9.70 404	0.29 596	9.95 052	**10**	
	51	9.65 481	9.70 435	0.29 565	9.95 046	9	
	52	9.65 506	9.70 466	0.29 534	9.95 039	8	
	53	9.65 531	9.70 498	0.29 502	9.95 033	7	
	54	9.65 556	9.70 529	0.29 471	9.95 027	6	
	55	9.65 580	9.70 560	0.29 440	9.95 020	5	
	56	9.65 605	9.70 592	0.29 408	9.95 014	4	
	57	9.65 630	9.70 623	0.29 377	9.95 007	3	
	58	9.65 655	9.70 654	0.29 346	9.95 001	2	
	59	9.65 680	9.70 685	0.29 315	9.94 995	1	
	60	9.65 705	9.70 717	0.29 283	9.94 988	**0**	
		L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′	

27°

'	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.65 705	9.70 717	0.29 283	9.94 988	**60**
1	9.65 729	9.70 748	0.29 252	9.94 982	59
2	9.65 754	9.70 779	0.29 221	9.94 975	58
3	9.65 779	9.70 810	0.29 190	9.94 969	57
4	9.65 804	9.70 841	0.29 159	9.94 962	56
5	9.65 828	9.70 873	0.29 127	9.94 956	55
6	9.65 853	9.70 904	0.29 096	9.94 949	54
7	9.65 878	9.70 935	0.29 065	9.94 943	53
8	9.65 902	9.70 966	0.29 034	9.94 936	52
9	9.65 927	9.70 997	0.29 003	9.94 930	51
10	9.65 952	9.71 028	0.28 972	9.94 923	**50**
11	9.65 976	9.71 059	0.28 941	9.94 917	49
12	9.66 001	9.71 090	0.28 910	9.94 911	48
13	9.66 025	9.71 121	0.28 879	9.94 904	47
14	9.66 050	9.71 153	0.28 847	9.94 898	46
15	9.66 075	9.71 184	0.28 816	9.94 891	45
16	9.66 099	9.71 215	0.28 785	9.94 885	44
17	9.66 124	9.71 246	0.28 754	9.94 878	43
18	9.66 148	9.71 277	0.28 723	9.94 871	42
19	9.66 173	9.71 308	0.28 692	9.94 865	41
20	9.66 197	9.71 339	0.28 661	9.94 858	**40**
21	9.66 221	9.71 370	0.28 630	9.94 852	39
22	9.66 246	9.71 401	0.28 599	9.94 845	38
23	9.66 270	9.71 431	0.28 569	9.94 839	37
24	9.66 295	9.71 462	0.28 538	9.94 832	36
25	9.66 319	9.71 493	0.28 507	9.94 826	35
26	9.66 343	9.71 524	0.28 476	9.94 819	34
27	9.66 368	9.71 555	0.28 445	9.94 813	33
28	9.66 392	9.71 586	0.28 414	9.94 806	32
29	9.66 416	9.71 617	0.28 383	9.94 799	31
30	9.66 441	9.71 648	0.28 352	9.94 793	**30**
31	9.66 465	9.71 679	0.28 321	9.94 786	29
32	9.66 489	9.71 709	0.28 291	9.94 780	28
33	9.66 513	9.71 740	0.28 260	9.94 773	27
34	9.66 537	9.71 771	0.28 229	9.94 767	26
35	9.66 562	9.71 802	0.28 198	9.94 760	25
36	9.66 586	9.71 833	0.28 167	9.94 753	24
37	9.66 610	9.71 863	0.28 137	9.94 747	23
38	9.66 634	9.71 894	0.28 106	9.94 740	22
39	9.66 658	9.71 925	0.28 075	9.94 734	21
40	9.66 682	9.71 955	0.28 045	9.94 727	**20**
41	9.66 706	9.71 986	0.28 014	9.94 720	19
42	9.66 731	9.72 017	0.27 983	9.94 714	18
43	9.66 755	9.72 048	0.27 952	9.94 707	17
44	9.66 779	9.72 078	0.27 922	9.94 700	16
45	9.66 803	9.72 109	0.27 891	9.94 694	15
46	9.66 827	9.72 140	0.27 860	9.94 687	14
47	9.66 851	9.72 170	0.27 830	9.94 680	13
48	9.66 875	9.72 201	0.27 799	9.94 674	12
49	9.66 899	9.72 231	0.27 769	9.94 667	11
50	9.66 922	9.72 262	0.27 738	9.94 660	**10**
51	9.66 946	9.72 293	0.27 707	9.94 654	9
52	9.66 970	9.72 323	0.27 677	9.94 647	8
53	9.66 994	9.72 354	0.27 646	9.94 640	7
54	9.67 018	9.72 384	0.27 616	9.94 634	6
55	9.67 042	9.72 415	0.27 585	9.94 627	5
56	9.67 066	9.72 445	0.27 555	9.94 620	4
57	9.67 090	9.72 476	0.27 524	9.94 614	3
58	9.67 113	9.72 506	0.27 494	9.94 607	2
59	9.67 137	9.72 537	0.27 463	9.94 600	1
60	9.67 161	9.72 567	0.27 433	9.94 593	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	'

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28°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.67 161	9.72 567	0.27 433	9.94 593	60
1	9.67 185	9.72 598	0.27 402	9.94 587	59
2	9.67 208	9.72 628	0.27 372	9.94 580	58
3	9.67 232	9.72 659	0.27 341	9.94 573	57
4	9.67 256	9.72 689	0.27 311	9.94 567	56
5	9.67 280	9.72 720	0.27 280	9.94 560	55
6	9.67 303	9.72 750	0.27 250	9.94 553	54
7	9.67 327	9.72 780	0.27 220	9.94 546	53
8	9.67 350	9.72 811	0.27 189	9.94 540	52
9	9.67 374	9.72 841	0.27 159	9.94 533	51
10	9.67 398	9.72 872	0.27 128	9.94 526	50
11	9.67 421	9.72 902	0.27 098	9.94 519	49
12	9.67 445	9.72 932	0.27 068	9.94 513	48
13	9.67 468	9.72 963	0.27 037	9.94 506	47
14	9.67 492	9 72 993	0.27 007	9.94 499	46
15	9.67 515	9.73 023	0.26 977	9.94 492	45
16	9.67 539	9.73 054	0.26 946	9.94 485	44
17	9.67 562	9.73 084	0.26 916	9.94 479	43
18	9.67 586	9.73 114	0.26 886	9.94 472	42
19	9.67 609	9.73 144	0.26 856	9.94 465	41
20	9.67 633	9.73 175	0.26 825	9.94 458	40
21	9.67 656	9.73 205	0.26 795	9.94 451	39
22	9.67 680	9.73 235	0.26 765	9.94 445	38
23	9.67 703	9.73 265	0.26 735	9.94 438	37
24	9.67 726	9.73 295	0.26 705	9.94 431	36
25	9.67 750	9.73 326	0.26 674	9.94 424	35
26	9.67 773	9.73 356	0.26 644	9.94 417	34
27	9.67 796	9.73 386	0.26 614	9.94 410	33
28	9.67 820	9.73 416	0.26 584	9.94 404	32
29	9.67 843	9.73 446	0.26 554	9.94 397	31
30	9.67 866	9.73 476	0.26 524	9.94 390	30
31	9.67 890	9.73 507	0.26 493	9.94 383	29
32	9.67 913	9.73 537	0.26 463	9.94 376	28
33	9.67 936	9.73 567	0.26 433	9.94 369	27
34	9.67 959	9.73 597	0.26 403	9.94 362	26
35	9.67 982	9.73 627	0.26 373	9.94 355	25
36	9.68 006	9.73 657	0.26 343	9.94 349	24
37	9.68 029	9.73 687	0.26 313	9.94 342	23
38	9.68 052	9.73 717	0.26 283	9.94 335	22
39	9.68 075	9.73 747	0.26 253	9.94 328	21
40	9.68 098	9.73 777	0.26 223	9.94 321	20
41	9.68 121	9.73 807	0.26 193	9.94 314	19
42	9.68 144	9.73 837	0.26 163	9.94 307	18
43	9.68 167	9.73 867	0.26 133	9.94 300	17
44	9.68 190	9.73 897	0.26 103	9.94 293	16
45	9.68 213	9.73 927	0.26 073	9.94 286	15
46	9.68 237	9.73 957	0.26 043	9.94 279	14
47	9.68 260	9.73 987	0.26 013	9.94 273	13
48	9.68 283	9.74 017	0.25 983	9.94 266	12
49	9.68 305	9.74 047	0.25 953	9.94 259	11
50	9.68 328	9.74 077	0.25 923	9.94 252	10
51	9.68 351	9.74 107	0.25 893	9.94 245	9
52	9.68 374	9.74 137	0.25 863	9.94 238	8
53	9.68 397	9.74 166	0.25 834	9.94 231	7
54	9.68 420	9.74 196	0.25 804	9.94 224	6
55	9.68 443	9.74 226	0.25 774	9.94 217	5
56	9.68 466	9.74 256	0.25 744	9.94 210	4
57	9.68 489	9.74 286	0.25 714	9.94 203	3
58	9.68 512	9.74 316	0.25 684	9.94 196	2
59	9.68 534	9.74 345	0.25 655	9.94 189	1
60	9.68 557	9.74 375	0.25 625	9.94 182	0
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

61°

29°	′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	′	60°
	0	9.68 557	9.74 375	0.25 625̄	9.94 182	**60**	
	1	9.68 580	9.74 405̄	0.25 595	9.94 175̄	59	
	2	9.68 603	9.74 435̄	0.25 565	9.94 168	58	
	3	9.68 625	9.74 465̄	0.25 535	9.94 161	57	
	4	9.68 648	9.74 494	0.25 506	9.94 154	56	
	5	9.68 671	9.74 524	0.25 476	9.94 147	55	
	6	9.68 694	9.74 554	0.25 446	9.94 140	54	
	7	9.68 716	9.74 583	0.25 417	9.94 133	53	
	8	9.68 739	9.74 613	0.25 387	9.94 126	52	
	9	9.68 762	9.74 643	0.25 357	9.94 119	51	
	10	9.68 784	9.74 673	0.25 327	9.94 112	**50**	
	11	9.68 807	9.74 702	0.25 298	9.94 105̄	49	
	12	9.68 829	9.74 732	0.25 268	9.94 098	48	
	13	9.68 852	9.74 762	0.25 238	9.94 090	47	
	14	9.68 875̄	9.74 791	0.25 209	9.94 083	46	
	15	9.68 897	9.74 821	0.25 179	9.94 076	45	
	16	9.68 920	9.74 851	0.25 149	9.94 069	44	
	17	9.68 942	9.74 880	0.25 120	9.94 062	43	
	18	9.68 965̄	9.74 910	0.25 090	9.94 055	42	
	19	9.68 987	9.74 939	0.25 061	9.94 048	41	
	20	9.69 010	9.74 969	0.25 031	9.94 041	**40**	
	21	9.69 032	9.74 998	0.25 002	9.94 034	39	
	22	9.69 055̄	9.75 028	0.24 972	9.94 027	38	
	23	9.69 077	9.75 058	0.24 942	9.94 020	37	
	24	9.69 100	9.75 087	0.24 913	9.94 012	36	
	25	9.69 122	9.75 117	0.24 883	9.94 005	35	
	26	9.69 144	9.75 146	0.24 854	9.93 998	34	
	27	9.69 167	9.75 176	0.24 824	9.93 991	33	
	28	9.69 189	9.75 205	0.24 795̄	9.93 984	32	
29°	29	9.69 212	9 75 235̄	0.24 765	9.93 977	31	**60°**
	30	9.69 234	9.75 264	0.24 736	9.93 970	**30**	
	31	9.69 256	9.75 294	0.24 706	9.93 963	29	
	32	9.69 279	9.75 323	0.24 677	9.93 955	28	
	33	9.69 301	9.75 353	0.24 647	9.93 948	27	
	34	9.69 323	9.75 382	0.24 618	9.93 941	26	
	35	9.69 345	9.75 411	0.24 589	9.93 934	25	
	36	9.69 368	9.75 441	0.24 559	9.93 927	24	
	37	9.69 390	9.75 470	0.24 530	9.93 920	23	
	38	9.69 412	9.75 5̄00	0.24 500	9.93 912	22	
	39	9.69 434	9.75 529	0.24 471	9.93 905	21	
	40	9.69 356	9.75 558	0.24 442	9.93 898	**20**	
	41	9.69 479	9.75 588	0.24 412	9.93 891	19	
	42	9.69 501	9.75 617	0.24 383	9.93 884	18	
	43	9.69 523	9.75 647	0.24 353	9.93 876	17	
	44	9.69 545	9.75 676	0.24 324	9.93 869	16	
	45	9.69 567	9.75 705	0.24 295̄	9.93 862	15	
	46	9.69 589	9.75 735̄	0.24 265	9.93 855̄	14	
	47	9.69 611	9.75 764	0.24 236	9.93 847	13	
	48	9.69 633	9.75 793	0.24 207	9.93 840	12	
	49	9.69 655	9.75 822	0.24 178	9.93 833	11	
	50	9.69 677	9.75 852	0.24 148	9.93 826	**10**	
	51	9.69 699	9.75 881	0.24 119	9.93 819	9	
	52	9.69 721	9.75 910	0.24 090	9.93 811	8	
	53	9.69 743	9.75 939	0.24 061	9.93 804	7	
	54	9.69 765	9.75 969	0.24 031	9.93 797	6	
	55	9.69 787	9.75 998	0.24 002	9.93 789	5	
	56	9.69 809	9.76 027	0.23 973	9.93 782	4	
	57	9.69 831	9.76 056	0.23 944	9.93 775̄	3	
	58	9.69 853	9.76 086	0.23 914	9.93 768	2	
	59	9.69 875	9.76 115̄	0.23 885	9.93 760	1	
	60	9.69 897	9.76 144	0.23 856	9.93 753	0	
		L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′	

30°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.69 897	9.76 144	0.23 856	9.93 753	**60**
1	9.69 919	9.76 173	0.23 827	9.93 746	59
2	9.69 941	9.76 202	0.23 798	9.93 738	58
3	9.69 963	9.76 231	0.23 769	9.93 731	57
4	9.69 984	9.76 261	0.23 739	9.93 724	56
5	9.70 006	9.76 290	0.23 710	9.93 717	55
6	9.70 028	9.76 319	0.23 681	9.93 709	54
7	9.70 05̄0	9.76 348	0.23 652	9.93 702	53
8	9.70 072	9.76 377	0.23 623	9.93 695̄	52
9	9.70 093	9.76 406	0.23 594	9.93 687	51
10	9.70 115	9.76 435	0.23 565̄	9.93 680	**50**
11	9.70 137	9.76 464	0.23 536	9.93 673	49
12	9.70 159	9.76 493	0.23 507	9.93 665	48
13	9.70 180	9.76 522	0.23 478	9.93 658	47
14	9.70 202	9.76 551	0.23 449	9.93 650	46
15	9.70 224	9.76 580	0.23 420	9.93 643	45
16	9.70 245	9.76 609	0.23 391	9.93 636	44
17	9.70 267	9.76 639	0.23 361	9.93 628	43
18	9.70 288	9.76 668	0.23 332	9.93 621	42
19	9.70 310	9.76 697	0.23 303	9.93 614	41
20	9.70 332	9.76 725	0.23 275̄	9.93 606	**40**
21	9.70 353	9.76 754	0.23 246	9.93 599	39
22	9.70 375̄	9.76 783	0.23 217	9.93 591	38
23	9.70 396	9.76 812	0.23 188	9.93 584	37
24	9.70 418	9.76 841	0.23 159	9.93 577	36
25	9.70 439	9.76 870	0.23 130	9.93 569	35
26	9.70 461	9.76 899	0.23 101	9.93 562	34
27	9.70 482	9.76 928	0.23 072	9.93 554	33
28	9.70 504	9.76 957	0.23 043	9.93 547	32
29	9.70 525	9.76 986	0.23 014	9.93 539	31
30	9.70 547	9.77 015̄	0.22 985	9.93 532	**30**
31	9.70 568	9.77 044	0 22 956	9.93 525̄	29
32	9.70 590	9.77 073	0.22 927	9.93 517	28
33	9.70 611	9.77 101	0.22 899	9.93 510	27
34	9.70 633	9.77 130	0.22 870	9.93 502	26
35	9.70 654	9.77 159	0.22 841	9.93 495̄	25
36	9.70 675	9.77 188	0.22 812	9.93 487	24
37	9.70 697	9.77 217	0.22 783	9.93 480	23
38	9.70 718	9.77 246	0.22 754	9.93 472	22
39	9.70 739	9.77 274	0.22 726	9.93 465̄	21
40	9.70 761	9.77 303	0.22 697	9.93 457	**20**
41	9.70 782	9.77 332	0.22 668	9.93 45̄0	19
42	9.70 803	9.77 361	0.22 639	9.93 442	18
43	9.70 824	9.77 390	0.22 610	9.93 435̄	17
44	9.70 846	9.77 418	0.22 582	9.93 427	16
45	9.70 867	9.77 447	0.22 553	9.93 420	15
46	9.70 888	9.77 476	0.22 524	9.93 412	14
47	9.70 909	9.77 505̄	0.22 495	9.93 405̄	13
48	9.70 931	9.77 533	0.22 467	9.93 397	12
49	9.70 952	9.77 562	0.22 438	9.93 390	11
50	9.70 973	9.77 591	0.22 409	9.93 382	**10**
51	9.70 994	9.77 619	0.22 381	9.93 375̄	9
52	9.71 015	9.77 648	0.22 352	9.93 367	8
53	9.71 036	9.77 677	0.22 323	9.93 360	7
54	9.71 058	9.77 706	0.22 294	9.93 352	6
55	9.71 079	9.77 734	0.22 266	9.93 344	5
56	9.71 100	9.77 763	0.22 237	9.93 337	4
57	9.71 121	9.77 791	0.22 209	9.93 329	3
58	9.71 142	9.77 820	0.22 180	9.93 322	2
59	9.71 163	9.77 849	0.22 151	9.93 314	1
60	9.71 184	9.77 877	0.22 123	9.93 307	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

59°

31°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.71 184	9.77 877	0.22 123	9.93 307	60
1	9.71 205	9.77 906	0.22 094	9.93 299	59
2	9.71 226	9.77 935	0.22 065	9.93 291	58
3	9.71 247	9.77 963	0.22 037	9.93 284	57
4	9.71 268	9.77 992	0.22 008	9.93 276	56
5	9.71 289	9.78 020	0.21 980	9.93 269	55
6	9.71 310	9.78 049	0.21 951	9.93 261	54
7	9.71 331	9.78 077	0.21 923	9.93 253	53
8	9.71 352	9.78 106	0.21 894	9.93 246	52
9	9.71 373	9.78 135	0.21 865	9.93 238	51
10	9.71 393	9.78 163	0.21 837	9.93 230	50
11	9.71 414	9.78 192	0.21 808	9.93 223	49
12	9.71 435	9.78 220	0.21 780	9.93 215	48
13	9.71 456	9.78 249	0.21 751	9.93 207	47
14	9.71 477	9.78 277	0.21 723	9.93 200	46
15	9.71 498	9.78 306	0.21 694	9.93 192	45
16	9.71 519	9.78 334	0.21 666	9.93 184	44
17	9.71 539	9.78 363	0.21 637	9.93 177	43
18	9.71 560	9.78 391	0.21 609	9.93 169	42
19	9.71 581	9.78 419	0.21 581	9.93 161	41
20	9.71 602	9.78 448	0.21 552	9.93 154	40
21	9.71 622	9.78 476	0.21 524	9.93 146	39
22	9.71 643	9.78 505	0.21 495	9.93 138	38
23	9.71 664	9.78 533	0.21 467	9.93 131	37
24	9.71 685	9.78 562	0.21 438	9.93 123	36
25	9.71 705	9.78 590	0.21 410	9.93 115	35
26	9.71 726	9.78 618	0.21 382	9.93 108	34
27	9.71 747	9.78 647	0.21 353	9.93 100	33
28	9.71 767	9.78 675	0.21 325	9.93 092	32
29	9.71 788	9.78 704	0.21 296	9.93 084	31
30	9.71 809	9.78 732	0.21 268	9.93 077	30
31	9.71 829	9.78 760	0.21 240	9.93 069	29
32	9.71 850	9.78 789	0.21 211	9.93 061	28
33	9.71 870	9.78 817	0.21 183	9.93 053	27
34	9.71 891	9.78 845	0.21 155	9.93 046	26
35	9.71 911	9.78 874	0.21 126	9.93 038	25
36	9.71 932	9.78 902	0.21 098	9.93 030	24
37	9.71 952	9.78 930	0.21 070	9.93 022	23
38	9.71 973	9.78 959	0.21 041	9.93 014	22
39	9.71 994	9.78 987	0.21 013	9.93 007	21
40	9.72 014	9.79 015	0.20 985	9.92 999	20
41	9.72 034	9.79 043	0.20 957	9.92 991	19
42	9.72 055	9.79 072	0.20 928	9.92 983	18
43	9.72 075	9.79 100	0.20 900	9.92 976	17
44	9.72 096	9.79 128	0.20 872	9.92 968	16
45	9.72 116	9.79 156	0.20 844	9.92 960	15
46	9.72 137	9.79 185	0.20 815	9.92 952	14
47	9.72 157	9.79 213	0.20 787	9.92 944	13
48	9.72 177	9.79 241	0.20 759	9.92 936	12
49	9.72 198	9.79 269	0.20 731	9.92 929	11
50	9.72 218	9.79 297	0.20 703	9.92 921	10
51	9.72 238	9.79 326	0.20 674	9.92 913	9
52	9.72 259	9.79 354	0.20 646	9.92 905	8
53	9.72 279	9.79 382	0.20 618	9.92 897	7
54	9.72 299	9.79 410	0.20 590	9.92 889	6
55	9.72 320	9.79 438	0.20 562	9.92 881	5
56	9.72 340	9.79 466	0.20 534	9.92 874	4
57	9.72 360	9.79 495	0.20 505	9.92 866	3
58	9.72 381	9.79 523	0.20 477	9.92 858	2
59	9.72 401	9.79 551	0.20 449	9.92 850	1
60	9.72 421	9.79 579	0.20 421	9.92 842	0
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

58°

32° | 57°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.72 421	9.79 579	0.20 421	9.92 842	**60**
1	9.72 441	9.79 607	0.20 393	9.92 834	59
2	9.72 461	9.79 635	0.20 36$\bar{5}$	9.92 826	58
3	9.72 482	9.79 663	0.20 337	9.92 818	57
4	9.72 502	9.79 691	0.20 309	9.92 810	56
5	9.72 522	9.79 719	0.20 281	9.92 803	55
6	9.72 542	9.79 747	0.20 253	9.92 79$\bar{5}$	54
7	9.72 562	9.79 776	0.20 224	9.92 787	53
8	9.72 582	9.79 804	0.20 196	9.92 779	52
9	9.72 602	9.79 832	0.20 168	9.92 771	51
10	9.72 622	9.79 860	0.20 140	9.92 763	**50**
11	9.72 643	9.79 888	0.20 112	9.92 75$\bar{5}$	49
12	9.72 663	9.79 916	0.20 084	9.92 747	48
13	9.72 683	9.79 944	0.20 056	9.92 739	47
14	9.72 703	9.79 972	0.20 028	9.92 731	46
15	9.72 723	9.80 000	0.20 000	9.92 723	45
16	9.72 743	9.80 028	0.19 972	9.92 715	44
17	9.72 763	9.80 056	0.19 944	9.92 707	43
18	9.72 783	9.80 084	0.19 916	9.92 699	42
19	9.72 803	9.80 112	0.19 888	9.92 691	41
20	9.72 823	9.80 140	0.19 860	9.92 683	**40**
21	9.72 843	9.80 168	0.19 832	9.92 675	39
22	9.72 863	9.80 195	0.19 80$\bar{5}$	9.92 667	38
23	9.72 883	9.80 223	0.19 777	9.92 659	37
24	9.72 902	9.80 251	0.19 749	9.92 651	36
25	9.72 922	9.80 279	0.19 721	9.92 643	35
26	9.72 942	9.80 307	0.19 693	9.92 635	34
27	9.72 962	9.80 335	0.19 66$\bar{5}$	9.92 627	33
28	9.72 982	9.80 363	0.19 637	9.92 619	32
29	9.73 002	9.80 391	0.19 609	9.92 611	31
30	9.73 022	9.80 419	0.19 581	9.92 603	**30**
31	9.73 041	9.80 447	0.19 553	9.92 59$\bar{5}$	29
32	9.73 061	9.80 474	0.19 526	9.92 587	28
33	9.73 081	9.80 502	0.19 498	9.92 579	27
34	9.73 101	9.80 530	0.19 470	9.92 571	26
35	9.73 121	9.80 558	0.19 442	9.92 563	25
36	9.73 140	9.80 586	0.19 414	9.92 55$\bar{5}$	24
37	9.73 160	9.80 614	0.19 386	9.92 546	23
38	9.73 180	9.80 642	0.19 358	9.92 538	22
39	9.73 200	9.80 669	0.19 331	9.92 530	21
40	9.73 219	9.80 697	0.19 303	9.92 522	**20**
41	9.73 239	9.80 72$\bar{5}$	0.19 275	9.92 514	19
42	9.73 259	9.80 753	0.19 247	9.92 506	18
43	9.73 278	9.80 781	0.19 219	9.92 498	17
44	9.73 298	9.80 808	0.19 192	9.92 490	16
45	9.73 318	9.80 836	0.19 164	9.92 482	15
46	9.73 337	9.80 864	0.19 136	9.92 473	14
47	9.73 357	9.80 892	0.19 108	9.92 465	13
48	9.73 377	9.80 919	0.19 081	9.92 457	12
49	9.73 396	9.80 947	0.19 053	9.92 449	11
50	9.73 416	9.80 97$\bar{5}$	0.19 025	9.92 441	**10**
51	9.73 435	9.81 003	0.18 997	9.92 433	9
52	9.73 45$\bar{5}$	9.81 030	0.18 970	9.92 42$\bar{5}$	8
53	9.73 474	9.81 058	0.18 942	9.92 416	7
54	9.73 494	9.81 086	0.18 914	9.92 408	6
55	9.73 513	9.81 113	0.18 887	9.92 400	5
56	9.73 533	9.81 141	0.18 859	9.92 392	4
57	9.73 552	9.81 169	0.18 831	9.92 384	3
58	9.73 572	9.81 196	0.18 804	9.92 376	2
59	9.73 591	9.81 224	0.18 776	9.92 367	1
60	9.73 611	9.81 252	0.18 748	9.92 359	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

	′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.		
	0	9.73 611	9.81 252	0.18 748	9.92 359	**60**	
	1	9.73 630	9.81 279	0.18 721	9.92 351	59	
	2	9.73 650	9.81 307	0.18 693	9.92 343	58	
	3	9.73 669	9.81 335	0.18 665	9.92 335	57	
	4	9.73 689	9.81 362	0.18 638	9.92 326	56	
	5	9.73 708	9.81 390	0.18 610	9.92 318	55	
	6	9.73 727	9.81 418	0.18 582	9.92 310	54	
	7	9.73 747	9.81 445	0.18 555	9.92 302	53	
	8	9.73 766	9.81 473	0.18 527	9.92 293	52	
	9	9.73 785	9.81 500	0.18 500	9.92 285	51	
	10	9.73 805	9.81 528	0.18 472	9.92 277	**50**	
	11	9.73 824	9.81 556	0.18 444	9.92 269	49	
	12	9.73 843	9.81 583	0.18 417	9.92 260	48	
	13	9.73 863	9.81 611	0.18 389	9.92 252	47	
	14	9.73 882	9.81 638	0.18 362	9.92 244	46	
	15	9.73 901	9.81 666	0.18 334	9.92 235	45	
	16	9.73 921	9.81 693	0.18 307	9.92 227	44	
	17	9.73 940	9.81 721	0.18 279	9.92 219	43	
	18	9.73 959	9.81 748	0.18 252	9.92 211	42	
	19	9.73 978	9.81 776	0.18 224	9.92 202	41	
	20	9.73 997	9.81 803	0.18 197	9.92 194	**40**	
	21	9.74 017	9.81 831	0.18 169	9.92 186	39	
	22	9.74 036	9.81 858	0.18 142	9.92 177	38	
	23	9.74 055	9.81 886	0.18 114	9.92 169	37	
	24	9.74 074	9.81 913	0.18 087	9.92 161	36	
	25	9.74 093	9.81 941	0.18 059	9.92 152	35	
	26	9.74 113	9.81 968	0.18 032	9.92 144	34	
	27	9.74 132	9.81 996	0.18 004	9.92 136	33	
	28	9.74 151	9.82 023	0.17 977	9.92 127	32	
33°	29	9.74 170	9.82 051	0.17 949	9.92 119	31	**56°**
	30	9.74 189	9.82 078	0.17 922	9.92 111	**30**	
	31	9.74 208	9.82 106	0.17 894	9.92 102	29	
	32	9.74 227	9.82 133	0.17 867	9.92 094	28	
	33	9.74 246	9.82 161	0.17 839	9.92 086	27	
	34	9.74 265	9.82 188	0.17 812	9.92 077	26	
	35	9.74 284	9.82 215	0.17 785	9.92 069	25	
	36	9.74 303	9.82 243	0.17 757	9.92 060	24	
	37	9.74 322	9.82 270	0.17 730	9.92 052	23	
	38	9.74 341	9.82 298	0.17 702	9.92 044	22	
	39	9.74 360	9.82 325	0.17 675	9.92 035	21	
	40	9.74 379	9.82 352	0.17 648	9.92 027	**20**	
	41	9.74 398	9.82 380	0.17 620	9.92 018	19	
	42	9.74 417	9.82 407	0.17 593	9.92 010	18	
	43	9.74 436	9.82 435	0.17 565	9.92 002	17	
	44	9.74 455	9.82 462	0.17 538	9.91 993	16	
	45	9.74 474	9.82 489	0.17 511	9.91 985	15	
	46	9.74 493	9.82 517	0.17 483	9.91 976	14	
	47	9.74 512	9.82 544	0.17 456	9.91 968	13	
	48	9.74 531	9.82 571	0.17 429	9.91 959	12	
	49	9.74 549	9.82 599	0.17 401	9.91 951	11	
	50	9.74 568	9.82 626	0.17 374	9.91 942	**10**	
	51	9.74 587	9.82 653	0.17 347	9.91 934	9	
	52	9.74 606	9.82 681	0.17 319	9.91 925	8	
	53	9.74 625	9.82 708	0.17 292	9.91 917	7	
	54	9.74 644	9.82 735	0.17 265	9.91 908	6	
	55	9.74 662	9.82 762	0.17 238	9.91 900	5	
	56	9.74 681	9.82 790	0.17 210	9.91 891	4	
	57	9.74 700	9.82 817	0.17 183	9.91 883	3	
	58	9.74 719	9.82 844	0.17 156	9.91 874	2	
	59	9.74 737	9.82 871	0.17 129	9.91 866	1	
	60	9.74 756	9.82 899	0.17 101	9.91 857	**0**	
		L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′	

34°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.74 756	9.82 899	0.17 101	9.91 857	60
1	9.74 775	9.82 926	0.17 074	9.91 849	59
2	9.74 794	9.82 953	0.17 047	9.91 840	58
3	9.74 812	9.82 980	0.17 020	9.91 832	57
4	9.74 831	9.83 008	0.16 992	9.91 823	56
5	9.74 850	9.83 035	0.16 965	9.91 815	55
6	9.74 868	9.83 062	0.16 938	9.91 806	54
7	9.74 887	9.83 089	0.16 911	9.91 798	53
8	9.74 906	9.83 117	0.16 883	9.91 789	52
9	9.74 924	9.83 144	0.16 856	9.91 781	51
10	9.74 943	9.83 171	0.16 829	9.91 772	50
11	9.74 961	9.83 198	0.16 802	9.91 763	49
12	9.74 980	9.83 225	0.16 775	9.91 755	48
13	9.74 999	9.83 252	0.16 748	9.91 746	47
14	9.75 017	9.83 280	0.16 720	9.91 738	46
15	9.75 036	9.83 307	0.16 693	9.91 729	45
16	9.75 054	9.83 334	0.16 666	9.91 720	44
17	9.75 073	9.83 361	0.16 639	9.91 712	43
18	9.75 091	9.83 388	0.16 612	9.91 703	42
19	9.75 110	9.83 415	0.16 585	9.91 695	41
20	9.75 128	9.83 442	0.16 558	9.91 686	40
21	9.75 147	9.83 470	0.16 530	9.91 677	39
22	9.75 165	9.83 497	0.16 503	9.91 669	38
23	9.75 184	9.83 524	0.16 476	9.91 660	37
24	9.75 202	9.83 551	0.16 449	9.91 651	36
25	9.75 221	9.83 578	0.16 422	9.91 643	35
26	9.75 239	9.83 605	0.16 395	9.91 634	34
27	9.75 258	9.83 632	0.16 368	9.91 625	33
28	9.75 276	9.83 659	0.16 341	9.91 617	32
29	9.75 294	9.83 686	0.16 314	9.91 608	31
30	9.75 313	9.83 713	0.16 287	9.91 599	30
31	9.75 331	9.83 740	0.16 260	9.91 591	29
32	9.75 350	9.83 768	0.16 232	9.91 582	28
33	9.75 368	9.83 795	0.16 205	9.91 573	27
34	9.75 386	9.83 822	0.16 178	9.91 565	26
35	9.75 405	9.83 849	0.16 151	9.91 556	25
36	9.75 423	9.83 876	0.16 124	9.91 547	24
37	9.75 441	9.83 903	0.16 097	9.91 538	23
38	9.75 459	9.83 930	0.16 070	9.91 530	22
39	9.75 478	9.83 957	0.16 043	9.91 521	21
40	9.75 496	9.83 984	0.16 016	9.91 512	20
41	9.75 514	9.84 011	0.15 989	9.91 504	19
42	9.75 533	9.84 038	0.15 962	9.91 495	18
43	9.75 551	9.84 065	0.15 935	9.91 486	17
44	9.75 569	9.84 092	0.15 908	9.91 477	16
45	9.75 587	9.84 119	0.15 881	9.91 469	15
46	9.75 605	9.84 146	0.15 854	9.91 460	14
47	9.75 624	9.84 173	0.15 827	9.91 451	13
48	9.75 642	9.84 200	0.15 800	9.91 442	12
49	9.75 660	9.84 227	0.15 773	9.91 433	11
50	9.75 678	9.84 254	0.15 746	9.91 425	10
51	9.75 696	9.84 280	0.15 720	9.91 416	9
52	9.75 714	9.84 307	0.15 693	9.91 407	8
53	9.75 733	9.84 334	0.15 666	9.91 398	7
54	9.75 751	9.84 361	0.15 639	9.91 389	6
55	9.75 769	9.84 388	0.15 612	9.91 381	5
56	9.75 787	9.84 415	0.15 585	9.91 372	4
57	9.75 805	9.84 442	0.15 558	9.91 363	3
58	9.75 823	9.84 469	0.15 531	9.91 354	2
59	9.75 841	9.84 496	0.15 504	9.91 345	1
60	9.75 859	9.84 523	0.15 477	9.91 336	0
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

55°

35°

'	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.75 859	9.84 523	0.15 477	9.91 336	60
1	9.75 877	9.84 550	0.15 450	9.91 328	59
2	9.75 895	9.84 576	0.15 424	9.91 319	58
3	9.75 913	9.84 603	0.15 397	9.91 310	57
4	9.75 931	9.84 630	0.15 370	9.91 301	56
5	9.75 949	9.84 657	0.15 343	9.91 292	55
6	9.75 967	9.84 684	0.15 316	9.91 283	54
7	9.75 985	9.84 711	0.15 289	9.91 274	53
8	9.76 003	9.84 738	0.15 262	9.91 266	52
9	9.76 021	9.84 764	0.15 236	9.91 257	51
10	9.76 039	9.84 791	0.15 209	9.91 248	50
11	9.76 057	9.84 818	0.15 182	9.91 239	49
12	9.76 075	9.84 845	0.15 155	9.91 230	48
13	9.76 093	9.84 872	0.15 128	9.91 221	47
14	9.76 111	9.84 899	0.15 101	9.91 212	46
15	9.76 129	9.84 925	0.15 075	9.91 203	45
16	9.76 146	9.84 952	0.15 048	9.91 194	44
17	9.76 164	9.84 979	0.15 021	9.91 185	43
18	9.76 182	9.85 006	0.14 994	9.91 176	42
19	9.76 200	9.85 033	0.14 967	9.91 167	41
20	9.76 218	9.85 059	0.14 941	9.91 158	40
21	9.76 236	9.85 086	0.14 914	9.91 149	39
22	9.76 253	9.85 113	0.14 887	9.91 141	38
23	9.76 271	9.85 140	0.14 860	9.91 132	37
24	9.76 289	9.85 166	0.14 834	9.91 123	36
25	9.76 307	9.85 193	0.14 807	9.91 114	35
26	9.76 324	9.85 220	0.14 780	9.91 105	34
27	9.76 342	9.85 247	0.14 753	9.91 096	33
28	9.76 360	9.85 273	0.14 727	9.91 087	32
29	9.76 378	9.85 300	0.14 700	9.91 078	31
30	9.76 395	9.85 327	0.14 673	9.91 069	30
31	9.76 413	9.85 354	0.14 646	9.91 060	29
32	9.76 431	9.85 380	0.14 620	9.91 051	28
33	9.76 448	9.85 407	0.14 593	9.91 042	27
34	9.76 466	9.85 434	0.14 566	9.91 033	26
35	9.76 484	9.85 460	0.14 540	9.91 023	25
36	9.76 501	9.85 487	0.14 513	9.91 014	24
37	9.76 519	9.85 514	0.14 486	9.91 005	23
38	9.76 537	9.85 540	0.14 460	9.90 996	22
39	9.76 554	9.85 567	0.14 433	9.90 987	21
40	9.76 572	9.85 594	0.14 406	9.90 978	20
41	9.76 590	9.85 620	0.14 380	9.90 969	19
42	9.76 607	9.85 647	0.14 353	9.90 960	18
43	9.76 625	9.85 674	0.14 326	9.90 951	17
44	9.76 642	9.85 700	0.14 300	9.90 942	16
45	9.76 660	9.85 727	0.14 273	9.90 933	15
46	9.76 677	9.85 754	0.14 246	9.90 924	14
47	9.76 695	9.85 780	0.14 220	9.90 915	13
48	9.76 712	9.85 807	0.14 193	9.90 906	12
49	9.76 730	9.85 834	0.14 166	9.90 896	11
50	9.76 747	9.85 860	0.14 140	9.90 887	10
51	9.76 765	9.85 887	0.14 113	9.90 878	9
52	9.76 782	9.85 913	0.14 087	9.90 869	8
53	9.76 800	9.85 940	0.14 060	9.90 860	7
54	9.76 817	9.85 967	0.14 033	9.90 851	6
55	9.76 835	9.85 993	0.14 007	9.90 842	5
56	9.76 852	9.86 020	0.13 980	9.90 832	4
57	9.76 870	9.86 046	0.13 954	9.90 823	3
58	9.76 887	9.86 073	0.13 927	9.90 814	2
59	9.76 904	9.86 100	0.13 900	9.90 805	1
60	9.76 922	9.86 126	0.13 874	9.90 796	0
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	'

54°

36° — 53°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.76 922	9.86 126	0.13 874	9.90 796	**60**
1	9.76 939	9.86 153	0.13 847	9.90 787	59
2	9.76 957	9.86 179	0.13 821	9.90 777	58
3	9.76 974	9.86 206	0.13 794	9.90 768	57
4	9.76 991	9.86 232	0.13 768	9.90 759	56
5	9.77 009	9.86 259	0.13 741	9.90 750	55
6	9.77 026	9.86 285	0.13 715	9.90 741	54
7	9.77 043	9.86 312	0.13 688	9.90 731	53
8	9.77 061	9.86 338	0.13 662	9.90 722	52
9	9.77 078	9.86 365	0.13 635	9.90 713	51
10	9.77 095	9.86 392	0.13 608	9.90 704	**50**
11	9.77 112	9.86 418	0.13 582	9.90 694	49
12	9.77 130	9.86 445	0.13 555	9.90 685	48
13	9.77 147	9.86 471	0.13 529	9.90 676	47
14	9.77 164	9.86 498	0.13 502	9.90 667	46
15	9.77 181	9.86 524	0.13 476	9.90 657	45
16	9.77 199	9.86 551	0.13 449	9.90 648	44
17	9.77 216	9.86 577	0.13 423	9.90 639	43
18	9.77 233	9.86 603	0.13 397	9.90 630	42
19	9.77 250	9.86 630	0.13 370	9.90 620	41
20	9.77 268	9.86 656	0.13 344	9.90 611	**40**
21	9.77 285	9.86 683	0.13 317	9.90 602	39
22	9.77 302	9.86 709	0.13 291	9.90 592	38
23	9.77 319	9.86 736	0.13 264	9.90 583	37
24	9.77 336	9.86 762	0.13 238	9.90 574	36
25	9.77 353	9.86 789	0.13 211	9.90 565	35
26	9.77 370	9.86 815	0.13 185	9.90 555	34
27	9.77 387	9.86 842	0.13 158	9.90 546	33
28	9.77 405	9.86 868	0.13 132	9.90 537	32
29	9.77 422	9.86 894	0.13 106	9.90 527	31
30	9.77 439	9.86 921	0.13 079	9.90 518	**30**
31	9.77 456	9.86 947	0.13 053	9.90 509	29
32	9.77 473	9.86 974	0.13 026	9.90 499	28
33	9.77 490	9.87 000	0.13 000	9.90 490	27
34	9.77 507	9.87 027	0.12 973	9.90 480	26
35	9.77 524	9.87 053	0.12 947	9.90 471	25
36	9.77 541	9.87 079	0.12 921	9.90 462	24
37	9.77 558	9.87 106	0.12 894	9.90 452	23
38	9.77 575	9.87 132	0.12 868	9.90 443	22
39	9.77 592	9.87 158	0.12 842	9.90 434	21
40	9.77 609	9.87 185	0.12 815	9.90 424	**20**
41	9.77 626	9.87 211	0.12 789	9.90 415	19
42	9.77 643	9.87 238	0.12 762	9.90 405	18
43	9.77 660	9.87 264	0.12 736	9.90 396	17
44	9.77 677	9.87 290	0.12 710	9.90 386	16
45	9.77 694	9.87 317	0.12 683	9.90 377	15
46	9.77 711	9.87 343	0.12 657	9.90 368	14
47	9.77 728	9.87 369	0.12 631	9.90 358	13
48	9.77 744	9.87 396	0.12 604	9.90 349	12
49	9.77 761	9.87 422	0.12 578	9.90 339	11
50	9.77 778	9.87 448	0.12 552	9.90 330	**10**
51	9.77 795	9.87 475	0.12 525	9.90 320	9
52	9.77 812	9.87 501	0.12 499	9.90 311	8
53	9.77 829	9.87 527	0.12 473	9.90 301	7
54	9.77 846	9.87 554	0.12 446	9.90 292	6
55	9.77 862	9.87 580	0.12 420	9.90 282	5
56	9.77 879	9.87 606	0.12 394	9.90 273	4
57	9.77 896	9.87 633	0.12 367	9.90 263	3
58	9.77 913	9.87 659	0.12 341	9.90 254	2
59	9.77 930	9.87 685	0.12 315	9.90 244	1
60	977 946	9.87 711	0.12 289	9.90 235	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

37°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.77 946	9.87 711	0.12 289	9.90 235	60
1	9.77 963	9.87 738	0.12 262	9.90 225	59
2	9.77 980	9.87 764	0.12 236	9.90 216	58
3	9.77 997	9.87 790	0.12 210	9.90 206	57
4	9.78 013	9.87 817	0.12 183	9.90 197	56
5	9.78 030	9.87 843	0.12 157	9.90 187	55
6	9.78 047	9.87 869	0.12 131	9.90 178	54
7	9.78 063	9.87 895	0.12 105	9.90 168	53
8	9.78 080	9.87 922	0.12 078	9.90 159	52
9	9.78 097	9.87 948	0.12 052	9.90 149	51
10	9.78 113	9.87 974	0.12 026	9.90 139	50
11	9.78 130	9.88 000	0.12 000	9.90 130	49
12	9.78 147	9.88 027	0.11 973	9.90 120	48
13	9.78 163	9.88 053	0.11 947	9.90 111	47
14	9.78 180	9.88 079	0.11 921	9.90 101	46
15	9.78 197	9.88 105	0.11 895	9.90 091	45
16	9.78 213	9.88 131	0.11 869	9.90 082	44
17	9.78 230	9.88 158	0.11 842	9.90 072	43
18	9.78 246	9.88 184	0.11 816	9.90 063	42
19	9.78 263	9.88 210	0.11 790	9.90 053	41
20	9.78 280	9.88 236	0.11 764	9.90 043	40
21	9.78 296	9.88 262	0.11 738	9.90 034	39
22	9.78 313	9.88 289	0.11 711	9.90 024	38
23	9.78 329	9.88 315	0.11 685	9.90 014	37
24	9.78 346	9.88 341	0.11 659	9.90 005	36
25	9.78 362	9.88 367	0.11 633	9.89 995	35
26	9.78 379	9.88 393	0.11 607	9.89 985	34
27	9.78 395	9.88 420	0.11 580	9.89 976	33
28	9.78 412	9.88 446	0.11 554	9.89 966	32
29	9.78 428	9.88 472	0.11 528	9.89 956	31
30	9.78 445	9.88 498	0.11 502	9.89 947	30
31	9.78 461	9.88 524	0.11 476	9.89 937	29
32	9.78 478	9.88 550	0.11 450	9.89 927	28
33	9.78 494	9.88 577	0.11 423	9.89 918	27
34	9.78 510	9.88 603	0.11 397	9.89 908	26
35	9.78 527	9.88 629	0.11 371	9.89 898	25
36	9.78 543	9.88 655	0.11 345	9.89 888	24
37	9.78 560	9.88 681	0.11 319	9.89 879	23
38	9.78 576	9.88 707	0.11 293	9.89 869	22
39	9.78 592	9.88 733	0.11 267	9.89 859	21
40	9.78 609	9.88 759	0.11 241	9.89 849	20
41	9.78 625	9.88 786	0.11 214	9.89 840	19
42	9.78 642	9.88 812	0.11 188	9.89 830	18
43	9.78 658	9.88 838	0.11 162	9.89 820	17
44	9.78 674	9.88 864	0.11 136	9.89 810	16
45	9.78 691	9.88 890	0.11 110	9.89 801	15
46	9.78 707	9.88 916	0.11 084	9.89 791	14
47	9.78 723	9.88 942	0.11 058	9.89 781	13
48	9.78 739	9.88 968	0.11 032	9.89 771	12
49	9.78 756	9.88 994	0.11 006	9.89 761	11
50	9.78 772	9.89 020	0.10 980	9.89 752	10
51	9.78 788	9.89 046	0.10 954	9.89 742	9
52	9.78 805	9.89 073	0.10 927	9.89 732	8
53	9.78 821	9.89 099	0.10 901	9.89 722	7
54	9.78 837	9.89 125	0.10 875	9.89 712	6
55	9.78 853	9.89 151	0.10 849	9.89 702	5
56	9.78 869	9.89 177	0.10 823	9.89 693	4
57	9.78 886	9.89 203	0.10 797	9.89 683	3
58	9.78 902	9.89 229	0.10 771	9.89 673	2
59	9.78 918	9.89 255	0.10 745	9.89 663	1
60	9.78 934	9.89 281	0.10 719	9.89 653	0
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	

52°

38° | 51°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.78 934	9.89 281	0.10 719	9.89 653	**60**
1	9.78 950	9.89 307	0.10 693	9.89 643	59
2	9.78 967	9.89 333	0.10 667	9.89 633	58
3	9.78 983	9.89 359	0.10 641	9.89 624	57
4	9.78 999	9.89 385	0.10 615	9.89 614	56
5	9.79 015	9.89 411	0.10 589	9.89 604	55
6	9.79 031	9.89 437	0.10 563	9.89 594	54
7	9.79 047	9.89 463	0.10 537	9.89 584	53
8	9.79 063	9.89 489	0.10 511	9.89 574	52
9	9.79 079	9.89 515	0.10 485	9.89 564	51
10	9.79 095	9.89 541	0.10 459	9.89 554	**50**
11	9.79 111	9.89 567	0.10 433	9.89 544	49
12	9.79 128	9.89 593	0.10 407	9.89 534	48
13	9.79 144	9.89 619	0.10 381	9.89 524	47
14	9.79 160	9.89 645	0.10 355	9.89 514	46
15	9.79 176	9.89 671	0.10 329	9.89 504	45
16	9.79 192	9.89 697	0.10 303	9.89 495	44
17	9.79 208	9.89 723	0.10 277	9.89 485	43
18	9.79 224	9.89 749	0.10 251	9.89 475	42
19	9.79 240	9.89 775	0.10 225	9.89 465	41
20	9.79 256	9.89 801	0.10 199	9.89 455	**40**
21	9.79 272	9.89 827	0.10 173	9.89 445	39
22	9.79 288	9.89 853	0.10 147	9.89 435	38
23	9.79 304	9.89 879	0.10 121	9.89 425	37
24	9.79 319	9.89 905	0.10 095	9.89 415	36
25	9.79 335	9.89 931	0.10 069	9.89 405	35
26	9.79 351	9.89 957	0.10 043	9.89 395	34
27	9.79 367	9.89 983	0.10 017	9.89 385	33
28	9.79 383	9.90 009	0.09 991	9.89 375	32
29	9.79 399	9.90 035	0.09 965	9.89 364	31
30	9.79 415	9.90 061	0.09 939	9.89 354	**30**
31	9.79 431	9.90 086	0.09 914	9.89 344	29
32	9.79 447	9.90 112	0.09 888	9.89 334	28
33	9.79 463	9.90 138	0.09 862	9.89 324	27
34	9.79 478	9.90 164	0.09 836	9.89 314	26
35	9.79 494	9.90 190	0.09 810	9.89 304	25
36	9.79 510	9.90 216	0.09 784	9.89 294	24
37	9.79 526	9.90 242	0.09 758	9.89 284	23
38	9.79 542	9.90 268	0.09 732	9.89 274	22
39	9.79 558	9.90 294	0.09 706	9.89 264	21
40	9.79 573	9.90 320	0.09 680	9.89 254	**20**
41	9.79 589	9.90 346	0.09 654	9.89 244	19
42	9.79 605	9.90 371	0.09 629	9.89 233	18
43	9.79 621	9.90 397	0.09 603	9.89 223	17
44	9.79 636	9.90 423	0.09 577	9.89 213	16
45	9.79 652	9.90 449	0.09 551	9.89 203	15
46	9.79 668	9.90 475	0.09 525	9.89 193	14
47	9.79 684	9.90 501	0.09 499	9.89 183	13
48	9.79 699	9.90 527	0.09 473	9.89 173	12
49	9.79 715	9.90 553	0.09 447	9.89 162	11
50	9.79 731	9.90 578	0.09 422	9.89 152	**10**
51	9.79 746	9.90 604	0.09 396	9.89 142	9
52	9.79 762	9.90 630	0.09 370	9.89 132	8
53	9.79 778	9.90 656	0.09 344	9.89 122	7
54	9.79 793	9.90 682	0.09 318	9.89 112	6
55	9.79 809	9.90 708	0.09 292	9.89 101	5
56	9.79 825	9.90 734	0.09 266	9.89 091	4
57	9.79 840	9.90 759	0.09 241	9.89 081	3
58	9.79 856	9.90 785	0.09 215	9.89 071	2
59	9.79 872	9.90 811	0.09 189	9.89 060	1
60	9.79 887	9.90 837	0.09 163	9.89 050	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

39° | 50°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.79 887	9.90 837	0.09 163	9.89 050	**60**
1	9.79 903	9.90 863	0.09 137	9.89 040	59
2	9.79 918	9.90 889	0.09 111	9.89 030	58
3	9.79 934	9.90 914	0.09 086	9.89 020	57
4	9.70 950	9.90 940	0.09 060	9.89 009	56
5	9.79 965	9.90 966	0.09 034	9.88 999	55
6	9.79 981	9.90 992	0.09 008	9.88 989	54
7	9.79 996	9.91 018	0.08 982	9.88 978	53
8	9.80 012	9.91 043	0.08 957	9.88 968	52
9	9.80 027	9.91 069	0.08 931	9.88 958	51
10	9.80 043	9.91 095	0.08 905	9.88 948	**50**
11	9.80 058	9.91 121	0.08 879	9.88 937	49
12	9.80 074	9.91 147	0.08 853	9.88 927	48
13	9.80 089	9.91 172	0.08 828	9.88 917	47
14	9.80 105	9.91 198	0.08 802	9.88 906	46
15	9.80 120	9.91 224	0.08 776	9.88 896	45
16	9.80 136	9.91 250	0.08 750	9.88 886	44
17	9.80 151	9.91 276	0.08 724	9.88 875	43
18	9.80 166	9.91 301	0.08 699	9.88 865	42
19	9.80 182	9.91 327	0.08 673	9.88 855	41
20	9.80 197	9.91 353	0.08 647	9.88 844	**40**
21	9.80 213	9.91 379	0.08 621	9.88 834	39
22	9.80 228	9.91 404	0.08 596	9.88 824	38
23	9.80 244	9.91 430	0.08 570	9.88 813	37
24	9.80 259	9.91 456	0.08 544	9.88 803	36
25	9.80 274	9.91 482	0.08 518	9.88 793	35
26	9.80 290	9.91 507	0.08 493	9.88 782	34
27	9.80 305	9.91 533	0.08 467	9.88 772	33
28	9.80 320	9.91 559	0.08 441	9.88 761	32
29	9.80 336	9.91 585	0.08 415	9.88 751	31
30	9.80 351	9.91 610	0.08 390	9.88 741	**30**
31	9.80 366	9.91 636	0.08 364	9.88 730	29
32	9.80 382	9.91 662	0.08 338	9.88 720	28
33	9.80 397	9.91 688	0.08 312	9.88 709	27
34	9.80 412	9.91 713	0.08 287	9.88 699	26
35	9.80 428	9.91 739	0.08 261	9.88 688	25
36	9.80 443	9.91 765	0.08 235	9.88 678	24
37	9.80 458	9.91 791	0.08 209	9.88 668	23
38	9.80 473	9.91 816	0.08 184	9.88 657	22
39	9.80 489	9.91 842	0.08 158	9.88 647	21
40	9.80 504	9.91 868	0.08 132	9.88 636	**20**
41	9.80 519	9.91 893	0.08 107	9.88 626	19
42	9.80 534	9.91 919	0.08 081	9.88 615	18
43	9.80 550	9.91 945	0.08 055	9.88 605	17
44	9.80 565	9.91 971	0.08 029	9.88 594	16
45	9.80 580	9.91 996	0.08 004	9.88 584	15
46	9.80 595	9.92 022	0.07 978	9.88 573	14
47	9.80 610	9.92 048	0.07 952	9.88 563	13
48	9.80 625	9.92 073	0.07 927	9.88 552	12
49	9.80 641	9.92 099	0.07 901	9.88 542	11
50	9.80 656	9.92 125	0.07 875	9.88 531	**10**
51	9.80 671	9.92 150	0.07 850	9.88 521	9
52	9.80 686	9.92 176	0.07 824	9.88 510	8
53	9.80 701	9.92 202	0.07 798	9.88 499	7
54	9.80 716	9.92 227	0.07 773	9.88 489	6
55	9.80 731	9.92 253	0.07 747	9.88 478	5
56	9.80 746	9.92 279	0.07 721	9.88 468	4
57	9.80 762	9.92 304	0.07 696	9.88 457	3
58	9.80 777	9.92 330	0.07 670	9.88 447	2
59	9.80 792	9.92 356	0.07 644	9.88 436	1
60	9.80 807	9.92 381	0.07 619	9.88 425	**0**
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

40°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.80 807	9.92 381	0.07 619	9.88 425	**60**
1	9.80 822	9.92 407	0.07 593	9.88 415	59
2	9.80 837	9.92 433	0.07 567	9.88 404	58
3	9.80 852	9.92 458	0.07 542	9.88 394	57
4	9.80 867	9.92 484	0.07 516	9.88 383	56
5	9.80 882	9.92 510	0.07 490	9.88 372	55
6	9.80 897	9.92 535	0.07 465	9.88 362	54
7	9.80 912	9.92 561	0.07 439	9.88 351	53
8	9.80 927	9.92 587	0.07 413	9.88 340	52
9	9.80 942	9.92 612	0.07 388	9.88 330	51
10	9.80 957	9.92 638	0.07 362	9.88 319	**50**
11	9.80 972	9.92 663	0.07 337	9.88 308	49
12	9.80 987	9.92 689	0.07 311	9.88 298	48
13	9.81 002	9.92 715	0.07 285	9.88 287	47
14	9.81 017	9.92 740	0.07 260	9.88 276	46
15	9.81 032	9.92 766	0.07 234	9.88 266	45
16	9.81 047	9.92 792	0.07 208	9.88 255	44
17	9.81 061	9.92 817	0.07 183	9.88 244	43
18	9.81 076	9.92 843	0.07 157	9.88 234	42
19	9.81 091	9.92 868	0.07 132	9.88 223	41
20	9.81 106	9.92 894	0.07 106	9.88 212	**40**
21	9.81 121	9.92 920	0.07 080	9.88 201	39
22	9.81 136	9.92 945	0.07 055	9.88 191	38
23	9.81 151	9.92 971	0.07 029	9.88 180	37
24	9.81 166	9.92 996	0.07 004	9.88 169	36
25	9.81 180	9.93 022	0.06 978	9.88 158	35
26	9.81 195	9.93 048	0.06 952	9.88 148	34
27	9.81 210	9.93 073	0.06 927	9.88 137	33
28	9.81 225	9.93 099	0.06 901	9.88 126	32
29	9.81 240	9.93 124	0.06 876	9.88 115	31
30	9.81 254	9.93 150	0.06 850	9.88 105	**30**
31	9.81 269	9.93 175	0.06 825	9.88 094	29
32	9.81 284	9.93 201	0.06 799	9.88 083	28
33	9.81 299	9.93 227	0.06 773	9.88 072	27
34	9.81 314	9.93 252	0.06 748	9.88 061	26
35	9.81 328	9.93 278	0.06 722	9.88 051	25
36	9.81 343	9.93 303	0.06 697	9.88 040	24
37	9.81 358	9.93 329	0.06 671	9.88 029	23
38	9.81 372	9.93 354	0.06 646	9.88 018	22
39	9.81 387	9.93 380	0.06 620	9.88 007	21
40	9.81 402	9.93 406	0.06 594	9.87 996	**20**
41	9.81 417	9.93 431	0.06 569	9.87 985	19
42	9.81 431	9.93 457	0.06 543	9.87 975	18
43	9.81 446	9.93 482	0.06 518	9.87 964	17
44	9.81 461	9.93 508	0.06 492	9.87 953	16
45	9.81 475	9.93 533	0.06 467	9.87 942	15
46	9.81 490	9.93 559	0.06 441	9.87 931	14
47	9.81 505	9.93 584	0 06 416	9.87 920	13
48	9.81 519	9.93 610	0.06 390	9.87 909	12
49	9.81 534	9.93 636	0.06 364	9.87 898	11
50	9.81 549	9.93 661	0.06 339	9.87 887	**10**
51	9.81 563	9.93 687	0.06 313	9.87 877	9
52	9.81 578	9.93 712	0.06 288	9.87 866	8
53	9.81 592	9.93 738	0.06 262	9.87 855	7
54	9.81 607	9.93 763	0.06 237	9.87 844	6
55	9.81 622	9.93 789	0.06 211	9.87 833	5
56	9.81 636	9.93 814	0.06 186	9.87 822	4
57	9.81 651	9.93 840	0.06 160	9.87 811	3
58	9.81 665	9.93 865	0.06 135	9.87 800	2
59	9.81 680	9.93 891	0.06 109	9.87 789	1
60	9.81 694	9.93 916	0.06 084	9 87 778	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

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41°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.81 694	9.93 916	0.06 084	9.87 778	**60**
1	9.81 709	9.93 942	0.06 058	9.87 767	59
2	9.81 723	9.93 967	0.06 033	9.87 756	58
3	9.81 738	9.93 993	0.06 007	9.87 745	57
4	9.81 752	9.94 018	0.05 982	9.87 734	56
5	9.81 767	9.94 044	0.05 956	9.87 723	55
6	9.81 781	9.94 069	0.05 931	9.87 712	54
7	9.81 796	9.94 095	0.05 905	9.87 701	53
8	9.81 810	9.94 120	0.05 880	9.87 690	52
9	9.81 825	9.94 146	0.05 854	9.87 679	51
10	9.81 839	9.94 171	0.05 829	9.87 668	**50**
11	9.81 854	9.94 197	0.05 803	9.87 657	49
12	9.81 868	9.94 222	0.05 778	9.87 646	48
13	9.81 882	9.94 248	0.05 752	9.87 635	47
14	9.81 897	9.94 273	0.05 727	9.87 624	46
15	9.81 911	9.94 299	0.05 701	9.87 613	45
16	9.81 926	9.94 324	0.05 676	9.87 601	44
17	9.81 940	9.94 350	0.05 650	9.87 590	43
18	9.81 955	9.94 375	0.05 625	9.87 579	42
19	9.81 969	9.94 401	0.05 599	9.87 568	41
20	9.81 983	9.94 426	0.05 574	9.87 557	**40**
21	9.81 998	9.94 452	0.05 548	9.87 546	39
22	9.82 012	9.94 477	0.05 523	9.87 535	38
23	9.82 026	9.94 503	0.05 497	9.87 524	37
24	9.82 041	9.94 528	0.05 472	9.87 513	36
25	9.82 055	9.94 554	0.05 446	9.87 501	35
26	9.82 069	9.94 579	0.05 421	9.87 490	34
27	9.82 084	9.94 604	0.05 396	9.87 479	33
28	9.82 098	9.94 630	0.05 370	9.87 468	32
29	9.82 112	9.94 655	0.05 345	9.87 457	31
30	9.82 126	9.94 681	0.05 319	9.87 446	**30**
31	9.82 141	9.94 706	0.05 294	9.87 434	29
32	9.82 155	9.94 732	0.05 268	9.87 423	28
33	9.82 169	9.94 757	0.05 243	9.87 412	27
34	9.82 184	9.94 783	0.05 217	9.87 401	26
35	9.82 198	9.94 808	0.05 192	9.87 390	25
36	9.82 212	9.94 834	0.05 166	9.87 378	24
37	9.82 226	9.94 859	0.05 141	9.87 367	23
38	9.82 240	9.94 884	0.05 116	9.87 356	22
39	9.82 255	9.94 910	0.05 090	9.87 345	21
40	9.82 269	9.94 935	0.05 065	9.87 334	**20**
41	9.82 283	9.94 961	0.05 039	9.87 322	19
42	9.82 297	9.94 986	0.05 014	9.87 311	18
43	9.82 311	9.95 012	0.04 988	9.87 300	17
44	9.82 326	9.95 037	0.04 963	9.87 288	16
45	9.82 340	9.95 062	0.04 938	9.87 277	15
46	9.82 354	9.95 088	0.04 912	9.87 266	14
47	9.82 368	9.95 113	0.04 887	9.87 255	13
48	9.82 382	9.95 139	0.04 861	9.87 243	12
49	9.82 396	9.95 164	0.04 836	9.87 232	11
50	9.82 410	9.95 190	0.04 810	9.87 221	**10**
51	9.82 424	9.95 215	0.04 785	9.87 209	9
52	9.82 439	9.95 240	0.04 760	9.87 198	8
53	9.82 453	9.95 266	0.04 734	9.87 187	7
54	9.82 467	9.95 291	0.04 709	9.87 175	6
55	9.82 481	9.95 317	0.04 683	9.87 164	5
56	9.82 495	9.95 342	0.04 658	9.87 153	4
57	9.82 509	9.95 368	0.04 632	9.87 141	3
58	9.82 523	9.95 393	0.04 607	9.87 130	2
59	9.82 537	9.95 418	0.04 582	9.87 119	1
60	9.82 551	9.95 444	0.04 556	9.87 107	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

48°

42° | 47°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.82 551	9.95 444	0.04 556	9.87 107	60
1	9.82 565	9.95 469	0.04 531	9.87 096	59
2	9.82 579	9.95 495	0.04 505	9.87 085	58
3	9.82 593	9.95 520	0.04 480	9.87 073	57
4	9.82 607	9.95 545	0.04 455	9.87 062	56
5	9.82 621	9.95 571	0.04 429	9.87 050	55
6	9.82 635	9.95 596	0.04 404	9.87 039	54
7	9.82 649	9.95 622	0.04 378	9.87 028	53
8	9.82 663	9.95 647	0.04 353	9.87 016	52
9	9.82 677	9.95 672	0.04 328	9.87 005	51
10	9.82 691	9.95 698	0.04 302	9.86 993	50
11	9.82 705	9.95 723	0.04 277	9.86 982	49
12	9.82 719	9.95 748	0.04 252	9.86 970	48
13	9.82 733	9.95 774	0.04 226	9.86 959	47
14	9.82 747	9.95 799	0.04 201	9.86 947	46
15	9.82 761	9.95 825	0.04 175	9.86 936	45
16	9.82 775	9.95 850	0.04 150	9.86 924	44
17	9.82 788	9.95 875	0.04 125	9.86 913	43
18	9.82 802	9.95 901	0.04 099	9.86 902	42
19	9.82 816	9.95 926	0.04 074	9.86 890	41
20	9.82 830	9.95 952	0.04 048	9.86 879	40
21	9.82 844	9.95 977	0.04 023	9.86 867	39
22	9.82 858	9.96 002	0.03 998	9.86 855	38
23	9.82 872	9.96 028	0.03 972	9.86 844	37
24	9.82 885	9.96 053	0.03 947	9.86 832	36
25	9.82 899	9.96 078	0.03 922	9.86 821	35
26	9.82 913	9.96 104	0.03 896	9.86 809	34
27	9.82 927	9.96 129	0.03 871	9.86 798	33
28	9.82 941	9.96 155	0.03 845	9.86 786	32
29	9.82 955	9.96 180	0.03 820	9.86 775	31
30	9.82 968	9.96 205	0.03 795	9.86 763	30
31	9.82 982	9.96 231	0.03 769	9.86 752	29
32	9.82 996	9.96 256	0.03 744	9.86 740	28
33	9.83 010	9.96 281	0.03 719	9.86 728	27
34	9.83 023	9.96 307	0.03 693	9.86 717	26
35	9.83 037	9.96 332	0.03 668	9.86 705	25
36	9.83 051	9.96 357	0.03 643	9.86 694	24
37	9.83 065	9.96 383	0.03 617	9.86 682	23
38	9.83 078	9.96 408	0.03 592	9.86 670	22
39	9.83 092	9.96 433	0.03 567	9.86 659	21
40	9.83 106	9.96 459	0.03 541	9.86 647	20
41	9.83 120	9.96 484	0.03 516	9.86 635	19
42	9.83 133	9.96 510	0.03 490	9.86 624	18
43	9.83 147	9.96 535	0.03 465	9.86 612	17
44	9.83 161	9.96 560	0.03 440	9.86 600	16
45	9.83 174	9.96 586	0.03 414	9.86 589	15
46	9.83 188	9.96 611	0.03 389	9.86 577	14
47	9.83 202	9.96 636	0.03 364	9.86 565	13
48	9.83 215	9.96 662	0.03 338	9.86 554	12
49	9.83 229	9.96 687	0.03 313	9.86 542	11
50	9.83 242	9.96 712	0.03 288	9.86 530	10
51	9.83 256	9.96 738	0.03 262	9.86 518	9
52	9.83 270	9.96 763	0.03 237	9.86 507	8
53	9.83 283	9.96 788	0.03 212	9.86 495	7
54	9.83 297	9.96 814	0.03 186	9.86 483	6
55	9.83 310	9.96 839	0.03 161	9.86 472	5
56	9.83 324	9.96 864	0.03 136	9.86 460	4
57	9.83 338	9.96 890	0.03 110	9.86 448	3
58	9.83 351	9.96 915	0.03 085	9.86 436	2
59	9.83 365	9.96 940	0.03 060	9.86 425	1
60	9.83 378	9.96 966	0.03 034	9.86 413	0
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	′

43° | 46°

'	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.83 378	9.96 966	0.03 034	9.86 413	60
1	9.83 392	9.96 991	0.03 009	9.86 401	59
2	9.83 405	9.97 016	0.02 984	9.86 389	58
3	9.83 419	9.97 042	0.02 958	9.86 377	57
4	9.83 432	9.97 067	0.02 933	9.86 366	56
5	9.83 446	9.97 092	0.02 908	9.86 354	55
6	9.83 459	9.97 118	0.02 882	9.86 342	54
7	9.83 473	9.97 143	0.02 857	9.86 330	53
8	9.83 486	9.97 168	0.02 832	9.86 318	52
9	9.83 500	9.97 193	0.02 807	9.86 306	51
10	9.83 513	9.97 219	0.02 781	9.86 295	50
11	9.83 527	9.97 244	0.02 756	9.86 283	49
12	9.83 540	9.97 269	0.02 731	9.86 271	48
13	9.83 554	9.97 295	0.02 705	9.86 259	47
14	9.83 567	9.97 320	0.02 680	9.86 247	46
15	9.83 581	9.97 345	0.02 655	9.86 235	45
16	9.83 594	9.97 371	0.02 629	9.86 223	44
17	9.83 608	9.97 396	0.02 604	9.86 211	43
18	9.83 621	9.97 421	0.02 579	9.86 200	42
19	9.83 634	9.97 447	0.02 553	9.86 188	41
20	9.83 648	9.97 472	0.02 528	9.86 176	40
21	9.83 661	9.97 497	0.02 503	9.86 164	39
22	9.83 674	9.97 523	0.02 477	9.86 152	38
23	9.83 688	9.97 548	0.02 452	9.86 140	37
24	9.83 701	9.97 573	0.02 427	9.86 128	36
25	9.83 715	9.97 598	0.02 402	9.86 116	35
26	9.83 728	9.97 624	0.02 376	9.86 104	34
27	9.83 741	9.97 649	0.02 351	9.86 092	33
28	9.83 755	9.97 674	0.02 326	9.86 080	32
29	9.83 768	9.97 700	0.02 300	9.86 068	31
30	9.83 781	9.97 725	0.02 275	9.86 056	30
31	9.83 595	9.97 750	0.02 250	9.86 044	29
32	9.83 808	9.97 776	0.02 224	9.86 032	28
33	9.83 821	9.97 801	0.02 199	9.86 020	27
34	9.83 834	9.97 826	0.02 174	9.86 008	26
35	9.83 848	9.97 851	0.02 149	9.85 996	25
36	9.83 861	9.97 877	0.02 123	9.85 984	24
37	9.83 874	9.97 902	0.02 098	9.85 972	23
38	9.83 887	9.97 927	0.02 073	9.85 960	22
39	9.83 901	9.97 953	0.02 047	9.85 948	21
40	9.83 914	9.97 978	0.02 022	9.85 936	20
41	9.83 927	9.98 003	0.01 997	9.85 924	19
42	9.83 940	9.98 029	0.01 971	9.85 912	18
43	9.83 954	9.98 054	0.01 946	9.85 900	17
44	9.83 967	9.98 079	0.01 921	9.85 888	16
45	9.83 980	9.98 104	0.01 896	9.85 876	15
46	9.83 993	9.98 130	0.01 870	9.85 864	14
47	9.84 006	9.98 155	0.01 845	9.85 851	13
48	9.84 020	9.98 180	0.01 820	9.85 839	12
49	9.84 033	9.98 206	0.01 794	9.85 827	11
50	9.84 046	9.98 231	0.01 769	9.85 815	10
51	9.84 059	9.98 256	0.01 744	9.85 803	9
52	9.84 072	9.98 281	0.01 719	9.85 791	8
53	9.84 085	9.98 307	0.01 693	9.85 779	7
54	9.84 098	9.98 332	0.01 668	9.85 766	6
55	9.84 112	9.98 357	0.01 643	9.85 754	5
56	9.84 125	9.98 383	0.00 617	9.85 742	4
57	9.84 138	9.98 408	0.01 592	9.85 730	3
58	9.84 151	9.98 433	0.01 567	9.85 718	2
59	9.84 164	9.98 458	0.01 542	9.85 706	1
60	9.84 177	9.98 484	0.01 516	9.85 693	0
	L. Cos.	L. Cotg.	L. Tang.	L. Sin.	'

44°

′	L. Sin.	L. Tang.	L. Cotg.	L. Cos.	
0	9.84 177	9.98 484	0.01 516	9.85 693	**60**
1	9.84 190	9.98 509	0.01 491	9.85 681	59
2	9.84 203	9.98 534	0.01 466	9.85 669	58
3	9.84 216	9.98 560	0.01 440	9.85 657	57
4	9.84 229	9.98 58$\bar{5}$	0.01 415	9.85 64$\bar{5}$	56
5	9.84 242	9.98 610	0.01 390	9.85 632	55
6	9.84 255	9.98 635	0.01 36$\bar{5}$	9.85 620	54
7	9.84 269	9.98 661	0.01 339	9.85 608	53
8	9.84 282	9.98 686	0.01 314	9.85 596	52
9	9.84 29$\bar{5}$	9.98 711	0.01 289	9.85 583	51
10	9.84 308	9.98 737	0.01 263	9.85 571	**50**
11	9.84 321	9.98 762	0.01 238	9.85 559	49
12	9.84 334	9.98 787	0.01 213	9.85 547	48
13	9.84 347	9.98 812	0.01 188	9.85 534	47
14	9.84 360	9.98 838	0.01 162	9.85 522	46
15	9.84 373	9.98 863	0.01 137	9.85 510	45
16	9.84 385	9.98 888	0.01 112	9.85 497	44
17	9.84 398	9.98 913	0.01 087	9.85 48$\bar{5}$	43
18	9.84 411	9.98 939	0.01 061	9.85 473	42
19	9.84 424	9.98 964	0.01 036	9.85 460	41
20	9.84 437	9.98 989	0.01 011	9.85 448	**40**
21	9.84 450	9.99 01$\bar{5}$	0.00 985	9.85 436	39
22	9.84 463	9.99 040	0.00 960	9.85 423	38
23	9.84 476	9.99 065	0.00 93$\bar{5}$	9.85 411	37
24	9.84 489	9.99 090	0.00 910	9.85 399	36
25	9.84 502	9.99 116	0.00 884	9.85 386	35
26	9.84 51$\bar{5}$	9.99 141	0.00 859	9.85 374	34
27	9.84 528	9.99 166	0.00 834	9.85 361	33
28	9.84 540	9.99 191	0.00 809	9.85 349	32
29	9.84 553	9.99 217	0.00 783	9.85 337	31
30	9.84 566	9.99 242	0.00 758	9.85 324	**30**
31	9.84 579	9.99 267	0.00 733	9.85 312	29
32	9.84 592	9.99 293	0.00 707	9.85 299	28
33	9.84 60$\bar{5}$	9.99 318	0.00 682	9.85 287	27
34	9.84 618	9.99 343	0.00 657	9.85 274	26
35	9.84 630	9.99 368	0.00 632	9.85 262	25
36	9.84 643	9.99 394	0.00 606	9.85 2$\bar{5}$0	24
37	9.84 656	9.99 419	0.00 581	9.85 237	23
38	9.84 669	9.99 444	0.00 556	9.85 22$\bar{5}$	22
39	9.84 682	9.99 469	0.00 531	9.85 212	21
40	9.84 694	9.99 49$\bar{5}$	0.00 505	9.85 200	**20**
41	9.84 707	9.99 520	0.00 480	9.85 187	19
42	9.84 720	9.99 545	0.00 45$\bar{5}$	9.85 17$\bar{5}$	18
43	9.84 733	9.99 570	0.00 430	9.85 162	17
44	9.84 745	9.99 596	0.00 404	9.85 1$\bar{5}$0	16
45	9.84 758	9.99 621	0.00 379	9.85 137	15
46	9.84 771	9.99 646	0.00 354	9.85 12$\bar{5}$	14
47	9.84 784	9.99 672	0.00 328	9.85 112	13
48	9.84 796	9.99 697	0.00 303	9.85 100	12
49	9.84 809	9.99 722	0.00 278	9.85 087	11
50	9.84 822	9.99 747	0.00 253	9.85 074	**10**
51	9.84 83$\bar{5}$	9.99 773	0.00 227	9.85 062	9
52	9.84 847	9.99 798	0.00 202	9.85 049	8
53	9.84 860	9.99 823	0.00 177	9.85 037	7
54	9.84 873	9.99 848	0.00 152	9.85 024	6
55	9.84 885	9.99 874	0.00 126	9.85 012	5
56	9.84 898	9.99 899	0.00 101	9.84 999	4
57	9.84 911	9.99 924	0.00 076	9.84 986	3
58	9.84 923	9.99 949	0.00 051	9.84 974	2
59	9.84 936	9.99 97$\bar{5}$	0.00 025	9.84 961	1
60	9.84 949	0.00 000	0.00 000	9.84 949	**0**
	L. Cos.	**L. Cotg.**	**L. Tang.**	**L. Sin.**	′

45°

TABLE IV

AUXILIARY FIVE-PLACE TABLE

FOR

SMALL ANGLES

	″	′	S	T	S′	T′	L. Sin.
0°	0	0	4.68557	4.68557	5.31443	5.31443	——
	60	1	.68557	.68557	.31443	.31443	6.46373
	120	2	.68557	.68557	.31443	.31443	.76476
	180	3	.68557	.68557	.31443	.31443	.94085
	240	4	.68557	.68558	.31443	.31442	7.06579
	300	5	4.68557	4.68558	5.31443	5.31442	7.16270
	360	6	.68557	.68558	.31443	.31442	.24188
	420	7	.68557	.68558	.31443	.31442	.30882
	480	8	.68557	.68558	.31443	.31442	.36682
	540	9	.68557	.68558	.31443	.31442	.41797
	600	10	4.68557	4.68558	5.31443	5.31442	7.46373
	660	11	.68557	.68558	.31443	.31442	.50512
	720	12	.68557	.68558	.31443	.31442	.54291
	780	13	.68557	.68558	.31443	.31442	.57767
	840	14	.68557	.68558	.31443	.31442	.60985
	900	15	4.68557	4.68558	5.31443	5.31442	7.63982
	960	16	.68557	.68558	.31443	.31442	.66784
	1020	17	.68557	.68558	.31443	.31442	.69417
	1080	18	.68557	.68558	.31443	.31442	.71900
	1140	19	.68557	.68558	.31443	.31442	.74248
	1200	20	4.68557	4.68558	5.31443	5.31442	7.76475
	1260	21	.68557	.68558	.31443	.31442	.78594
	1320	22	.68557	.68558	.31443	.31442	.80615
	1380	23	.68557	.68558	.31443	.31442	.82545
	1440	24	.68557	.68558	.31443	.31442	.84393
	1500	25	4.68557	4.68558	5.31443	5.31442	7.86166
	1560	26	.68557	.68558	.31443	.31442	.87870
	1620	27	.68557	.68558	.31443	.31442	.89509
	1680	28	.68557	.68558	.31443	.31442	.91088
	1740	29	.68557	.68559	.31443	.31441	.92612
	1800	30	4.68557	4.68559	5.31443	5.31441	7.94084
	1860	31	.68557	.68559	.31443	.31441	.95508
	1920	32	.68557	.68559	.31443	.31441	.96887
	1980	33	.68557	.68559	.31443	.31441	.98223
	2040	34	.68557	.68559	.31443	.31441	.99520
	2100	35	4.68557	4.68559	5.31443	5.31441	8.00779
	2160	36	.68557	.68559	.31443	.31441	.02002
	2220	37	.68557	.68559	.31443	.31441	.03192
	2280	38	.68557	.68559	.31443	.31441	.04350
	2340	39	.68557	.68559	.31443	.31441	.05478
	2400	40	4.68557	4.68559	5.31443	5.31441	8.06578
	2460	41	.68556	.68560	.31444	.31440	.07650
	2520	42	.68556	.68560	.31444	.31440	.08696
	2580	43	.68556	.68560	.31444	.31440	.09718
	2640	44	.68556	.68560	.31444	.31440	.10717
	2700	45	4.68556	4.68560	5.31444	5.31440	8.11693
	2760	46	.68556	.68560	.31444	.31440	.12647
	2820	47	.68556	.68560	.31444	.31440	.13581
	2880	48	.68556	.68560	.31444	.31440	.14495
	2940	49	.68556	.68560	.31444	.31440	.15391
	3000	50	4.68556	4.68561	5.31444	5.31439	8.16268
	3060	51	.68556	.68561	.31444	.31439	.17128
	3120	52	.68556	.68561	.31444	.31439	.17971
	3180	53	.68556	.68561	.31444	.31439	.18798
	3240	54	.68556	.68561	.31444	.31439	.19610
	3300	55	4.68556	4.68561	5.31444	5.31439	8.20407
	3360	56	.68556	.68561	.31444	.31439	.21189
	3420	57	.68555	.68561	.31445	.31439	.21958
	3480	58	.68555	.68562	.31445	.31438	.22713
	3540	59	.68555	.68562	.31445	.31438	.23456
	3600	60	4.68555	4.68562	5.31445	5.31438	8.24186

	″	′	S	T	S′	T′	L. Sin.
	3600	0	4.68555	4.68562	5.31445	5.31438	8.24186
	3660	1	.68555	.68562	.31445	.31438	.24903
	3720	2	.68555	.68562	.31445	.31438	.25609
	3780	3	.68555	.68562	.31445	.31438	.26304
	3840	4	.68555	.68563	.31445	.31437	.26988
	3900	5	4.68555	4.68563	5.31445	5.31437	8.27661
	3960	6	.68555	.68563	.31445	.31437	.28324
	4020	7	.68555	.68563	.31445	.31437	.28977
	4080	8	.68555	.68563	.31445	.31437	.29621
	4140	9	.68555	.68563	.31445	.31437	.30255
	4200	10	4.68554	4.68563	5.31446	5.31437	8.30879
	4260	11	.68554	.68564	.31446	.31436	.31495
	4320	12	.68554	.68564	.31446	.31436	.32103
	4380	13	.68554	.68564	.31446	.31436	.32702
	4440	14	.68554	.68564	.31446	.31436	.33292
	4500	15	4.68554	4.68564	5.31446	5.31436	8.33875
	4560	16	.68554	.68565	.31446	.31435	.34450
	4620	17	.68554	.68565	.31446	.31435	.35018
	4680	18	.68554	.68565	.31446	.31435	.35578
	4740	19	.68554	.68565	.31446	.31435	.36131
	4800	20	4.68554	4.68565	5.31446	5.31435	8.36678
	4860	21	.68553	.68566	.31447	.31434	.37217
	4920	22	.68553	.68566	.31447	.31434	.37750
	4980	23	.68553	.68566	.31447	.31434	.38276
	5040	24	.68553	.68566	.31447	.31434	.38796
	5100	25	4.68553	4.68566	5.31447	5.31434	8.39310
	5160	26	.68553	.68567	.31447	.31433	.39818
	5220	27	.68553	.68567	.31447	.31433	.40320
	5280	28	.68553	.68567	.31447	.31433	.40816
1°	5340	29	.68553	.68567	.31447	.31433	.41307
	5400	30	4.68553	4.68567	5.31447	5.31433	8.41792
	5460	31	.68552	.68568	.31448	.31432	.42272
	5520	32	.68552	.68568	.31448	.31432	.42746
	5580	33	.68552	.68568	.31448	.31432	.43216
	5640	34	.68552	.68568	.31448	.31432	.43680
	5700	35	4.68552	4.68569	5.31448	5.31431	8.44139
	5760	36	.68552	.68569	.31448	.31431	.44594
	5820	37	.68552	.68569	.31448	.31431	.45044
	5880	38	.68552	.68569	.31448	.31431	.45489
	5940	39	.68551	.68569	.31449	.31431	.45930
	6000	40	4.68551	4.68570	5.31449	5.31430	8.46366
	6060	41	.68551	.68570	.31449	.31430	.46799
	6120	42	.68551	.68570	.31449	.31430	.47226
	6180	43	.68551	.68570	.31449	.31430	.47650
	6240	44	.68551	.68571	.31449	.31429	.48069
	6300	45	4.68551	4.68571	5.31449	5.31429	8.48485
	6360	46	.68551	.68571	.31449	.31429	.48896
	6420	47	.68550	.68572	.31450	.31428	.49304
	6480	48	.68550	.68572	.31450	.31428	.49708
	6540	49	.68550	.68572	.31450	.31428	.50108
	6600	50	4.68550	4.68572	5.31450	5.31428	8.50504
	6660	51	.68550	.68573	.31450	.31427	.50897
	6720	52	.68550	.68573	.31450	.31427	.51287
	6780	53	.68550	.68573	.31450	.31427	.51673
	6840	54	.68550	.68573	.31450	.31427	.52055
	6900	55	4.68549	4.68574	5.31451	5.31426	8.52434
	6960	56	.68549	.68574	.31451	.31426	.52810
	7020	57	.68549	.68574	.31451	.31426	.53183
	7080	58	.68549	.68575	.31451	.31425	.53552
	7140	59	.68549	.68575	.31451	.31425	.53919
	7200	60	4.68549	4.68575	5.31451	5.31425	8.54282

TABLE V

FOUR-PLACE TABLE

OF THE

NATURAL SINE, COSINE, TANGENT, AND COTANGENT

FOR

EVERY 10′ OF THE QUADRANT

° ′	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0 00	.0000	.0000	∞	1.0000	00 90
10	.0029	.0029	343.77	1.0000	50
20	.0058	.0058	171.89	1.0000	40
30	.0087	.0087	114.59	1.0000	30
40	.0116	.0116	85.940	.9999	20
50	.0145	.0145	68.750	.9999	10
1 00	.0175	.0175	57.290	.9998	00 89
10	.0204	.0204	49.104	.9998	50
20	.0233	.0233	42.964	.9997	40
30	.0262	.0262	38.188	.9997	30
40	.0291	.0291	34.368	.9996	20
50	.0320	.0320	31.242	.9995	10
2 00	.0349	.0349	28.636	.9994	00 88
10	.0378	.0378	26.432	.9993	50
20	.0407	.0407	24.542	.9992	40
30	.0436	.0437	22.904	.9990	30
40	.0465	.0466	21.470	.9989	20
50	.0494	.0495	20.206	.9988	10
3 00	.0523	.0524	19.081	.9986	00 87
10	.0552	.0553	18.075	.9985	50
20	.0581	.0582	17.169	.9983	40
30	.0610	.0612	16.350	.9981	30
40	.0640	.0641	15.605	.9980	20
50	.0669	.0670	14.924	.9978	10
4 00	.0698	.0699	14.301	.9976	00 86
10	.0727	.0729	13.727	.9974	50
20	.0756	.0758	13.197	.9971	40
30	.0785	.0787	12.706	.9969	30
40	.0814	.0816	12.251	.9967	20
50	.0843	0846	11.826	.9964	10
5 00	.0872	.0875	11.430	.9962	00 85
10	.0901	.0904	11.059	.9959	50
20	.0929	.0934	10.712	.9957	40
30	.0958	.0963	10.385	.9954	30
40	.0987	.0992	10.078	.9951	20
50	.1016	.1022	9.7882	.9948	10
6 00	.1045	.1051	9.5144	.9945	00 84
10	.1074	.1080	9.2553	.9942	50
20	.1103	.1110	9.0098	.9939	40
30	.1132	.1139	8.7769	.9936	30
40	.1161	.1169	8.5555	.9932	20
50	.1190	.1198	8.3450	.9929	10
7 00	.1219	.1228	8.1443	.9925	00 83
10	.1248	.1257	7.9530	.9922	50
20	.1276	.1287	7.7704	.9918	40
30	.1305	.1317	7.5958	.9914	30
40	.1334	.1346	7.4287	.9911	20
50	.1363	.1376	7.2687	.9907	10
8 00	.1392	.1405	7.1154	.9903	00 82
10	.1421	.1435	6.9682	.9899	50
20	.1449	.1465	6.8269	.9894	40
30	.1478	.1495	6.6912	.9890	30
40	.1507	.1524	6.5606	.9886	20
50	.1536	.1554	6.4348	.9881	10
9 00	.1564	.1584	6.3138	.9877	00 81
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	′ °

° ′	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
9 00	.1564	.1584	6.3138	.9877	00 81
10	.1593	.1614	6.1970	.9872	50
20	.1622	.1644	6.0844	.9868	40
30	.1650	.1673	5.9758	.9863	30
40	.1679	.1703	5.8708	.9858	20
50	.1708	.1733	5.7694	.9853	10
10 00	.1736	.1763	5.6713	.9848	00 80
10	.1765	.1793	5.5764	.9843	50
20	.1794	.1823	5.4845	.9838	40
30	.1822	.1853	5.3955	.9833	30
40	.1851	.1883	5.3093	.9827	20
50	.1880	.1914	5.2257	.9822	10
11 00	.1908	.1944	5.1446	.9816	00 79
10	.1937	.1974	5.0658	.9811	50
20	.1965	.2004	4.9894	.9805	40
30	.1994	.2035	4.9152	.9799	30
40	.2022	.2065	4.8430	.9793	20
50	.2051	.2095	4.7729	.9787	10
12 00	.2079	.2126	4.7046	.9781	00 78
10	.2108	.2156	4.6382	.9775	50
20	.2136	.2186	4.5736	.9769	40
30	.2164	.2217	4.5107	.9763	30
40	.2193	.2247	4.4494	.9757	20
50	.2221	.2278	4.3897	.9750	10
13 00	.2250	.2309	4.3315	.9744	00 77
10	.2278	.2339	4.2747	.9737	50
20	.2306	.2370	4.2193	.9730	40
30	.2334	.2401	4.1653	.9724	30
40	.2363	.2432	4.1126	.9717	20
50	.2391	.2462	4.0611	.9710	10
14 00	.2419	.2493	4.0108	.9703	00 76
10	.2447	.2524	3.9617	.9696	50
20	.2476	.2555	3.9136	.9689	40
30	.2504	.2586	3.8667	.9681	30
40	.2532	.2617	3.8208	.9674	20
50	.2560	.2648	3.7760	.9667	10
15 00	.2588	.2679	3.7321	.9659	00 75
10	.2616	.2711	3.6891	.9652	50
20	.2644	.2742	3.6470	.9644	40
30	.2672	.2773	3.6059	.9636	30
40	.2700	.2805	3.5656	.9628	20
50	.2728	.2836	3.5261	.9621	10
16 00	.2756	.2867	3.4874	.9613	00 74
10	.2784	.2899	3.4495	.9605	50
20	.2812	.2931	3.4124	.9596	40
30	.2840	.2962	3.3759	.9588	30
40	.2868	.2994	3 3402	.9580	20
50	.2896	.3026	3.3052	.9572	10
17 00	.2924	.3057	3.2709	.9563	00 73
10	.2952	.3089	3.2371	.9555	50
20	.2979	.3121	3.2041	.9546	40
30	.3007	.3153	3.1716	.9537	30
40	.3035	.3185	3.1397	.9528	20
50	.3062	.3217	3.1084	.9520	10
18 00	.3090	.3249	3.0777	.9511	00 72
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	′ °

°	′	N. Sin.	N. Tan.	N. Cot.	N. Cos.		
18	00	.3090	.3249	3.0777	.9511	00	72
	10	.3118	.3281	3.0475	.9502	50	
	20	.3145	.3314	3.0178	.9492	40	
	30	.3173	.3346	2.9887	.9483	30	
	40	.3201	.3378	2.9600	.9474	20	
	50	.3228	.3411	2.9319	.9465	10	
19	00	.3256	.3443	2.9042	.9455	00	71
	10	.3283	.3476	2.8770	.9446	50	
	20	.3311	.3508	2.8502	.9436	40	
	30	.3338	.3541	2.8239	.9426	30	
	40	.3365	.3574	2.7980	.9417	20	
	50	.3393	.3607	2.7725	.9407	10	
20	00	.3420	.3640	2.7475	.9397	00	70
	10	.3448	.3673	2.7228	.9387	50	
	20	.3475	.3706	2.6985	.9377	40	
	30	.3502	.3739	2.6746	.9367	30	
	40	.3529	.3772	2.6511	.9356	20	
	50	.3557	.3805	2.6279	.9346	10	
21	00	.3584	.3839	2.6051	.9336	00	69
	10	.3611	.3872	2.5826	.9325	50	
	20	.3638	.3906	2.5605	.9315	40	
	30	.3665	.3939	2.5386	.9304	30	
	40	.3692	.3973	2.5172	.9293	20	
	50	.3719	.4006	2.4960	.9283	10	
22	00	.3746	.4040	2.4751	.9272	00	68
	10	.3773	.4074	2.4545	.9261	50	
	20	.3800	.4108	2.4342	.9250	40	
	30	.3827	.4142	2.4142	.9239	30	
	40	.3854	.4176	2.3945	.9228	20	
	50	.3881	.4210	2.3750	.9216	10	
23	00	.3907	.4245	2.3559	.9205	00	67
	10	.3934	.4279	2.3369	.9194	50	
	20	.3961	.4314	2.3183	.9182	40	
	30	.3987	.4348	2.2998	.9171	30	
	40	.4014	.4383	2.2817	.9159	20	
	50	.4041	.4417	2.2637	.9147	10	
24	00	.4067	.4452	2.2460	.9135	00	66
	10	.4094	.4487	2.2286	.9124	50	
	20	.4120	.4522	2.2113	.9112	40	
	30	.4147	.4557	2.1943	.9100	30	
	40	.4173	.4592	2.1775	.9088	20	
	50	.4200	.4628	2.1609	.9075	10	
25	00	.4226	.4663	2.1445	.9063	00	65
	10	.4253	.4699	2.1283	.9051	50	
	20	.4279	.4734	2.1123	.9038	40	
	30	.4305	4770	2.0965	.9026	30	
	40	.4331	.4806	2.0809	.9013	20	
	50	.4358	.4841	2.0655	.9001	10	
26	00	.4384	.4877	2.0503	.8988	00	64
	10	.4410	.4913	2.0353	.8975	50	
	20	.4436	.4950	2.0204	.8962	40	
	30	.4462	.4986	2.0057	.8949	30	
	40	.4488	.5022	1.9912	.8936	20	
	50	.4514	.5059	1.9768	.8923	10	
27	00	.4540	.5095	1.9626	.8910	00	63
		N. Cos.	N. Cot.	N. Tan.	N. Sin.	′	°

°	′	N. Sin.	N. Tan.	N. Cot.	N. Cos.	′	°
27	00	.4540	.5095	1.9626	.8910	00	63
	10	.4566	.5132	1.9486	.8897	50	
	20	.4592	.5169	1.9347	.8884	40	
	30	.4617	.5206	1.9210	.8870	30	
	40	.4643	.5243	1.9074	.8857	20	
	50	.4669	.5280	1.8940	.8843	10	
28	00	.4695	.5317	1.8807	.8829	00	62
	10	.4720	.5354	1.8676	.8816	50	
	20	.4746	.5392	1.8546	.8802	40	
	30	.4772	.5430	1.8418	.8788	30	
	40	.4797	.5467	1.8291	.8774	20	
	50	.4823	.5505	1.8165	.8760	10	
29	00	.4848	.5543	1.8040	.8746	00	61
	10	.4874	.5581	1.7917	.8732	50	
	20	.4899	.5619	1.7796	.8718	40	
	30	.4924	.5658	1.7675	.8704	30	
	40	.4950	.5696	1.7556	.8689	20	
	50	.4975	.5735	1.7437	.8675	10	
30	00	.5000	.5774	1.7321	.8660	00	60
	10	.5025	.5812	1.7205	.8646	50	
	20	.5050	.5851	1.7090	.8631	40	
	30	.5075	.5890	1.6977	.8616	30	
	40	.5100	.5930	1.6864	.8601	20	
	50	.5125	.5969	1.6753	.8587	10	
31	00	.5150	.6009	1.6643	.8572	00	59
	10	.5175	.6048	1.6534	.8557	50	
	20	.5200	.6088	1.6426	.8542	40	
	30	.5225	.6128	1.6319	.8526	30	
	40	.5250	.6168	1.6212	.8511	20	
	50	.5275	.6208	1.6107	.8496	10	
32	00	.5299	.6249	1.6003	.8480	00	58
	10	.5324	.6289	1.5900	.8465	50	
	20	.5348	.6330	1.5798	.8450	40	
	30	.5373	.6371	1.5697	.8434	30	
	40	.5398	.6412	1.5597	.8418	20	
	50	.5422	.6453	1.5497	.8403	10	
33	00	.5446	.6494	1.5399	.8387	00	57
	10	.5471	.6536	1.5301	.8371	50	
	20	.5495	.6577	1.5204	.8355	40	
	30	.5519	.6619	1.5108	.8339	30	
	40	.5544	.6661	1.5013	.8323	20	
	50	.5568	.6703	1.4919	.8307	10	
34	00	.5592	.6745	1.4826	.8290	00	56
	10	.5616	.6787	1.4733	.8274	50	
	20	.5640	.6830	1.4641	.8258	40	
	30	.5664	.6873	1.4550	.8241	30	
	40	.5688	.6916	1.4460	.8225	20	
	50	.5712	.6959	1.4370	.8208	10	
35	00	.5736	.7002	1.4281	.8192	00	55
	10	.5760	.7046	1.4193	.8175	50	
	20	.5783	.7089	1.4106	.8158	40	
	30	.5807	.7133	1.4019	.8141	30	
	40	.5831	.7177	1.3934	.8124	20	
	50	.5854	.7221	1.3848	.8107	10	
36	00	.5878	.7265	1.3764	.8090	00	54
		N. Cos.	N. Cot.	N. Tan.	N. Sin.	′	°

°	′	N. Sin.	N. Tan.	N. Cot.	N. Cot.	′	°
36	00	.5878	.7265	1.3764	.8090	00	54
	10	.5901	.7310	1.3680	.8073	50	
	20	.5925	.7355	1.3597	.8056	40	
	30	.5948	.7400	1.3514	.8039	30	
	40	.5972	.7445	1.3432	.8021	20	
	50	.5995	.7490	1.3351	.8004	10	
37	00	.6018	.7536	1.3270	.7986	00	53
	10	.6041	.7581	1.3190	.7969	50	
	20	.6065	.7627	1.3111	.7951	40	
	30	.6088	.7673	1.3032	.7934	30	
	40	.6111	.7720	1.2954	.7916	20	
	50	.6134	.7766	1.2876	.7898	10	
38	00	.6157	.7813	1.2799	.7880	00	52
	10	.6180	.7860	1.2723	.7862	50	
	20	.6202	.7907	1.2647	.7844	40	
	30	.6225	.7954	1.2572	.7826	30	
	40	.6248	.8002	1.2497	.7808	20	
	50	.6271	.8050	1.2423	.7790	10	
39	00	.6293	.8098	1.2349	.7771	00	51
	10	.6316	.8146	1.2276	.7753	50	
	20	.6338	.8195	1.2203	.7735	40	
	30	.6361	.8243	1.2131	.7716	30	
	40	.6383	.8292	1.2059	.7698	20	
	50	.6406	.8342	1.1988	.7679	10	
40	00	.6428	.8391	1.1918	.7660	00	50
	10	.6450	.8441	1.1847	.7642	50	
	20	.6472	.8491	1.1778	.7623	40	
	30	.6494	.8541	1.1708	.7604	30	
	40	.6517	.8591	1.1640	.7585	20	
	50	.6539	.8642	1.1571	.7566	10	
41	00	.6561	.8693	1.1504	.7547	00	49
	10	.6583	.8744	1.1436	.7528	50	
	20	.6604	.8796	1.1369	.7509	40	
	30	.6626	.8847	1.1303	.7490	30	
	40	.6648	.8899	1.1237	.7470	20	
	50	.6670	.8952	1.1171	.7451	10	
42	00	.6691	.9004	1.1106	.7431	00	48
	10	.6713	.9057	1.1041	.7412	50	
	20	.6734	.9110	1.0977	.7392	40	
	30	.6756	.9163	1.0913	.7373	30	
	40	.6777	.9217	1.0850	.7353	20	
	50	.6799	.9271	1.0786	.7333	10	
43	00	.6820	.9325	1.0724	.7314	00	47
	10	.6841	.9380	1.0661	.7294	50	
	20	.6862	.9435	1.0599	.7274	40	
	30	.6884	.9490	1.0538	.7254	30	
	40	.6905	.9545	1.0477	.7234	20	
	50	.6926	.9601	1.0416	.7214	10	
44	00	.6947	.9657	1.0355	.7193	00	46
	10	.6967	.9713	1.0295	.7173	50	
	20	.6988	.9770	1.0235	.7153	40	
	30	.7009	.9827	1.0176	.7133	30	
	40	.7030	.9884	1.0117	.7112	20	
	50	.7050	.9942	1.0058	.7092	10	
45	00	.7071	1.0000	1.0000	.7071	00	45
		N. Cos.	N. Cot.	N. Tan.	N. Sin.	′	°

TABLE VI

FOUR-PLACE LOGARITHMS

OF

NUMBERS 1–2000

N.	0	1	2	3	4	5	6	7	8	9
0	0000	0000	3010	4771	6021	6990	7782	8451	9031	9542
1	0000	0414	0792	1139	1461	1761	2041	2304	2553	2788
2	3010	3222	3424	3617	3802	3979	4150	4314	4472	4624
3	4771	4914	5051	5185	5315	5441	5563	5682	5798	5911
4	6021	6128	6232	6335	6435	6532	6628	6721	6812	6902
5	6990	7076	7160	7243	7324	7404	7482	7559	7634	7709
6	7782	7853	7924	7993	8062	8129	8195	8261	8325	8388
7	8451	8513	8573	8633	8692	8751	8808	8865	8921	8976
8	9031	9085	9138	9191	9243	9294	9345	9395	9445	9494
9	9542	9590	9638	9685	9731	9777	9823	9868	9912	9956
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5900
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N.	0	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039
N.	0	1	2	3	4	5	6	7	8	9

N.	0	1	2	3	4	5	6	7	8	9
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039
101	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082
102	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124
103	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166
104	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208
105	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249
106	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290
107	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330
108	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370
109	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410
110	0414	0418	0422	0426	0430	0434	0438	0441	0445	0449
111	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488
112	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527
113	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565
114	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603
115	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641
116	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678
117	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715
118	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752
119	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788
120	0792	0795	0799	0803	0806	0810	0813	0817	0821	0824
121	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860
122	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896
123	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931
124	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966
125	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000
126	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035
127	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069
128	1072	1075	1079	1082	1086	1089	1092	1096	1099	1103
129	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136
130	1139	1143	1146	1149	1153	1156	1159	1163	1166	1169
131	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202
132	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235
133	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268
134	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300
135	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332
136	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364
137	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396
138	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427
139	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458
140	1461	1464	1467	1471	1474	1477	1480	1483	1486	1489
141	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520
142	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550
143	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581
144	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611
145	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641
146	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670
147	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700
148	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729
149	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758
150	1761	1764	1767	1770	1772	1775	1778	1781	1784	1787
N.	0	1	2	3	4	5	6	7	8	9

N.	0	1	2	3	4	5	6	7	8	9
150	1761	1764	1767	1770	1772	1775	1778	1781	1784	1787
151	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816
152	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844
153	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872
154	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901
155	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928
156	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956
157	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984
158	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011
159	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038
160	2041	2044	2047	2049	2052	2055	2057	2060	2063	2066
161	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092
162	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119
163	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146
164	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172
165	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198
166	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225
167	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251
168	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276
169	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302
170	2304	2307	2310	2312	2315	2317	2320	2322	2325	2327
171	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353
172	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378
173	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403
174	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428
175	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453
176	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477
177	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502
178	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526
179	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550
180	2553	2555	2558	2560	2562	2565	2567	2570	2572	2574
181	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598
182	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622
183	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646
184	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669
185	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693
186	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716
187	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739
188	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762
189	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785
190	2788	2790	2792	2794	2797	2799	2801	2804	2806	2808
191	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831
192	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853
193	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876
194	2878	2880	2883	2885	2887	2889	2891	2894	2896	2898
195	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920
196	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942
197	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964
198	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986
199	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008
200	3010	3012	3015	3017	3019	3021	3023	3025	3028	3030
N.	0	1	2	3	4	5	6	7	8	9

TABLE VII

FOUR-PLACE LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS

FOR THE

DECIMALLY DIVIDED DEGREE

L. Sin.	0	1	2	3	4	5	6	7	8	9		
0°.0	−∞	6.2419	5429	7190	8439	9408	*0200	*0870	*1450	*1961	*2419	89.9
0.1	7.2419	2833	3211	3558	3880	4180	4460	4723	4971	5206	5429	89.8
0.2	7.5429	5641	5843	6036	6221	6398	6568	6732	6890	7043	7190	89.7
0.3	7.7190	7332	7470	7604	7734	7859	7982	8101	8217	8329	8439	89.6
0.4	7.8439	8547	8651	8753	8853	8951	9046	9140	9231	9321	9408	89.5
0.5	7.9408	9494	9579	9661	9743	9822	9901	9977	*0053	*0127	*0200	89.4
0.6	8.0200	0272	0343	0412	0480	0548	0614	0679	0744	0807	0870	89.3
0.7	8.0870	0931	0992	1052	1111	1169	1227	1284	1340	1395	1450	89.2
0.8	8.1450	1503	1557	1609	1661	1713	1764	1814	1863	1912	1961	89.1
0.9	8.1961	2009	2056	2103	2150	2196	2241	2286	2331	2375	2419	89°.0
1°.0	8.2419	2462	2505	2547	2589	2630	2672	2712	2753	2793	2832	88.9
1.1	8.2832	2872	2911	2949	2988	3025	3063	3100	3137	3174	3210	88.8
1.2	8.3210	3246	3282	3317	3353	3388	3422	3456	3491	3524	3558	88.7
1.3	8.3558	3591	3624	3657	3689	3722	3754	3786	3817	3848	3880	88.6
1.4	8.3880	3911	3941	3972	4002	4032	4062	4091	4121	4150	4179	88.5
1.5	8.4179	4208	4237	4265	4293	4322	4349	4377	4405	4432	4459	88.4
1.6	8.4459	4486	4513	4540	4567	4593	4619	4645	4671	4697	4723	88.3
1.7	8.4723	4748	4773	4799	4824	4848	4873	4898	4922	4947	4971	88.2
1.8	8.4971	4995	5019	5043	5066	5090	5113	5136	5160	5183	5206	88.1
1.9	8.5206	5228	5251	5274	5296	5318	5340	5363	5385	5406	5428	88°.0
2°.0	8.5428	5450	5471	5493	5514	5535	5557	5578	5598	5619	5640	87.9
2.1	8.5640	5661	5681	5702	5722	5742	5762	5782	5802	5822	5842	87.8
2.2	8.5842	5862	5881	5901	5920	5939	5959	5978	5997	6016	6035	87.7
2.3	8.6035	6054	6072	6091	6110	6128	6147	6165	6183	6201	6220	87.6
2.4	8.6220	6238	6256	6274	6291	6309	6327	6344	6362	6379	6397	87.5
2.5	8.6397	6414	6431	6449	6466	6483	6500	6517	6534	6550	6567	87.4
2.6	8.6567	6584	6600	6617	6633	6650	6666	6682	6699	6715	6731	87.3
2.7	8.6731	6747	6763	6779	6795	6810	6826	6842	6858	6873	6889	87.2
2.8	8.6889	6904	6920	6935	6950	6965	6981	6996	7011	7026	7041	87.1
2.9	8.7041	7056	7071	7086	7100	7115	7130	7144	7159	7174	7188	87°.0
3°.0	8.7188	7202	7217	7231	7245	7260	7274	7288	7302	7316	7330	86.9
3.1	8.7330	7344	7358	7372	7386	7400	7413	7427	7441	7454	7468	86.8
3.2	8.7468	7482	7495	7508	7522	7535	7549	7562	7575	7588	7602	86.7
3.3	8.7602	7615	7628	7641	7654	7667	7680	7693	7705	7718	7731	86.6
3.4	8.7731	7744	7756	7769	7782	7794	7807	7819	7832	7844	7857	86.5
3.5	8.7857	7869	7881	7894	7906	7918	7930	7943	7955	7967	7979	86.4
3.6	8.7979	7991	8003	8015	8027	8039	8051	8062	8074	8086	8098	86.3
3.7	8.8098	8109	8121	8133	8144	8156	8168	8179	8191	8202	8213	86.2
3.8	8.8213	8225	8236	8248	8259	8270	8281	8293	8304	8315	8326	86.1
3.9	8.8326	8337	8348	8359	8370	8381	8392	8403	8414	8425	8436	86°.0
4°.0	8.8436	8447	8457	8468	8479	8490	8500	8511	8522	8532	8543	85.9
4.1	8.8543	8553	8564	8575	8585	8595	8606	8616	8627	8637	8647	85.8
4.2	8.8647	8658	8668	8678	8688	8699	8709	8719	8729	8739	8749	85.7
4.3	8.8749	8759	8769	8780	8790	8799	8809	8819	8829	8839	8849	85.6
4.4	8.8849	8859	8869	8878	8888	8898	8908	8917	8927	8937	8946	85.5
4.5	8.8946	8956	8966	8975	8985	8994	9004	9013	9023	9032	9042	85.4
4.6	8.9042	9051	9060	9070	9079	9089	9098	9107	9116	9126	9135	85.3
4.7	8.9135	9144	9153	9162	9172	9181	9190	9199	9208	9217	9226	85.2
4.8	8.9226	9235	9244	9253	9262	9271	9280	9289	9298	9307	9315	85.1
4.9	8.9315	9324	9333	9342	9351	9359	9368	9377	9386	9394	9403	85°.0
		9	8	7	6	5	4	3	2	1	0	L. Cos.

L. Sin.	0	1	2	3	4	5	6	7	8	9		
5°.0	8.9403	9412	9420	9429	9437	9446	9455	9463	9472	9480	9489	84.9
5.1	8.9489	9497	9506	9514	9523	9531	9539	9548	9556	9565	9573	84.8
5.2	8.9573	9581	9589	9598	9606	9614	9623	9631	9639	9647	9655	84.7
5.3	8.9655	9664	9672	9680	9688	9696	9704	9712	9720	9728	9736	84.6
5.4	8.9736	9744	9752	9760	9768	9776	9784	9792	9800	9808	9816	84.5
5.5	8.9816	9824	9831	9839	9847	9855	9863	9870	9878	9886	9894	84.4
5.6	8.9894	9901	9909	9917	9925	9932	9940	9948	9955	9963	9970	84.3
5.7	8.9970	9978	9986	9993	*0001	*0008	*0016	*0023	*0031	*0038	*0046	84.2
5.8	9.0046	0053	0061	0068	0075	0083	0090	0098	0105	0112	0120	84.1
5.9	9.0120	0127	0134	0142	0149	0156	0163	0171	0178	0185	0192	84°.0
6°.0	9.0192	0200	0207	0214	0221	0228	0235	0243	0250	0257	0264	83.9
6.1	9.0264	0271	0278	0285	0292	0299	0306	0313	0320	0327	0334	83.8
6.2	9.0334	0341	0348	0355	0362	0369	0376	0383	0390	0397	0403	83.7
6.3	9.0403	0410	0417	0424	0431	0438	0444	0451	0458	0465	0472	83.6
6.4	9.0472	0478	0485	0492	0498	0505	0512	0519	0525	0532	0539	83.5
6.5	9.0539	0545	0552	0558	0565	0572	0578	0585	0591	0598	0605	83.4
6.6	9.0605	0611	0618	0624	0631	0637	0644	0650	0657	0663	0670	83.3
6.7	9.0670	0676	0683	0689	0695	0702	0708	0715	0721	0727	0734	83.2
6.8	9.0734	0740	0746	0753	0759	0765	0772	0778	0784	0790	0797	83.1
6.9	9.0797	0803	0809	0816	0822	0828	0834	0840	0847	0853	0859	83°.0
7°.0	9.0859	0865	0871	0877	0884	0890	0896	0902	0908	0914	0920	82.9
7.1	9.0920	0926	0932	0938	0945	0951	0957	0963	0969	0975	0981	82.8
7.2	9.0981	0987	0993	0999	1005	1011	1017	1022	1028	1034	1040	82.7
7.3	9.1040	1046	1052	1058	1064	1070	1076	1081	1087	1093	1099	82.6
7.4	9.1099	1105	1111	1116	1122	1128	1134	1140	1145	1151	1157	82.5
7.5	9.1157	1163	1168	1174	1180	1186	1191	1197	1203	1208	1214	82.4
7.6	9.1214	1220	1226	1231	1237	1242	1248	1254	1259	1265	1271	82.3
7.7	9.1271	1276	1282	1287	1293	1299	1304	1310	1315	1321	1326	82.2
7.8	9.1326	1332	1337	1343	1348	1354	1359	1365	1370	1376	1381	82.1
7.9	9.1381	1387	1392	1398	1403	1409	1414	1419	1425	1430	1436	82°.0
8°.0	9.1436	1441	1446	1452	1457	1462	1468	1473	1478	1484	1489	81.9
8.1	9.1489	1494	1500	1505	1510	1516	1521	1526	1532	1537	1542	81.8
8.2	9.1542	1547	1553	1558	1563	1568	1574	1579	1584	1589	1594	81.7
8.3	9.1594	1600	1605	1610	1615	1620	1625	1631	1636	1641	1646	81.6
8.4	9.1646	1651	1656	1661	1666	1672	1677	1682	1687	1692	1697	81.5
8.5	9.1697	1702	1707	1712	1717	1722	1727	1732	1737	1742	1747	81.4
8.6	9.1747	1752	1757	1762	1767	1772	1777	1782	1787	1792	1797	81.3
8.7	9.1797	1802	1807	1812	1817	1822	1827	1832	1837	1842	1847	81.2
8.8	9.1847	1851	1856	1861	1866	1871	1876	1881	1886	1890	1895	81.1
8.9	9.1895	1900	1905	1910	1915	1919	1924	1929	1934	1939	1943	81°.0
9°.0	9.1943	1948	1953	1958	1962	1967	1972	1977	1981	1986	1991	80.9
9.1	9.1991	1996	2000	2005	2010	2015	2019	2024	2029	2033	2038	80.8
9.2	9.2038	2043	2047	2052	2057	2061	2066	2071	2075	2080	2085	80.7
9.3	9.2085	2089	2094	2098	2103	2108	2112	2117	2121	2126	2131	80.6
9.4	9.2131	2135	2140	2144	2149	2153	2158	2162	2167	2172	2176	80.5
9.5	9.2176	2181	2185	2190	2194	2199	2203	2208	2212	2217	2221	80.4
9.6	9.2221	2226	2230	2235	2239	2243	2248	2252	2257	2261	2266	80.3
9.7	9.2266	2270	2275	2279	2283	2288	2292	2297	2301	2305	2310	80.2
9.8	9.2310	2314	2319	2323	2327	2332	2336	2340	2345	2349	2353	80.1
9.9	9.2353	2358	2362	2367	2371	2375	2379	2384	2388	2392	2397	80°.0
		9	8	7	6	5	4	3	2	1	0	L. Cos.

L. Sin.	0	1	2	3	4	5	6	7	8	9		
											−∞	90°
0°	−∞	7.2419	5429	7190	8439	9408	*0200	*0870	*1450	*1961	*2419	89
1	8.2419	2832	3210	3558	3880	4179	4459	4723	4971	5206	5428	88
2	8.5428	5640	5842	6035	6220	6397	6567	6731	6889	7041	7188	87
3	8.7188	7330	7468	7602	7731	7857	7979	8098	8213	8326	8436	86
4	8.8436	8543	8647	8749	8849	8946	9042	9135	9226	9315	9403	85
5	8.9403	9489	9573	9655	9736	9816	9894	9970	*0046	*0120	*0192	84
6	9.0192	0264	0334	0403	0472	0539	0605	0670	0734	0797	0859	83
7	9.0859	0920	0981	1040	1099	1157	1214	1271	1326	1381	1436	82
8	9.1436	1489	1542	1594	1646	1697	1747	1797	1847	1895	1943	81
9	9.1943	1991	2038	2085	2131	2176	2221	2266	2310	2353	2397	80°
10°	9.2397	2439	2482	2524	2565	2606	2647	2687	2727	2767	2806	79
11	9.2806	2845	2883	2921	2959	2997	3034	3070	3107	3143	3179	78
12	9.3179	3214	3250	3284	3319	3353	3387	3421	3455	3488	3521	77
13	9.3521	3554	3586	3618	3650	3682	3713	3745	3775	3806	3837	76
14	9.3837	3867	3897	3927	3957	3986	4015	4044	4073	4102	4130	75
15	9.4130	4158	4186	4214	4242	4269	4296	4323	4350	4377	4403	74
16	9.4403	4430	4456	4482	4508	4533	4559	4584	4609	4634	4659	73
17	9.4659	4684	4709	4733	4757	4781	4805	4829	4853	4876	4900	72
18	9.4900	4923	4946	4969	4992	5015	5037	5060	5082	5104	5126	71
19	9.5126	5148	5170	5192	5213	5235	5256	5278	5299	5320	5341	70°
20°	9.5341	5361	5382	5402	5423	5443	5463	5484	5504	5523	5543	69
21	9.5543	5563	5583	5602	5621	5641	5660	5679	5698	5717	5736	68
22	9.5736	5754	5773	5792	5810	5828	5847	5865	5883	5901	5919	67
23	9.5919	5937	5954	5972	5990	6007	6024	6042	6059	6076	6093	66
24	9.6093	6110	6127	6144	6161	6177	6194	6210	6227	6243	6259	65
25	9.6259	6276	6292	6308	6324	6340	6356	6371	6387	6403	6418	64
26	9.6418	6434	6449	6465	6480	6495	6510	6526	6541	6556	6570	63
27	9.6570	6585	6600	6615	6629	6644	6659	6673	6687	6702	6716	62
28	9.6716	6730	6744	6759	6773	6787	6801	6814	6828	6842	6856	61
29	9.6856	6869	6883	6896	6910	6923	6937	6950	6963	6977	6990	60°
30°	9.6990	7003	7016	7029	7042	7055	7068	7080	7093	7106	7118	59
31	9.7118	7131	7144	7156	7168	7181	7193	7205	7218	7230	7242	58
32	9.7242	7254	7266	7278	7290	7302	7314	7326	7338	7349	7361	57
33	9.7361	7373	7384	7396	7407	7419	7430	7442	7453	7464	7476	56
34	9.7476	7487	7498	7509	7520	7531	7542	7553	7564	7575	7586	55
35	9.7586	7597	7607	7618	7629	7640	7650	7661	7671	7682	7692	54
36	9.7692	7703	7713	7723	7734	7744	7754	7764	7774	7785	7795	53
37	9.7795	7805	7815	7825	7835	7844	7854	7864	7874	7884	7893	52
38	9.7893	7903	7913	7922	7932	7941	7951	7960	7970	7979	7989	51
39	9.7989	7998	8007	8017	8026	8035	8044	8053	8063	8072	8081	50°
40°	9.8081	8090	8099	8108	8117	8125	8134	8143	8152	8161	8169	49
41	9.8169	8178	8187	8195	8204	8213	8221	8230	8238	8247	8255	48
42	9.8255	8264	8272	8280	8289	8297	8305	8313	8322	8330	8338	47
43	9.8338	8346	8354	8362	8370	8378	8386	8394	8402	8410	8418	46
44	9.8418	8426	8433	8441	8449	8457	8464	8472	8480	8487	8495	45°
45°	9.8495											
		9	8	7	6	5	4	3	2	1	0	L. Cos.

L. Sin.	0	1	2	3	4	5	6	7	8	9		
											9.8495	45°
45°	9.8495	8502	8510	8517	8525	8532	8540	8547	8555	8562	8569	44
46	9.8569	8577	8584	8591	8598	8606	8613	8620	8627	8634	8641	43
47	9.8641	8648	8655	8662	8669	8676	8683	8690	8697	8704	8711	42
48	9.8711	8718	8724	8731	8738	8745	8751	8758	8765	8771	8778	41
49	9.8778	8784	8791	8797	8804	8810	8817	8823	8830	8836	8843	40°
50°	9.8843	8849	8855	8862	8868	8874	8880	8887	8893	8899	8905	39
51	9.8905	8911	8917	8923	8929	8935	8941	8947	8953	8959	8965	38
52	9.8965	8971	8977	8983	8989	8995	9000	9006	9012	9018	9023	37
53	9.9023	9029	9035	9041	9046	9052	9057	9063	9069	9074	9080	36
54	9.9080	9085	9091	9096	9101	9107	9112	9118	9123	9128	9134	35
55	9.9134	9139	9144	9149	9155	9160	9165	9170	9175	9181	9186	34
56	9.9186	9191	9196	9201	9206	9211	9216	9221	9226	9231	9236	33
57	9.9236	9241	9246	9251	9255	9260	9265	9270	9275	9279	9284	32
58	9.9284	9289	9294	9298	9303	9308	9312	9317	9322	9326	9331	31
59	9.9331	9335	9340	9344	9349	9353	9358	9362	9367	9371	9375	30°
60°	9.9375	9380	9384	9388	9393	9397	9401	9406	9410	9414	9418	29
61	9.9418	9422	9427	9431	9435	9439	9443	9447	9451	9455	9459	28
62	9.9459	9463	9467	9471	9475	9479	9483	9487	9491	9495	9499	27
63	9.9499	9503	9506	9510	9514	9518	9522	9525	9529	9533	9537	26
64	9.9537	9540	9544	9548	9551	9555	9558	9562	9566	9569	9573	25
65	9.9573	9576	9580	9583	9587	9590	9594	9597	9601	9604	9607	24
66	9.9607	9611	9614	9617	9621	9624	9627	9631	9634	9637	9640	23
67	9.9640	9643	9647	9650	9653	9656	9659	9662	9666	9669	9672	22
68	9.9672	9675	9678	9681	9684	9687	9690	9693	9696	9699	9702	21
69	9.9702	9704	9707	9710	9713	9716	9719	9722	9724	9727	9730	20°
70°	9.9730	9733	9735	9738	9741	9743	9746	9749	9751	9754	9757	19
71	9.9757	9759	9762	9764	9767	9770	9772	9775	9777	9780	9782	18
72	9.9782	9785	9787	9789	9792	9794	9797	9799	9801	9804	9806	17
73	9.9806	9808	9811	9813	9815	9817	9820	9822	9824	9826	9828	16
74	9.9828	9831	9833	9835	9837	9839	9841	9843	9845	9847	9849	15
75	9.9849	9851	9853	9855	9857	9859	9861	9863	9865	9867	9869	14
76	9.9869	9871	9873	9875	9876	9878	9880	9882	9884	9885	9887	13
77	9.9887	9889	9891	9892	9894	9896	9897	9899	9901	9902	9904	12
78	9.9904	9906	9907	9909	9910	9912	9913	9915	9916	9918	9919	11
79	9.9919	9921	9922	9924	9925	9927	9928	9929	9931	9932	9934	10°
80°	9.9934	9935	9936	9937	9939	9940	9941	9943	9944	9945	9946	9
81	9.9946	9947	9949	9950	9951	9952	9953	9954	9955	9956	9958	8
82	9.9958	9959	9960	9961	9962	9963	9964	9965	9966	9967	9968	7
83	9.9968	9968	9969	9970	9971	9972	9973	9974	9975	9975	9976	6
84	9.9976	9977	9978	9978	9979	9980	9981	9981	9982	9983	9983	5
85	9.9983	9984	9985	9985	9986	9987	9987	9988	9988	9989	9989	4
86	9.9989	9990	9990	9991	9991	9992	9992	9993	9993	9994	9994	3
87	9.9994	9994	9995	9995	9996	9996	9996	9996	9997	9997	9997	2
88	9.9997	9998	9998	9998	9998	9999	9999	9999	9999	9999	9999	1
89	9.9999	9999	*0000	*0000	*0000	*0000	*0000	*0000	*0000	0000	*0000	0°
90°	0.0000											
		9	8	7	6	5	4	3	2	1	0	L. Cos.

L. Tang.	0	1	2	3	4	5	6	7	8	9		
0°.0	— ∞	6.2419	5429	7190	8439	9408	*0200	*0870	*1450	*1961	*2419	89.9
0.1	7.2419	2833	3211	3558	3880	4180	4460	4723	4972	5206	5429	89.8
0.2	7.5429	5641	5843	6036	6221	6398	6569	6732	6890	7043	7190	89.7
0.3	7.7190	7332	7470	7604	7734	7860	7982	8101	8217	8329	8439	89.6
0.4	7.8439	8547	8651	8754	8853	8951	9046	9140	9231	9321	9409	89.5
0.5	7.9409	9495	9579	9662	9743	9823	9901	9978	*0053	*0127	*0200	89.4
0.6	8.0200	0272	0343	0412	0481	0548	0614	0680	0744	0807	0870	89.3
0.7	8.0870	0932	0992	1052	1111	1170	1227	1284	1340	1395	1450	89.2
0.8	8.1450	1504	1557	1610	1662	1713	1764	1814	1864	1913	1962	89.1
0.9	8.1962	2010	2057	2104	2150	2196	2242	2287	2331	2376	2419	89°.0
1°.0	8.2419	2462	2505	2548	2590	2631	2672	2713	2754	2794	2833	88.9
1.1	8.2833	2873	2912	2950	2988	3026	3064	3101	3138	3175	3211	88.8
1.2	8.3211	3247	3283	3318	3354	3389	3423	3458	3492	3525	3559	88.7
1.3	8.3559	3592	3625	3658	3691	3723	3755	3787	3818	3850	3881	88.6
1.4	8.3881	3912	3943	3973	4003	4033	4063	4093	4122	4152	4181	88.5
1.5	8.4181	4210	4238	4267	4295	4323	4351	4379	4406	4434	4461	88.4
1.6	8.4461	4488	4515	4542	4568	4595	4621	4647	4673	4699	4725	88.3
1.7	8.4725	4750	4775	4801	4826	4851	4875	4900	4924	4949	4973	88.2
1.8	8.4973	4997	5021	5045	5068	5092	5115	5139	5162	5185	5208	88.1
1.9	8.5208	5231	5253	5276	5298	5321	5343	5365	5387	5409	5431	88°.0
2°.0	8.5431	5453	5474	5496	5517	5538	5559	5580	5601	5622	5643	87.9
2.1	8.5643	5664	5684	5705	5725	5745	5765	5785	5805	5825	5845	87.8
2.2	8.5845	5865	5884	5904	5923	5943	5962	5981	6000	6019	6038	87.7
2.3	8.6038	6057	6076	6095	6113	6132	6150	6169	6187	6205	6223	87.6
2.4	8.6223	6242	6260	6277	6295	6313	6331	6348	6366	6384	6401	87.5
2.5	8.6401	6418	6436	6453	6470	6487	6504	6521	6538	6555	6571	87.4
2.6	8.6571	6588	6605	6621	6638	6654	6671	6687	6703	6719	6736	87.3
2.7	8.6736	6752	6768	6784	6800	6815	6831	6847	6863	6878	6894	87.2
2.8	8.6894	6909	6925	6940	6956	6971	6986	7001	7016	7031	7046	87.1
2.9	8.7046	7061	7076	7091	7106	7121	7136	7150	7165	7179	7194	87°.0
3°.0	8.7194	7208	7223	7237	7252	7266	7280	7294	7308	7323	7337	86.9
3.1	8.7337	7351	7365	7379	7392	7406	7420	7434	7448	7461	7475	86.8
3.2	8.7475	7488	7502	7515	7529	7542	7556	7569	7582	7596	7609	86.7
3.3	8.7609	7622	7635	7648	7661	7674	7687	7700	7713	7726	7739	86.6
3.4	8.7739	7751	7764	7777	7790	7802	7815	7827	7840	7852	7865	86.5
3.5	8.7865	7877	7890	7902	7914	7927	7939	7951	7963	7975	7988	86.4
3.6	8.7988	8000	8012	8024	8036	8048	8059	8071	8083	8095	8107	86.3
3.7	8.8107	8119	8130	8142	8154	8165	8177	8188	8200	8212	8223	86.2
3.8	8.8223	8234	8246	8257	8269	8280	8291	8302	8314	8325	8336	86.1
3.9	8.8336	8347	8358	8370	8381	8392	8403	8414	8425	8436	8446	86°.0
4°.0	8.8446	8457	8468	8479	8490	8501	8511	8522	8533	8543	8554	85.9
4.1	8.8554	8565	8575	8586	8596	8607	8617	8628	8638	8649	8659	85.8
4.2	8.8659	8669	8680	8690	8700	8711	8721	8731	8741	8751	8762	85.7
4.3	8.8762	8772	8782	8792	8802	8812	8822	8832	8842	8852	8862	85.6
4.4	8.8862	8872	8882	8891	8901	8911	8921	8931	8940	8950	8960	85.5
4.5	8.8960	8970	8979	8989	8998	9008	9018	9027	9037	9046	9056	85.4
4.6	8.9056	9065	9075	9084	9093	9103	9112	9122	9131	9140	9150	85.3
4.7	8.9150	9159	9168	9177	9186	9196	9205	9214	9223	9232	9241	85.2
4.8	8.9241	9250	9260	9269	9278	9287	9296	9305	9313	9322	9331	85.1
4.9	8.9331	9340	9349	9358	9367	9376	9384	9393	9402	9411	9420	85°.0
		9	8	7	6	5	4	3	2	1	0	L. Cot.

L. Tang.	0	1	2	3	4	5	6	7	8	9		
5°.0	8.9420	9428	9437	9446	9454	9463	9472	9480	9489	9497	9506	84.9
5.1	8.9506	9515	9523	9532	9540	9549	9557	9565	9574	9582	9591	84.8
5.2	8.9591	9599	9608	9616	9624	9633	9641	9649	9657	9666	9674	84.7
5.3	8.9674	9682	9690	9699	9707	9715	9723	9731	9739	9747	9756	84.6
5.4	8.9756	9764	9772	9780	9788	9796	9804	9812	9820	9828	9836	84.5
5.5	8.9836	9844	9852	9860	9867	9875	9883	9891	9899	9907	9915	84.4
5.6	8.9915	9922	9930	9938	9946	9953	9961	9969	9977	9984	9992	84.3
5.7	8.9992	*0000	*0007	*0015	*0022	*0030	*0038	*0045	*0053	*0060	*0068	84.2
5.8	9.0068	0075	0083	0090	0098	0105	0113	0120	0128	0135	0143	84.1
5.9	9.0143	0150	0157	0165	0172	0180	0187	0194	0202	0209	0216	84°.0
6°.0	9.0216	0223	0231	0238	0245	0253	0260	0267	0274	0281	0289	83.9
6.1	9.0289	0296	0303	0310	0317	0324	0331	0338	0346	0353	0360	83.8
6.2	9.0360	0367	0374	0381	0388	0395	0402	0409	0416	0423	0430	83.7
6.3	9.0430	0437	0444	0451	0457	0464	0471	0478	0485	0492	0499	83.6
6.4	9.0499	0506	0512	0519	0526	0533	0540	0546	0553	0560	0567	83.5
6.5	9.0567	0573	0580	0587	0593	0600	0607	0614	0620	0627	0633	83.4
6.6	9.0633	0640	0647	0653	0660	0667	0673	0680	0686	0693	0699	83.3
6.7	9.0699	0706	0712	0719	0725	0732	0738	0745	0751	0758	0764	83.2
6.8	9.0764	0771	0777	0784	0790	0796	0803	0809	0816	0822	0828	83.1
6.9	9 0828	0835	0841	0847	0854	0860	0866	0873	0879	0885	0891	83°.0
7°.0	9.0891	0898	0904	0910	0916	0923	0929	0935	0941	0947	0954	82.9
7.1	9.0954	0960	0966	0972	0978	0984	0991	0997	1003	1009	1015	82.8
7.2	9.1015	1021	1027	1033	1039	1045	1051	1058	1064	1070	1076	82.7
7.3	9.1076	1082	1088	1094	1100	1106	1112	1117	1123	1129	1135	82.6
7.4	9.1135	1141	1147	1153	1159	1165	1171	1177	1183	1188	1194	82.5
7.5	9.1194	1200	1206	1212	1218	1223	1229	1235	1241	1247	1252	82.4
7.6	9.1252	1258	1264	1270	1276	1281	1287	1293	1299	1304	1310	82.3
7.7	9.1310	1316	1321	1327	1333	1338	1344	1350	1355	1361	1367	82.2
7.8	9.1367	1372	1378	1384	1389	1395	1400	1406	1412	1417	1423	82.1
7.9	9.1423	1428	1434	1439	1445	1450	1456	1461	1467	1473	1478	82°.0
8°.0	9.1478	1484	1489	1494	1500	1505	1511	1516	1522	1527	1533	81.9
8.1	9.1533	1538	1544	1549	1554	1560	1565	1571	1576	1581	1587	81.8
8.2	9.1587	1592	1597	1603	1608	1613	1619	1624	1629	1635	1640	81.7
8.3	9.1640	1645	1651	1656	1661	1667	1672	1677	1682	1688	1693	81.6
8.4	9.1693	1698	1703	1709	1714	1719	1724	1729	1735	1740	1745	81.5
8.5	9.1745	1750	1755	1761	1766	1771	1776	1781	1786	1791	1797	81.4
8.6	9.1797	1802	1807	1812	1817	1822	1827	1832	1837	1842	1848	81.3
8.7	9.1848	1853	1858	1863	1868	1873	1878	1883	1888	1893	1898	81.2
8.8	9.1898	1903	1908	1913	1918	1923	1928	1933	1938	1943	1948	81.1
8.9	9.1948	1953	1958	1963	1968	1973	1977	1982	1987	1992	1997	81°.0
9°.0	9.1997	2002	2007	2012	2017	2022	2026	2031	2036	2041	2046	80.9
9.1	9.2046	2051	2056	2060	2065	2070	2075	2080	2085	2089	2094	80.8
9.2	9.2094	2099	2104	2109	2113	2118	2123	2128	2132	2137	2142	80.7
9.3	9.2142	2147	2151	2156	2161	2166	2170	2175	2180	2185	2189	80.6
9.4	9.2189	2194	2199	2203	2208	2213	2217	2222	2227	2231	2236	80.5
9.5	9.2236	2241	2245	2250	2255	2259	2264	2269	2273	2278	2282	80.4
9.6	9.2282	2287	2292	2296	2301	2305	2310	2315	2319	2324	2328	80.3
9.7	9.2328	2333	2337	2342	2346	2351	2356	2360	2365	2369	2374	80.2
9.8	9.2374	2378	2383	2387	2392	2396	2401	2405	2410	2414	2419	80.1
9.9	9.2419	2423	2428	2432	2437	2441	2445	2450	2454	2459	2463	80°.0
		9	8	7	6	5	4	3	2	1	0	L. Cot.

L. Tang.	0	1	2	3	4	5	6	7	8	9		
											−∞	90°
0°	−∞	7.2419	5429	7190	8439	9409	*0200	*0870	*1450	*1962	*2419	89
1	8.2419	2833	3211	3559	3881	4181	4461	4725	4973	5208	5431	88
2	8.5431	5643	5845	6038	6223	6401	6571	6736	6894	7046	7194	87
3	8.7194	7337	7475	7609	7739	7865	7988	8107	8223	8336	8446	86
4	8.8446	8554	8659	8762	8862	8960	9056	9150	9241	9331	9420	85
5	8.9420	9506	9591	9674	9756	9836	9915	9992	*0068	*0143	*0216	84
6	9.0216	0289	0360	0430	0499	0567	0633	0699	0764	0828	0891	83
7	9.0891	0954	1015	1076	1135	1194	1252	1310	1367	1423	1478	82
8	9.1478	1533	1587	1640	1693	1745	1797	1848	1898	1948	1997	81
9	9.1997	2046	2094	2142	2189	2236	2282	2328	2374	2419	2463	80°
10°	9.2463	2507	2551	2594	2637	2680	2722	2764	2805	2846	2887	79
11	9.2887	2927	2967	3006	3046	3085	3123	3162	3200	3237	3275	78
12	9.3275	3312	3349	3385	3422	3458	3493	3529	3564	3599	3634	77
13	9.3634	3668	3702	3736	3770	3804	3837	3870	3903	3935	3968	76
14	9.3968	4000	4032	4064	4095	4127	4158	4189	4220	4250	4281	75
15	9.4281	4311	4341	4371	4400	4430	4459	4488	4517	4546	4575	74
16	9.4575	4603	4632	4660	4688	4716	4744	4771	4799	4826	4853	73
17	9.4853	4880	4907	4934	4961	4987	5014	5040	5066	5092	5118	72
18	9.5118	5143	5169	5195	5220	5245	5270	5295	5320	5345	5370	71
19	9.5370	5394	5419	5443	5467	5491	5516	5539	5563	5587	5611	70°
20°	9.5611	5634	5658	5681	5704	5727	5750	5773	5796	5819	5842	69
21	9.5842	5864	5887	5909	5932	5954	5976	5998	6020	6042	6064	68
22	9.6064	6086	6108	6129	6151	6172	6194	6215	6236	6257	6279	67
23	9.6279	6300	6321	6341	6362	6383	6404	6424	6445	6465	6486	66
24	9.6486	6506	6527	6547	6567	6587	6607	6627	6647	6667	6687	65
25	9.6687	6706	6726	6746	6765	6785	6804	6824	6843	6863	6882	64
26	9.6882	6901	6920	6939	6958	6977	6996	7015	7034	7053	7072	63
27	9.7072	7090	7109	7128	7146	7165	7183	7202	7220	7238	7257	62
28	9.7257	7275	7293	7311	7330	7348	7366	7384	7402	7420	7438	61
29	9.7438	7455	7473	7491	7509	7526	7544	7562	7579	7597	7614	60°
30°	9.7614	7632	7649	7667	7684	7701	7719	7736	7753	7771	7788	59
31	9.7788	7805	7822	7839	7856	7873	7890	7907	7924	7941	7958	58
32	9.7958	7975	7992	8008	8025	8042	8059	8075	8092	8109	8125	57
33	9.8125	8142	8158	8175	8191	8208	8224	8241	8257	8274	8290	56
34	9.8290	8306	8323	8339	8355	8371	8388	8404	8420	8436	8452	55
35	9.8452	8468	8484	8501	8517	8533	8549	8565	8581	8597	8613	54
36	9.8613	8629	8644	8660	8676	8692	8708	8724	8740	8755	8771	53
37	9.8771	8787	8803	8818	8834	8850	8865	8881	8897	8912	8928	52
38	9.8928	8944	8959	8975	8990	9006	9022	9037	9053	9068	9084	51
39	9.9084	9099	9115	9130	9146	9161	9176	9192	9207	9223	9238	50°
40°	9.9238	9254	9269	9284	9300	9315	9330	9346	9361	9376	9392	49
41	9.9392	9407	9422	9438	9453	9468	9483	9499	9514	9529	9544	48
42	9.9544	9560	9575	9590	9605	9621	9636	9651	9666	9681	9697	47
43	9.9697	9712	9727	9742	9757	9772	9788	9803	9818	9833	9848	46
44	9.9848	9864	9879	9894	9909	9924	9939	9955	9970	9985	*0000	45°
45°	0.0000											
		9	8	7	6	5	4	3	2	1	0	L. Cot.

L. Tang.	0	1	2	3	4	5	6	7	8	9		
											0.0000	45°
45°	0.0000	0015	0030	0045	0061	0076	0091	0106	0121	0136	0152	44
46	0152	0167	0182	0197	0212	0228	0243	0258	0273	0288	0303	43
47	0303	0319	0334	0349	0364	0379	0395	0410	0425	0440	0456	42
48	0456	0471	0486	0501	0517	0532	0547	0562	0578	0593	0608	41
49	0608	0624	0639	0654	0670	0685	0700	0716	0731	0746	0762	40°
50°	0.0762	0777	0793	0808	0824	0839	0854	0870	0885	0901	0916	39
51	0916	0932	0947	0963	0978	0994	1010	1025	1041	1056	1072	38
52	1072	1088	1103	1119	1135	1150	1166	1182	1197	1213	1229	37
53	1229	1245	1260	1276	1292	1308	1324	1340	1356	1371	1387	36
54	1387	1403	1419	1435	1451	1467	1483	1499	1516	1532	1548	35
55	1548	1564	1580	1596	1612	1629	1645	1661	1677	1694	1710	34
56	1710	1726	1743	1759	1776	1792	1809	1825	1842	1858	1875	33
57	1875	1891	1908	1925	1941	1958	1975	1992	2008	2025	2042	32
58	2042	2059	2076	2093	2110	2127	2144	2161	2178	2195	2212	31
59	2212	2229	2247	2264	2281	2299	2316	2333	2351	2368	2386	30°
60°	0.2386	2403	2421	2438	2456	2474	2491	2509	2527	2545	2562	29
61	2562	2580	2598	2616	2634	2652	2670	2689	2707	2725	2743	28
62	2743	2762	2780	2798	2817	2835	2854	2872	2891	2910	2928	27
63	2928	2947	2966	2985	3004	3023	3042	3061	3080	3099	3118	26
64	3118	3137	3157	3176	3196	3215	3235	3254	3274	3294	3313	25
65	3313	3333	3353	3373	3393	3413	3433	3453	3473	3494	3514	24
66	3514	3535	3555	3576	3596	3617	3638	3659	3679	3700	3721	23
67	3721	3743	3764	3785	3806	3828	3849	3871	3892	3914	3936	22
68	3936	3958	3980	4002	4024	4046	4068	4091	4113	4136	4158	21
69	4158	4181	4204	4227	4250	4273	4296	4319	4342	4366	4389	20°
70°	0.4389	4413	4437	4461	4484	4509	4533	4557	4581	4606	4630	19
71	4630	4655	4680	4705	4730	4755	4780	4805	4831	4857	4882	18
72	4882	4908	4934	4960	4986	5013	5039	5066	5093	5120	5147	17
73	5147	5174	5201	5229	5256	5284	5312	5340	5368	5397	5425	16
74	5425	5454	5483	5512	5541	5570	5600	5629	5659	5689	5719	15
75	5719	5750	5780	5811	5842	5873	5905	5936	5968	6000	6032	14
76	6032	6065	6097	6130	6163	6196	6230	6264	6298	6332	6366	13
77	6366	6401	6436	6471	6507	6542	6578	6615	6651	6688	6725	12
78	6725	6763	6800	6838	6877	6915	6954	6994	7033	7073	7113	11
79	7113	7154	7195	7236	7278	7320	7363	7406	7449	7493	7537	10°
80°	0.7537	7581	7626	7672	7718	7764	7811	7858	7906	7954	8003	9
81	8003	8052	8102	8152	8203	8255	8307	8360	8413	8467	8522	8
82	8522	8577	8633	8690	8748	8806	8865	8924	8985	9046	9109	7
83	9109	9172	9236	9301	9367	9433	9501	9570	9640	9711	9784	6
84	0.9784	9857	9932	*0008	*0085	*0164	*0244	*0326	*0409	*0494	*0580	5
85	1.0580	0669	0759	0850	0944	1040	1138	1238	1341	1446	1554	4
86	1554	1664	1777	1893	2012	2135	2261	2391	2525	2663	2806	3
87	2806	2954	3106	3264	3429	3599	3777	3962	4155	4357	4569	2
88	4569	4792	5027	5275	5539	5819	6119	6441	6789	7167	7581	1
89	1.7581	8038	8550	9130	9800	*0591	*1561	*2810	*4571	*7581	∞	0°
90°	∞											
		9	8	7	6	5	4	3	2	1	0	L. Cot.

L. Tang.	0	1	2	3	4	5	6	7	8	9		
80°.0	0.7537	7541	7546	7550	7555	7559	7563	7568	7572	7577	7581	9.9
80.1	7581	7586	7590	7595	7599	7604	7608	7613	7617	7622	7626	9.8
80.2	7626	7631	7635	7640	7644	7649	7654	7658	7663	7667	7672	9.7
80.3	7672	7676	7681	7685	7690	7695	7699	7704	7708	7713	7718	9.6
80.4	7718	7722	7727	7731	7736	7741	7745	7750	7755	7759	7764	9.5
80.5	7764	7769	7773	7778	7783	7787	7792	7797	7801	7806	7811	9.4
80.6	7811	7815	7820	7825	7830	7834	7839	7844	7849	7853	7858	9.3
80.7	7858	7863	7868	7872	7877	7882	7887	7891	7896	7901	7906	9.2
80.8	7906	7911	7915	7920	7925	7930	7935	7940	7944	7949	7954	9.1
80.9	7954	7959	7964	7969	7974	7978	7983	7988	7993	7998	8003	9°.0
81°.0	0.8003	8008	8013	8018	8023	8027	8032	8037	8042	8047	8052	8.9
81.1	8052	8057	8062	8067	8072	8077	8082	8087	8092	8097	8102	8.8
81.2	8102	8107	8112	8117	8122	8127	8132	8137	8142	8147	8152	8.7
81.3	8152	8158	8163	8168	8173	8178	8183	8188	8193	8198	8203	8.6
81.4	8203	8209	8214	8219	8224	8229	8234	8239	8245	8250	8255	8.5
81.5	8255	8260	8265	8271	8276	8281	8286	8291	8297	8302	8307	8.4
81.6	8307	8312	8318	8323	8328	8333	8339	8344	8349	8355	8360	8.3
81.7	8360	8365	8371	8376	8381	8387	8392	8397	8403	8408	8413	8.2
81.8	8413	8419	8424	8429	8435	8440	8446	8451	8456	8462	8467	8.1
81.9	8467	8473	8478	8484	8489	8495	8500	8506	8511	8516	8522	8°.0
82°.0	0.8522	8527	8533	8539	8544	8550	8555	8561	8566	8572	8577	7.9
82.1	8577	8583	8588	8594	8600	8605	8611	8616	8622	8628	8633	7.8
82.2	8633	8639	8645	8650	6856	8662	8667	8673	8679	8684	8690	7.7
82.3	8690	8696	8701	8707	8713	8719	8724	8730	8736	8742	8748	7.6
82.4	8748	8753	8759	8765	8771	8777	8782	8788	8794	8800	8806	7.5
82.5	8806	8812	8817	8823	8829	8835	8841	8847	8853	8859	8865	7.4
82.6	8865	8871	8877	8883	8888	8894	8900	8906	8912	8918	8924	7.3
82.7	8924	8930	8936	8942	8949	8955	8961	8967	8973	8979	8985	7.2
82.8	8985	8991	8997	9003	9009	9016	9022	9028	9034	9040	9046	7.1
82.9	9046	9053	9059	9065	9071	9077	9084	9090	9096	9102	9109	7°.0
83°.0	0.9109	9115	9121	9127	9134	9140	9146	9153	9159	9165	9172	6.9
83.1	9172	9178	9184	9191	9197	9204	9210	9216	9223	9229	9236	6.8
83.2	9236	9242	9249	9255	9262	9268	9275	9281	9288	9294	9301	6.7
83.3	9301	9307	9314	9320	9327	9333	9340	9347	9353	9360	9367	6.6
83.4	9367	9373	9380	9386	9393	9400	9407	9413	9420	9427	9433	6.5
83.5	9433	9440	9447	9454	9460	9467	9474	9481	9488	9494	9501	6.4
83.6	9501	9508	9515	9522	9529	9536	9543	9549	9556	9563	9570	6.3
83.7	9570	9577	9584	9591	9598	9605	9612	9619	9626	9633	9640	6.2
83.8	9640	9647	9654	9662	9669	9676	9683	9690	9697	9704	9711	6.1
83.9	9711	9719	9726	9733	9740	9747	9755	9762	9769	9777	9784	6°.0
84°.0	0.9784	9791	9798	9806	9813	9820	9828	9835	9843	9850	9857	5.9
84.1	9857	9865	9872	9880	9887	9895	9902	9910	9917	9925	9932	5.8
84.2	0.9932	9940	9947	9955	9962	9970	9978	9985	9993	*0000	*0008	5.7
84.3	1.0008	0016	0023	0031	0039	0047	0054	0062	0070	0078	0085	5.6
84.4	0085	0093	0101	0109	0117	0125	0133	0140	0148	0156	0164	5.5
84.5	0164	0172	0180	0188	0196	0204	0212	0220	0228	0236	0244	5.4
84.6	0244	0253	0261	0269	0277	0285	0293	0301	0310	0318	0326	5.3
84.7	0326	0334	0343	0351	0359	0367	0376	0384	0392	0401	0409	5.2
84.8	0409	0418	0426	0435	0443	0451	0460	0468	0477	0485	0494	5.1
84.9	1.0494	0503	0511	0520	0528	0537	0546	0554	0563	0572	0580	5°.0
		9	8	7	6	5	4	3	2	1	0	L. Cot.

L. Tang.	0	1	2	3	4	5	6	7	8	9		
85°.0	1.0580	0589	0598	0607	0616	0624	0633	0642	0651	0660	0669	4.9
85.1	0669	0678	0687	0695	0704	0713	0722	0731	0740	0750	0759	4.8
85.2	0759	0768	0777	0786	0795	0804	0814	0823	0832	0841	0850	4.7
85.3	0850	0860	0869	0878	0888	0897	0907	0916	0925	0935	0944	4.6
85.4	0944	0954	0963	0973	0982	0992	1002	1011	1021	1030	1040	4.5
85.5	1040	1050	1060	1069	1079	1089	1099	1109	1118	1128	1138	4.4
85.6	1138	1148	1158	1168	1178	1188	1198	1208	1218	1228	1238	4.3
85.7	1238	1249	1259	1269	1279	1289	1300	1310	1320	1331	1341	4.2
85.8	1341	1351	1362	1372	1383	1393	1404	1414	1425	1435	1446	4.1
85.9	1446	1457	1467	1478	1489	1499	1510	1521	1532	1543	1554	4°.0
86°.0	1.1554	1564	1575	1586	1597	1608	1619	1630	1642	1653	1664	3.9
86.1	1664	1675	1686	1698	1709	1720	1731	1743	1754	1766	1777	3.8
86.2	1777	1788	1800	1812	1823	1835	1846	1858	1870	1881	1893	3.7
86.3	1893	1905	1917	1929	1941	1952	1964	1976	1988	2000	2012	3.6
86.4	2012	2025	2037	2049	2061	2073	2086	2098	2110	2123	2135	3.5
86.5	2135	2148	2160	2173	2185	2198	2210	2223	2236	2249	2261	3.4
86.6	2261	2274	2280	2300	2313	2326	2339	2352	2365	2378	2391	3.3
86.7	2391	2404	2418	2431	2444	2458	2471	2485	2498	2512	2525	3.2
86.8	2525	2539	2552	2566	2580	2594	2608	2621	2635	2649	2663	3.1
86.9	2663	2677	2692	2706	2720	2734	2748	2763	2777	2792	2806	3°.0
87°.0	1.2806	2821	2835	2850	2864	2879	2894	2909	2924	2939	2954	2.9
87.1	2954	2969	2984	2999	3014	3029	3044	3060	3075	3091	3106	2.8
87.2	3106	3122	3137	3153	3169	3185	3200	3216	3232	3248	3264	2.7
87.3	3264	3281	3297	3313	3329	3346	3362	3379	3395	3412	3429	2.6
87.4	3429	3445	3462	3479	3496	3513	3530	3547	3564	3582	3599	2.5
87.5	3599	3616	3634	3652	3669	3687	3705	3723	3740	3758	3777	2.4
87.6	3777	3795	3813	3831	3850	3868	3887	3905	3924	3943	3962	2.3
87.7	3962	3981	4000	4019	4038	4057	4077	4096	4116	4135	4155	2.2
87.8	4155	4175	4195	4215	4235	4255	4275	4295	4316	4336	4357	2.1
87.9	4357	4378	4399	4420	4441	4462	4483	4504	4526	4547	4569	2°.0
88°.0	1.4569	4591	4613	4635	4657	4679	4702	4724	4747	4769	4792	1.9
88.1	4792	4815	4838	4861	4885	4908	4932	4955	4979	5003	5027	1.8
88.2	5027	5051	5076	5100	5125	5149	5174	5199	5225	5250	5275	1.7
88.3	5275	5301	5327	5353	5379	5405	5432	5458	5485	5512	5539	1.6
88.4	5539	5566	5594	5621	5649	5677	5705	5733	5762	5790	5819	1.5
88.5	5819	5848	5878	5907	5937	5967	5997	6027	6057	6088	6119	1.4
88.6	6119	6150	6182	6213	6245	6277	6309	6342	6375	6408	6441	1.3
88.7	6441	6475	6508	6542	6577	6611	6646	6682	6717	6753	6789	1.2
88.8	6789	6825	6862	6899	6936	6974	7012	7050	7088	7127	7167	1.1
88.9	7167	7206	7246	7287	7328	7369	7410	7452	7495	7538	7581	1°.0
89°.0	1.7581	7624	7669	7713	7758	7804	7850	7896	7943	7990	8038	0.9
89.1	8038	8087	8136	8186	8236	8287	8338	8390	8443	8496	8550	0.8
89.2	8550	8605	8660	8716	8773	8830	8889	8948	9008	9068	9130	0.7
89.3	9130	9193	9256	9320	9386	9452	9519	9588	9657	9728	9800	0.6
89.4	1.9800	9873	9947	*0022	*0099	*0177	*0257	*0338	*0421	*0505	*0591	0.5
89.5	2.0591	0679	0769	0860	0954	1049	1147	1246	1349	1453	1561	0.4
89.6	1561	1671	1783	1899	2018	2140	2266	2396	2530	2668	2810	0.3
89.7	2810	2957	3110	3268	3431	3602	3779	3964	4157	4359	4571	0.2
89.8	4571	4794	5028	5277	5540	5820	6120	6442	6789	7167	7581	0.1
89.9	2.7581	8039	8550	9130	9800	*0592	*1561	*2810	*4571	*7581	−∞	0°.0
		9	8	7	6	5	4	3	2	1	0	L. Cot.

TABLE VIII

Conversion of ′ ″ into Decimal Parts of a Degree

1′	0.016°	11′	0.183°	21′	0.350°	31′	0.516°	41′	0.683°	51′	0.850°
2′	.033	12′	.200	22′	.366	32′	.533	42′	.700	52′	.866
3′	.050	13′	.216	23′	.383	33′	.550	43′	.716	53′	.883
4′	.066	14′	.233	24′	.400	34′	.566	44′	.733	54′	.900
5′	.083	15′	.250	25′	.416	35′	.583	45′	.750	55′	.916
6′	.100	16′	.266	26′	.433	36′	.600	46′	.766	56′	.933
7′	.116	17′	.283	27′	.450	37′	.616	47′	.783	57′	.950
8′	.133	18′	.300	28′	.466	38′	.633	48′	.800	58′	.966
9′	.150	19′	.316	29′	.483	39′	.650	49′	.816	59′	.983
10′	.166	20′	.333	30′	.500	40′	.666	50′	.833	60′	1.000

1″	0.00028°	6″	0.00166°	10″	0.00277°
2″	.00056	7″	.00194	20″	.00555
3″	.00083	8″	.00222	30″	.00833
4″	.00111	9″	.00250	40″	.01111
5″	.00138			50″	.01388

TABLE IX

Conversion of Decimal Parts of a Degree into ′ ″

0.01°	0′ 36″	0.11°	6′ 36″	0.21°	12′ 36″	0.31°	18′ 36″
.02	1′ 12″	.12	7′ 12″	.22	13′ 12″	.32	19′ 12″
.03	1′ 48″	.13	7′ 48″	.23	13′ 48″	.33	19′ 48″
.04	2′ 24″	.14	8′ 24″	.24	14′ 24″	.34	20′ 24″
.05	3′	.15	9′	.25	15′	.35	21′
.06	3′ 36″	.16	9′ 36″	.26	15′ 36″	.36	21′ 36″
.07	4′ 12″	.17	10′ 12″	.27	16′ 12″	.37	22′ 12″
.08	4′ 48″	.18	10′ 48″	.28	16′ 48″	.38	22′ 48″
.09	5′ 24″	.19	11′ 24″	.29	17′ 24″	.39	23′ 24″
.10	6′	.20	12′	.30	18′	.40	24′
0.41°	24′ 36″	0.51°	30′ 36″	0.61°	36′ 36″	0.71°	42′ 36″
.42	25′ 12″	.52	31′ 12″	.62	37′ 12″	.72	43′ 12″
.43	25′ 48″	.53	31′ 48″	.63	37′ 48″	.73	43′ 48″
.44	26′ 24″	.54	32′ 24″	.64	38′ 24″	.74	44′ 24″
.45	27′	.55	33′	.65	39′	.75	45′
.46	27′ 36″	.56	33′ 36″	.66	39′ 36″	.76	45′ 36″
.47	28′ 12″	.57	34′ 12″	.67	40′ 12″	.77	46′ 12″
.48	28′ 48″	.58	34′ 48″	.68	40′ 48″	.78	46′ 48″
.49	29′ 24″	.59	35′ 24″	.69	41′ 24″	.79	47′ 24″
.50	30′	.60	36′	.70	42′	.80	48′
0.81°	48′ 36″	0.91°	54′ 36″	0.001°	3.6″		
.82	49′ 12″	.92	55′ 12″	.002	7.2″		
.83	49′ 48″	.93	55′ 48″	.003	10.8″		
.84	50′ 24″	.94	56′ 24″	.004	14.4″		
.85	51′	.95	57′	.005	18 ″		
.86	51′ 36″	.96	57′ 36″	.006	21.6″		
.87	52′ 12″	.97	58′ 12″	.007	25.2″		
.88	52′ 48″	.98	58′ 48″	.008	28.8″		
.89	53′ 24″	.99	59′ 24″	.009	32.4″		
.90	54′	1.00	60′				

ANSWERS

Exercise 1

1. $\log_3 9 = 2.$ $\log_3 27 = 3.$ $\log_4 64 = 4.$ $\log_4 \frac{1}{16} = -2.$ $\log_3 \frac{1}{9} = -2.$ $\log_3 \frac{1}{81} = -4.$ $\log_{10} \frac{1}{10} = -1.$ $\log_{10} .01 = -2.$ $\log_{10} .001 = -3.$

2. $\log_2 32 = 5.$ $\log_2 \frac{1}{32} = -5.$ $\log_4 8 = \frac{3}{2}.$ $\log_2 \frac{1}{128} = -7.$ $\log_8 16 = \frac{4}{3}.$

3. 1. **9.** $\sqrt[3]{64} = 4.$ $\sqrt[5]{1024} = 4.$ $\sqrt[4]{4096} = 8.$

Exercise 2

1. 2. **3.** 2. **5.** 0. **7.** 0. **9.** -3. **11.** 0. **13.** -4. **15.** 1.
2. 4. **4.** 1. **6.** -2. **8.** 0. **10.** -5. **12.** 3. **14.** 2.

16. $3 = 4.$ $2 = 3.$ $5 = 6.$ $1 = 2.$ $0 = 1.$ $4 = 5.$ $8 - 10 = 1.$ $7 - 10 = 2.$ $9 - 10 = 0.$

Exercise 3

1. 3.88235. **8.** 1.82751. **15.** 1.93952. **22.** 8.27135 − 10.
2. 3.82737. **9.** 0.52410. **16.** 9.88081 − 10. **23.** 4.51427.
3. 1.91381. **10.** 7.82737 − 10. **17.** 6.09691 − 10. **24.** 3.51427.
4. 3.89553. **11.** 4.84510 − 10. **18.** 2.00109. **25.** 2.51427.
5. 1.87506. **12.** 5.60206. **19.** 1.24622. **26.** 1.51427.
6. 2.19590. **13.** 1.16505 − 10. **20.** 1.62325. **27.** 0.51427.
7. 4.55965. **14.** 7.35550. **21.** 4.0000 − 10.

28. $\log 200 = 2.30103.$ $\log 3000 = 3.47712.$ $\log 50 = 1.69897.$ $\log 100\,\pi = 2.49715.$ $\log 20 = 1.30103.$ $\log .002 = 7.30103 - 10.$ $\log 30 = 1.47712.$ $\log .0005 = 6.69897 - 10.$ $\log \frac{\pi}{100} = 8.49715 - 10.$ $\log .3 = 9.47712 - 10.$ $\log .2 = 9.30103 - 10.$ $\log 10\,\pi = 1.49715.$ $\log 20000 = 4.30103.$

29. 1.1028. **35.** .0011. **40.** 2.9847. **45.** 4.4619.
30. 2.8824. **36.** 1.3923. **41.** 0.1666. **46.** 1.2916.
31. 1.6302. **37.** 9.0459 − 10. **42.** 0.2462. **47.** 9.9358 − 10.
32. .0887. **38.** 1.0676. **43.** 5.5655 − 10. **48.** 8.0012 − 10.
33. 8.4200 − 10. **39.** 7.1030 − 10. **44.** 7.4213 − 10. **49.** 0.3474.
34. 7.1030 − 10.

Exercise 4

1. 26.22. **11.** 221.705. **20.** 25.6. **29.** 454.44.
2. 157.6. **12.** .01569. **21.** 541. **30.** .0000022337.
3. 9.627. **13.** 10.88375. **22.** 1712. **31.** 657.166.
4. 48323333.3. **14.** .50742. **23.** .14277. **32.** 201.409.
5. .16719. **15.** 1647.3. **24.** 107.8. **33.** .3625.
6. .00026827. **16.** 1008581.4. **25.** 10.315. **34.** 9.6968 − 10.
7. 3896545.45. **17.** .78488. **26.** .0106725. **35.** 3.1443.
8. .000055855. **18.** 96988. **27.** .001309. **36.** 49.25.
9. 100925581.4. **19.** .69781. **28.** .000010044. **37.** .2285.
10. .37029.

Exercise 5

1. 53295. **2.** 1383.62. **3.** 211820. **4.** 8.3552. **5.** 514.055. **6.** 19.033913. **7.** 1.492. **8.** .01141. **9.** 5.3921. **10.** .96518. **11.** -1.8583. **12.** $-.059439$. **13.** $-.34526$. **14.** \$33945. **15.** \$491.04.

17. $\frac{200}{376} = .53191.$ $\frac{100\,\pi}{58} = 5.4165.$ $\frac{300 \times 500}{\pi} = 47746.67.$

18. 1.3774 A., 3.4435 A., 45.9134 A.

19. 33.38. **20.** 6.727. **21.** .4171. **22.** 2034.3. **23.** 3261. **24.** 1.16467. **25.** 3.908. **26.** 3.413. **27.** .0939. **30.** \$213.47. **31.** \$325.60. **32.** \$5874.75.

Exercise 6

1. .972.
2. 99.266.
3. 8.9254.
4. .182916.
5. 1602.4
6. 2.37242.
7. 218.51.
6.6943.
7.1845.
8. 500 m. = 1640.5 ft.
7294 m. = 23931.11 ft.
300 m. = 984.26 ft.
9. 2.34667.
10. $-.0447$.
11. -1.5793.
12. 24.1394.
13. 19.85.
14. 24.035
15. 189.66.
16. .12246.
17. 13306.06.
18. 1029.4.
19. .11069.
20. 2519.6.
21. 7061.67.
22. 65.97 = 66 yr.
23. .5342.
24. 1.6167.
25. 1.1377.
26. 22.33.
27. 10695.
28. .1705.
29. 6080000.
30. 4.245.
31. 17.49.
32. 1.272.
33. .4163.
34. 12.07.
35. 5.77.
36. 2316.8.

Exercise 7

1. 2.544. **2.** 1.2445. **3.** 2.495. **4.** $-.053474$. **5.** 1.465. **6.** .65959. **7.** -29.78. **8.** 5.9837. **9.** $-.46187$. **10.** .64509. **11.** $-.4167$. **12.** .29414. **13.** 3. **14.** 5. **15.** -2. **16.** $-\frac{3}{4}$. **17.** -3. **18.** -4. **19.** 2. **20.** 81. **21.** 25. **22.** $\frac{8}{27}$. **23.** 32. **24.** 17.677. 11.894. **25.** 5%.

Exercise 8

1.		$\sin B = \frac{b}{c}$,	$\tan B = \frac{b}{a}$,	$\sec B = \frac{c}{a}$,	$\cos B = \frac{a}{c}$,	$\cot B = \frac{a}{b}$,	$\csc B = \frac{c}{b}$.
2.		$\sin A = \frac{4}{5}$,	$\tan A = \frac{4}{3}$,	$\sec A = \frac{5}{3}$,	$\cos A = \frac{3}{5}$,	$\cot A = \frac{3}{4}$,	$\csc A = \frac{5}{4}$.
3.		$\sin A = \frac{3}{5}$,	$\tan A = \frac{3}{4}$,	$\sec A = \frac{5}{4}$,	$\cos A = \frac{4}{5}$,	$\cot A = \frac{4}{3}$,	$\csc A = \frac{5}{3}$.
4.		$\sin A = \frac{8}{17}$,	$\tan A = \frac{8}{15}$,	$\sec A = \frac{17}{15}$,	$\cos A = \frac{15}{17}$,	$\cot A = \frac{15}{8}$,	$\csc A = \frac{17}{8}$.
5.		$\sin A = \frac{12}{13}$,	$\tan A = \frac{12}{5}$,	$\sec A = \frac{13}{5}$,	$\cos A = \frac{5}{13}$,	$\cot A = \frac{5}{12}$,	$\csc A = \frac{13}{12}$.
6.		$\sin A = \frac{39}{89}$,	$\tan A = \frac{39}{80}$,	$\sec A = \frac{89}{80}$,	$\cos A = \frac{80}{89}$,	$\cot A = \frac{80}{39}$,	$\csc A = \frac{89}{39}$.
7.		$\sin A = \frac{9}{41}$,	$\tan A = \frac{9}{40}$,	$\sec A = \frac{41}{40}$,	$\cos A = \frac{40}{41}$,	$\cot A = \frac{40}{9}$,	$\csc A = \frac{41}{9}$.
8.		$\sin A = \frac{119}{169}$,	$\tan A = \frac{119}{120}$,	$\sec A = \frac{169}{120}$,	$\cos A = \frac{120}{169}$,	$\cot A = \frac{120}{119}$,	$\csc A = \frac{169}{119}$.
9.	III.	$\sin B = \frac{4}{5}$,	$\tan B = \frac{4}{3}$,	$\sec B = \frac{5}{3}$,	$\cos B = \frac{3}{5}$,	$\cot B = \frac{3}{4}$,	$\csc B = \frac{5}{4}$.
	IV.	$\sin B = \frac{15}{17}$,	$\tan B = \frac{15}{8}$,	$\sec B = \frac{17}{8}$,	$\cos B = \frac{8}{17}$,	$\cot B = \frac{8}{15}$,	$\csc B = \frac{17}{15}$.
	V.	$\sin B = \frac{5}{13}$,	$\tan B = \frac{5}{12}$,	$\sec B = \frac{13}{12}$,	$\cos B = \frac{12}{13}$,	$\cot B = \frac{12}{5}$,	$\csc B = \frac{13}{5}$.
	VI.	$\sin B = \frac{80}{89}$,	$\tan B = \frac{80}{39}$,	$\sec B = \frac{89}{39}$,	$\cos B = \frac{39}{89}$,	$\cot B = \frac{39}{80}$,	$\csc B = \frac{89}{80}$.
	VII.	$\sin B = \frac{40}{41}$,	$\tan B = \frac{40}{9}$,	$\sec B = \frac{41}{9}$,	$\cos B = \frac{9}{41}$,	$\cot B = \frac{9}{40}$,	$\csc B = \frac{41}{40}$.
	VIII.	$\sin B = \frac{120}{169}$,	$\tan B = \frac{120}{119}$,	$\sec B = \frac{169}{119}$,	$\cos B = \frac{119}{169}$,	$\cot B = \frac{119}{120}$,	$\csc B = \frac{169}{120}$.

10. (1) $\frac{16}{15}$. (2) 1. (3) 1. (4) $\frac{25}{9}$. (5) 1. (6) 1. (7) 0. (8) 1.

22. $AD = 218.4.$ $CD = 358.7.$ $DB = 181.3.$ $AB = 283.86.$
23. .854. 24. 56.75.

27. $\sin A = \dfrac{\sqrt{p^2+q^2}}{p+q}$, $\cos A = \dfrac{\sqrt{2pq}}{p+q}$.

28. $\sin A = \dfrac{2mn}{m^2+n^2}$, $\sec A = \dfrac{m^2+n^2}{m^2-n^2}$, $\tan A = \dfrac{2mn}{m^2-n^2}$.

29. $\sin A = \dfrac{p^2-q^2}{p^2+q^2}$, $\tan A = \dfrac{p^2-q^2}{2pq}$, $\csc A = \dfrac{p^2+q^2}{p^2-q^2}$.

30. $\sin B = \dfrac{\sqrt{mn+n^2}}{m+n}$, $\tan B = \sqrt{\dfrac{mn+n^2}{mn+m^2}}$, $\sec B = \dfrac{m+n}{\sqrt{m^2+mn}}$,

$\cos B = \dfrac{\sqrt{m^2+mn}}{m+n}$, $\cot B = \dfrac{\sqrt{mn+m^2}}{\sqrt{mn+n^2}}$, $\csc B = \dfrac{m+n}{\sqrt{mn+n^2}}$.

31. $\sin B = \dfrac{m-n}{m+n}$, $\tan B = \dfrac{(m-n)\sqrt{mn}}{2mn}$, $\sec B = \dfrac{(m+n)\sqrt{mn}}{2mn}$,

$\cos B = \dfrac{2\sqrt{mn}}{m+n}$, $\cot B = \dfrac{2\sqrt{mn}}{m-n}$, $\csc B = \dfrac{m+n}{m-n}$.

32. $\sec A = \frac{61}{11}$, $\tan B = \frac{11}{60}$, $\cot B = \frac{60}{11}$, $\sin A = \frac{60}{61}$.
33. $\sin B = \frac{264}{265}$, $\tan B = \frac{264}{23}$, $\sec B = \frac{265}{23}$, $\cos B = \frac{23}{265}$, $\cot B = \frac{23}{264}$, $\csc B = \frac{265}{264}$.
34. $\sin A = \frac{2}{5}\sqrt{5}$, $\tan A = 2$, $\sec A = \sqrt{5}$, $\cos A = \frac{1}{5}\sqrt{5}$, $\cot A = \frac{1}{2}$, $\csc A = \dfrac{\sqrt{5}}{2}$.

35. $\sin A = \dfrac{\sqrt{5}}{3}$, $\tan A = \dfrac{\sqrt{5}}{2}$, $\sec A = \frac{3}{2}$, $\cos A = \frac{2}{3}$, $\cot A = \dfrac{2\sqrt{5}}{5}$, $\csc A = \frac{3}{5}\sqrt{5}$.

36. $\sin B = \dfrac{4-\sqrt{2}}{6}$, $\tan B = \dfrac{9-4\sqrt{2}}{7}$, $\sec B = \dfrac{3(4-\sqrt{2})}{7}$.

$\cos B = \dfrac{4+\sqrt{2}}{6}$, $\cot B = \dfrac{9+4\sqrt{2}}{7}$, $\csc B = \dfrac{3(4+\sqrt{2})}{7}$.

37. $\sin A = \frac{12}{13}$, $\tan A = \frac{12}{5}$, $\sec A = \frac{13}{5}$, $\cos A = \frac{5}{13}$, $\cot A = \frac{5}{12}$, $\csc A = \frac{13}{12}$.
38. $\sin B = \frac{3}{5}$, $\tan B = \frac{3}{4}$, $\sec B = \frac{5}{4}$, $\cos B = \frac{4}{5}$, $\cot B = \frac{4}{3}$, $\csc B = \frac{5}{3}$.
41. 1.62. 42. $\frac{3}{5}$, $\frac{3}{5}$.

Exercise 9

1. $\cos 30°$.	4. $\tan 34° 24'$.	7. $\cos 1'$.	10. $\frac{1}{2}$.
2. $\sin 75°$.	5. $\sec 68° 35' 30''$.	8. $\sin 88° 42'$.	11. $\dfrac{x}{y}$.
3. $\cot 24° 36'$.	6. $\csc 5° 44'$.	9. $\sqrt{3}$.	12. p.

Exercise 10

1. $\tan A = \frac{15}{8}$, $\sec A = \frac{17}{8}$, $\cos A = \frac{8}{17}$, $\cot A = \frac{8}{15}$, $\csc A = \frac{17}{15}$.
2. $\sin A = \frac{12}{13}$, $\sec A = \frac{13}{5}$, $\cos A = \frac{5}{13}$, $\cot A = \frac{5}{12}$, $\csc A = \frac{13}{12}$.
3. $\sin A = \frac{40}{41}$, $\tan A = \frac{40}{9}$, $\cos A = \frac{9}{41}$, $\cot A = \frac{9}{40}$, $\csc A = \frac{41}{40}$.
4. $\sin A = \dfrac{\sqrt{5}}{3}$, $\tan A = \dfrac{\sqrt{5}}{2}$, $\sec A = \frac{3}{2}$, $\cot A = \frac{2}{5}\sqrt{5}$, $\csc A = \frac{3}{5}\sqrt{5}$.

5. $\sin A = \dfrac{\sqrt{m^2+1}}{m^2+1}$, $\tan A = \dfrac{1}{m}$, $\cos A = \dfrac{m\sqrt{m^2+1}}{m^2+1}$, $\sec A = \dfrac{\sqrt{1+m^2}}{m}$.
$\csc A = \sqrt{m^2+1}$.

6. $\sin A = \frac{\sqrt{5}}{5}$, $\tan A = \frac{1}{2}$, $\sec A = \frac{\sqrt{5}}{2}$, $\cos A = \frac{2\sqrt{5}}{5}$, $\cot A = 2$.

7. $\tan A = 0$, $\sec A = 1$, $\cos A = 1$, $\cot A = \infty$, $\csc A = \infty$.

8. $\sin A = 1$, $\tan A = \infty$, $\sec A = \infty$, $\cot A = 0$, $\csc A = 1$.

9. $\sin A = 0$, $\sec A = 1$, $\cos A = 1$, $\cot A = \infty$, $\csc A = \infty$.

10. $\tan A = \infty$, $\sec A = \infty$, $\cos A = 0$, $\cot A = 0$, $\csc A = 1$.

11. $\sin A = 1$, $\tan A = \infty$, $\cos A = 0$, $\cot A = 0$, $\csc A = 1$.

12. $\tan x = \frac{5p}{\sqrt{1-25p^2}}$, $\sec x = \frac{1}{\sqrt{1-25p^2}}$, $\cos x = \sqrt{1-25p^2}$,
$\cot x = \frac{\sqrt{1-25p^2}}{5p}$, $\csc x = \frac{1}{5p}$.

13. $\sin A = \frac{3}{5}$, $\sec A = \frac{5}{4}$, $\cos A = \frac{4}{5}$, $\cot A = \frac{4}{3}$, $\csc A = \frac{5}{3}$,

14. $\sin A = \frac{12}{13}$, $\tan A = \frac{12}{5}$, $\sec A = \frac{13}{5}$, $\cot A = \frac{5}{12}$, $\csc A = \frac{13}{12}$.

15. $\sin A = \frac{15}{17}$, $\tan A = \frac{15}{8}$, $\sec A = \frac{17}{8}$, $\cos A = \frac{8}{17}$, $\cot A = \frac{8}{15}$.

16. $\sin A = \frac{2\sqrt{13}}{13}$, $\tan A = \frac{2}{3}$, $\sec A = \frac{\sqrt{13}}{3}$, $\cos A = \frac{3\sqrt{13}}{13}$, $\csc A = \frac{\sqrt{13}}{2}$.

17. $\tan A = \frac{1}{3}\sqrt{3}$, $\sec A = \frac{2}{3}\sqrt{3}$, $\cos A = \frac{\sqrt{3}}{2}$, $\cot A = \sqrt{3}$, $\csc A = 2$.

18. $\sin A = \frac{\sqrt{15}}{4}$, $\tan A = \sqrt{15}$, $\cos A = \frac{1}{4}$, $\cot A = \frac{1}{15}\sqrt{15}$, $\csc A = \frac{4}{15}\sqrt{15}$.

19. $\sin A = \frac{m\sqrt{m^2+1}}{m^2+1}$, $\cos A = \frac{\sqrt{m^2+1}}{m^2+1}$, $\cot A = \frac{1}{m}$, $\sec A = \sqrt{m^2+1}$,
$\csc A = \frac{\sqrt{m^2+1}}{m}$.

20. $\sin A = \frac{\sqrt{2}}{2}$, $\tan A = 1$, $\sec A = \sqrt{2}$, $\cos A = \frac{\sqrt{2}}{2}$, $\cot A = 1$, $\csc A = \sqrt{2}$.

21. $\sin x = 0$, $\tan x = 0$, $\sec x = 1$, $\cot x = \infty$, $\csc x = \infty$.

22. $\sin A = \frac{40}{41}$, $\tan A = \frac{40}{9}$, $\sec A = \frac{41}{9}$, $\cos A = \frac{9}{41}$, $\cot A = \frac{9}{40}$,

23. $\sin A = \frac{2mn}{m^2+n^2}$, $\cos A = \frac{m^2-n^2}{m^2+n^2}$, $\cot A = \frac{m^2-n^2}{2mn}$, $\sec A = \frac{m^2+n^2}{m^2-n^2}$,
$\csc A = \frac{m^2+n^2}{2mn}$.

24. $\sin A = \frac{1}{2}\sqrt{2-\sqrt{2}}$, $\tan A = \sqrt{2}-1$, $\cos A = \frac{1}{2}\sqrt{2+\sqrt{2}}$,
$\sec A = \sqrt{4-2\sqrt{2}}$, $\csc A = \sqrt{4+2\sqrt{2}}$.

25. $\tan A = \infty$, $\sec A = \infty$, $\cos A = 0$, $\cot A = 0$, $\csc A = 1$.

26. $\sin 22\frac{1}{2}° = \frac{1}{2}\sqrt{2-\sqrt{2}}$, $\cos 22\frac{1}{2}° = \frac{\sqrt{2+\sqrt{2}}}{2}$, $\cot 22\frac{1}{2}° = \sqrt{2}+1$,
$\sec 22\frac{1}{2}° = \sqrt{4-2\sqrt{2}}$, $\csc 22\frac{1}{2}° = \sqrt{4+2\sqrt{2}}$.

27. $\sin A = \frac{\sqrt{39}}{8}$, $\tan A = \frac{\sqrt{39}}{5}$, $\sec A = \frac{8}{5}$, $\cot A = \frac{5}{39}\sqrt{39}$, $\csc A = \frac{8}{39}\sqrt{39}$.

28. $\sin A = \frac{\sqrt{2+\sqrt{3}}}{2}$, $\tan A = 2+\sqrt{3}$, $\cos A = \frac{\sqrt{6}-\sqrt{2}}{4}$, $\cot A = 2-\sqrt{3}$,
$\csc A = 2\sqrt{2-\sqrt{3}}$.

29. $\sin A = \sqrt{1-K^2}$, $\tan A = \dfrac{\sqrt{1-K^2}}{K}$, $\cot A = \dfrac{K\sqrt{(1-K^2)}}{1-K^2}$, $\sec A = \dfrac{1}{K}$,

$\csc A = \dfrac{\sqrt{1-K^2}}{1-K^2}$.

30. $\sin 15° = \dfrac{\sqrt{2-\sqrt{3}}}{2}$, $\tan 15° = 2-\sqrt{3}$, $\cos 15° = \dfrac{\sqrt{2+\sqrt{3}}}{2}$,

$\sec 15° = 2\sqrt{2-\sqrt{3}}$, $\csc 15° = 2\sqrt{2+\sqrt{3}}$.

31. $\cos A = \sqrt{1-\sin^2 A}$, $\tan A = \dfrac{\sin A}{\sqrt{1-\sin^2 A}}$, $\csc A = \dfrac{1}{\sin A}$,

$\cot A = \dfrac{\sqrt{1-\sin^2 A}}{\sin A}$, $\sec A = \dfrac{1}{\sqrt{1-\sin^2 A}}$.

32. $\sin A = \sqrt{1-\cos^2 A}$, $\tan A = \dfrac{\sqrt{1-\cos^2 A}}{\cos A}$, $\cot A = \dfrac{\cos A}{\sqrt{1-\cos^2 A}}$,

$\sec A = \dfrac{1}{\cos A}$, $\csc A = \dfrac{1}{\sqrt{1-\cos^2 A}}$.

33. $\sin A = \dfrac{\tan A}{\sqrt{1+\tan^2 A}}$, $\cos A = \dfrac{1}{\sqrt{1+\tan^2 A}}$, $\cot A = \dfrac{1}{\tan A}$,

$\sec A = \sqrt{1+\tan^2 A}$, $\csc A = \dfrac{\sqrt{1+\tan^2 A}}{\tan A}$.

34. $\tan A = \dfrac{1}{\cot A}$, $\csc A = \sqrt{1+\cot^2 A}$, $\sin A = \dfrac{1}{\sqrt{1+\cot^2 A}}$,

$\cos A = \dfrac{\cot A}{\sqrt{1+\cot^2 A}}$, $\sec A = \dfrac{\sqrt{1+\cot^2 A}}{\cot A}$.

35. $\cos A = \dfrac{1}{\sec A}$, $\tan A = \sqrt{\sec^2 A - 1}$, $\cot A = \dfrac{1}{\sqrt{\sec^2 A - 1}}$,

$\csc A = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$, $\sin A = \dfrac{\sqrt{\sec^2 A - 1}}{\sec A}$.

36. $\sin A = \dfrac{1}{\csc A}$, $\cos A = \dfrac{\sqrt{\csc^2 A - 1}}{\csc A}$, $\tan A = \dfrac{1}{\sqrt{\csc^2 A - 1}}$,

$\sec A = \dfrac{\csc A}{\sqrt{\csc^2 A - 1}}$, $\cot A = \sqrt{\csc^2 A - 1}$.

37. $\cos A = 1 - \text{vers } A$, $\sec A = \dfrac{1}{1-\text{vers } A}$,

$\tan A = \dfrac{\sqrt{2 \text{ vers } A - 2 \text{ vers}^2 A}}{1 - \text{vers } A}$, $\cot A = \dfrac{1-\text{vers } A}{\sqrt{2 \text{ vers } A - \text{vers}^2 A}}$,

$\sin A = \sqrt{2 \text{ vers } A - \text{vers}^2 A}$, $\csc A = \dfrac{1}{\sqrt{2 \text{ vers } A - \text{vers}^2 A}}$.

38. $\dfrac{\sqrt{7}}{3}$.

39. $\frac{80}{4879}\sqrt{4879}$.

40. $\frac{1}{2}\sqrt{3}$.

41. $\frac{8}{39}\sqrt{39}$.

42. $\frac{7}{24}$.

43. $\frac{1}{2}\sqrt{2-\sqrt{2}}$.

44. $\frac{1}{6}\sqrt{42}$.

45. $1-\cos^2 A + \cos A$.

46. $\dfrac{1}{\cos^3 A}$.

47. $\dfrac{1}{\sin A \cos A}$.

48. $2\sin^2 x + \sin x = 1$.

49. $\tan^2 x - 2\tan x = 1$.

Exercise 12

13. $2\frac{1}{2}$. **14.** $\frac{1}{3}\sqrt{3}(b+c)$. **15.** $2+\sqrt{2}$. **16.** $1-2\sqrt{3}$. **17.** $-1-\sqrt{2}$. **18.** $-6\frac{1}{3}$. **20.** $\frac{1}{2}(\sqrt{2}-1)$. **21.** $\frac{4}{3}$. **22.** $\frac{1}{2}\sqrt{6}$. **23.** 5. **35.** 86.6. **36.** 150; 259.8. **38.** 961.3+. **39.** 165.

Exercise 13

1. 60°. **2.** 60°. **3.** 30°. **4.** 60°. **5.** 0°. **6.** 45°. **7.** 45°. **8.** 45°. **9.** 30°. **10.** 60°. **11.** 45°. **12.** 30°, 90°. **13.** 60°. **14.** 30°. **15.** 45°. **16.** 30°. **17.** 45°. **18.** 45°. **19.** 60°. **20.** 90°. **21.** 0°. **22.** 27° 13′ 12″. **23.** 15°. **24.** 10°. **25.** 60°. **26.** $22\frac{1}{2}$°. **27.** $\dfrac{90°}{n+1}$. **28.** 18°. **29.** 45°. **30.** 38° 50′. **33.** 30°. **34.** 60°. **35.** 30°.

Exercise 14

1. 9.64647 – 10. **2.** 9.98997 – 10. **3.** 9.86603 – 10. **4.** 9.38699 – 10. **5.** 0.15908. **6.** 9.43707 – 10. **7.** 8.73767 – 10. **8.** 9.86126 – 10. **9.** 8.95017 – 10. **10.** 9.97991 – 10. **11.** 0.11532. **12.** 9.99194 – 10. **13.** 1.24820. **14.** 8.91931 – 10. **15.** 9.84324 – 10. **16.** 9.74610 – 10. **19.** 6.1493. **20.** 14.991. **21.** 9.4214 – 10. **22.** 9.8297 – 10. **23.** 0.1759. **24.** 0.7033. **25.** 9.6622 – 10. **26.** 9.9523 – 10. **27.** 0.3076. **28.** 0.6489. **29.** 9.8832 – 10. **30.** 0.2522. **31.** 0.6413. **32.** 15.24.

Exercise 15

1. 23° 15′. **2.** 28° 40′. **3.** 35° 43′. **4.** 40° 23′. **5.** 66° 15′ 24″. **6.** 70° 16′ 21″. **7.** 70° 0′ 26″. **8.** 85° 5′ 15″. **9.** 65° 10′ 20″. **10.** 5° 20′ 29″. **11.** 4° 0′ 47″. **12.** 85° 59′ 13″. **13.** 26.5°. **14.** 50.2°. **15.** 28.7°. **16.** 18.5°. **17.** 56.26°. **18.** 70.14°. **19.** 64.43°. **20.** 46.11°. **21.** 61.07°. **22.** 0.541°. **23.** 88.465°. **24.** 65.67°. **25.** 78.14°. **26.** 14.47°.

Exercise 16

1. 8.21421 – 10. **2.** 8.34812 – 10. **3.** 8.49128 – 10. **4.** 1.72220. **5.** 1.64078. **6.** 8.18538 – 10. **7.** 8.28456 – 10. **8.** 8.47866 – 10. **9.** 0° 26′ 10″. **10.** 88° 53′ 6″. **11.** 0° 42′ 53″. **12.** 89° 32′ 27″. **13.** 89° 57′. **14.** 0° 4′ 31″. **15.** 0° 2′ 39″. **16.** 89° 45′ 6″. **17.** 42° 5′ 26″. **18.** 82° 52′ 1″. **19.** 83° 24′ 25″. **20.** 0° 17′ 7.3″. **21.** 0° 17′ 7.1″. **22.** 89° 54′ 15″. **23.** 8.245. **24.** .1504. **25.** 1.6687. **26.** 8.3353 – 10. **27.** 8.1238 – 10. **28.** 8.1070 – 10. **29.** 8.2701 – 10. **30.** 1.6657. **31.** 1.8744. **32.** 8.3446 – 10. **33.** 7.9686 – 10. **34.** 89.266°. **35.** 1.036°. **36.** 89.216°. **37.** .634°. **38.** 89.553°. **39.** .507°. **40.** 4.662°. **41.** 84.35°. **42.** 8.3638 – 10. **43.** 1.6362. **44.** 89.266°. **45.** .613°. **46.** 89.285°. **47.** .624°. **48.** 1.6375. **49.** 2.792. **50.** 112.82. **51.** .7348. **52.** .026694.

Exercise 17

1. Sine $A = \frac{8}{17}$. Cosine $A = \frac{15}{17}$. Cotangent $A = \frac{15}{8}$. Secant $A = \frac{17}{15}$. Cosecant $A = \frac{17}{8}$. $b = 30$. $c = 34$.
2. $-\frac{5625}{128}$.
5. $\sin 49° > \cos 49°$.
6. $A < 45°$.
7. $A > 60°$.
8. $\cot 37° > \tan 37°$.
19. $x = 45°$.
20. $x = 60°$.
21. $x = 45°$.
22. 1.
23. $\frac{4}{3}\sqrt{3} - \frac{1}{2}\sqrt{2} - \frac{3}{4}$.
24. $\dfrac{11 - 3\sqrt{3}}{2}$.
25. $\cot A = \frac{24}{7}$, $\csc A = \frac{25}{7}$.
26. $\dfrac{p}{r}$.
27. .3056.
28. 300.
29. 270.12

Exercise 18

4. $B = 62°$.
   $a = 6.3804$.
   $c = 13.591$.
5. $B = 12°$.
   $a = 26.15$.
   $b = 5.5585$.
6. $B = 43° 42'$.
   $a = 50.78$.
   $c = 70.24$.
7. $B = 61° 43'$.
   $a = 11.448$.
   $b = 21.276$.
8. $A = 35° 17'$.
   $a = 648.67$.
   $b = 916.7$.
9. $A = 52° 41'$.
   $a = 385.436$.
   $c = 484.644$.
10. $B = 51° 43' 36''$.
    $a = 2.2478$.
    $b = 2.849$.
11. $A = 17° 43' 18''$.
    $b = 70.985$.
    $c = 74.5217$.
12. .23661.
13. .282726.
14. $B = 26° 31' 20''$.
    $b = 127.976$.
    $c = 286.5875$.
15. $A = 2° 43' 30''$.
    $a = 13.85129$.
    $b = .674616$.
16. $B = 38° 50' 54''$.
    $a = .153254$.
    $b = .12343$.
17. $B = 63° 41' 24''$.
    $b = 256.406$.
    $c = 286.033$.
18. .96565.
19. 164.93.
20. 1416.13.
21. 1614.26 yd. = depth of cañon. 5521.125 yd. = distance of river.
24. $B = 57.4°$.
    $a = 11.5125$.
    $c = 21.37$.
25. $B = 34°$.
    $a = 2.22$.
    $b = 1.4976$.
26. $A = 51.8°$.
    $a = .604$.
    $b = .4753$.
27. $A = 7.5°$.
    $b = 95.42$.
    $c = 96.225$.
28. $B = 62.33°$.
    $a = 77.43$.
    $b = 52.33$.
29. $A = 13.75°$.
    $b = 3.7845$.
    $c = 3.89583$.
30. $B = 68.68°$.
    $b = 41.65$.
    $c = 44.71$.
31. $A = 23.73°$.
    $a = .003824$.
    $c = .009504$.
32. .3907.
33. .11388.
34. 50.933.
35. $B = 1.83°$.
    $a = 13.125$.
    $b = .4194$.
36. $A = 47.84°$.
    $b = .4757$.
    $c = .7086$.
37. 129.15.
38. 1.081.
39. 352.1.
41. $B = 60°$.
    $a = \frac{7}{3}\sqrt{3} = 4.0425$.
    $c = \frac{14}{3}\sqrt{3} = 8.083$.
42. $a = b = 6\sqrt{2} = 8.484$.
43. $a = \frac{25}{3}\sqrt{3} = 14.43$.
    $c = \frac{50}{3}\sqrt{3} = 28.86$.
44. $b = \frac{1000}{3}\sqrt{3} = 577.4$.
    $c = \frac{2000}{3}\sqrt{3} = 1154.7$.
45. $b = \frac{2000}{3}\sqrt{3} = 1154.8$.
    $c = \frac{4000}{3}\sqrt{3} = 2309.5$.
46. $a = 600\sqrt{3} = 1039.25$.
    $b = 600$.
47. $a = 200$.
    $c = 200\sqrt{2} = 282.8$.
48. $a = 10\,d$.
    $b = 10\,d\sqrt{3} = 17.32\,d$.

49. Same as the respective answers for numbers 6 and 7 in this exercise.

51. $DB = 50$. $BC = 25$. $DC = \frac{25}{2}\sqrt{3} = 21.65\,x$.

Exercise 19

1. $A = 35° 33' 27''$. $b = 14.969$.
2. $A = 33° 18' 3''$. $b = 31.147$.
3. $A = 42° 24' 43''$. $b = 29.2557$.
4. $A = 39° 48' 20''$. $c = 7.81016$.
5. $A = 49° 44' 5''$. $b = .579587$.
6. $A = 49°$. $a = 16.3608$.
7. $A = 52° 12' 25''$. $c = .079471$.
8. $A = 43° 52'$. $b = .184875$.
9. $53° 7' 48''$.
10. $21° 53' 58''$.
11. $42° 24' 39''$.
12. $c = 8.48$.
13. $25° 48' 40''$.
14. $B = 16° 11' 7''$. $b = 32.702$.
15. $A = 8° 31' 31''$. $a = 53.666$.
16. $B = 17° 56' 5''$. $b = 8.6188$.
17. $13° 7' 18''$.
18. $\angle = 67° 22' 48''$, $\therefore 7' 12''$ too small.
21. $A = 41.49°$. $b = 17.755$.
22. $A = 45.17°$. $a = .39855$.
23. $A = 50.66°$. $c = 43.04$.
24. $A = 32.02°$. $c = 9.432$.
25. $A = 46.31°$. $a = 7.015$.
26. $A = 48.43°$. $c = .19107$.
27. $A = 40.67°$. $a = 86.64$.
28. $A = 40.95°$. $b = .0839$.
29. $A = 52.33°$. $c = 2987.33$.
30. $A = 43.44°$.
31. 50.43.
32. $A = 18.96°$. $a = 50.91$.
33. $B = 7.812°$. $b = 117.166$.
34. $57.26°$.
35. $26.77°$.
37. $A = B = 45°$. $c = 13\sqrt{2} = 18.384$.
38. $A = 30°$. $b = 9\sqrt{3} = 15.888$.
39. $B = 30°$. $a = 100\sqrt{3} = 173.2$.
40. $B = 30°$. $c = 2$.
41. $A = 60°$. $b = 3$.
42. $A = 45°$. $b = 1$.
43. $A = 60°$. $b = 50$.
44. $A = 30°$. $a = 6$. $c = 12$.

Exercise 20

1. Leg $= 120$. Vertex $\angle = 60°$.
2. Base $= 353.87$.
3. Base $= 9.6837$. Vertex $\angle = 67° 24'$.
4. Leg $= 50.699$. Base $= 79.578$. Vertex $\angle = 103° 24' 20''$.
5. Vertex $\angle = 69° 23' 12''$. Leg $= 927.84$. Base $= 1056.225$.
6. Leg $= 8.8204$. Base $\angle = 62° 10'$. Vertex $\angle = 55° 40''$.
7. Base $\angle = 33° 21' 30''$. Leg $= .075978$.
8. Base $\angle = 46° 16' 41''$. Vertex $\angle = 87° 26' 38''$. Leg $= 6690.16$.
9. $r = 8.2583$. $R = 10.208$. Perimeter $= 60$. Area $= 247.75$.
10. $r = 1.5388$. $R = 1.618$. Perimeter $= 10$. Area $= 7.694$.
11. Side $= 8.282$. $r = 15.455$. Area $= 768$.
12. Side $= 9.112$. $r = 17$. Area $= 929.24$.

13. Side = 8.6524.
r = 5.9546.
Perimeter = 43.262.
Area = 128.8.

14. Perimeter = 4.70498.
Area = 1.6417.

15. $h = l \sin D.$
$m = 2\,l \cos D.$
$C = 180° - 2\,D.$

16. $\tan D = \frac{2\,h}{m}.$
$l = \sqrt{h^2 + \left(\frac{m}{2}\right)^2}.$
$\tan \frac{1}{2}\,C = \frac{m}{2\,h}.$

17. $\sin D = \frac{h}{l}.$
$\cos \frac{1}{2}\,C = \frac{h}{l}.$
$m = 2\sqrt{l^2 - h^2}.$

18. $C = 180° - 2\,D.$
$h = \frac{1}{2}\,m \tan D.$
$l = \frac{m}{2} \sec D.$

19. $D = 90° - \frac{1}{2}\,C.$
$h = \frac{m}{2} \cot \frac{1}{2}\,C.$
$l = \frac{m}{2} \csc \frac{1}{2}\,C.$

20. Base = 3.889.
Base ∠ = 42° 15′ 34″.
Vertex ∠ = 95° 28′ 52″.

21. 12.7001.
22. 95.94.
23. 15.1848.
26. 8.1183.
27. 48.2055.
28. Base = 61.86.
Vertex ∠ = 114.8°.
29. Leg = 2081.5.
Vertex ∠ = 45.2°.
30. Leg = 34.47.
Base = 59.026.
Base ∠ = 31.14°.
31. Base ∠ = 52.86°.
Leg = .61014.
32. Base ∠ = 61.1°.
Base = 124.4.
33. Base = 114.2.
Vertex ∠ = 114.54°.

34. Base = .0588.
Leg = .12027.
35. Base ∠ = 54.275°.
Leg = 26.77.
36. Base = .8462.
Base ∠ = 14.15°.
37. r = 16.9.
Area = 946.5.
38. Perimeter = 143.166.
39. Side = 1.0878.
r = 1.6737.
40. Side = 20.22.
r = 21.
R = 23.3.
Area = 1486.34.
Perimeter = 141.54.

41. Side = 9.318.
r = 17.387.
Area = 972.
42. 22.025.
43. 111.4.
44. Altitude $= \frac{25}{3}\sqrt{3}$
= 14.435.
Base ∠ = 30°.
45. Base ∠ = 30°.
Base $= 50\sqrt{3}$
= 86.6.
46. $\frac{20}{3}\sqrt{3} = 11.547$
= leg = base.
47. Base ∠ = 45°.
Vertex ∠ = 90°.
Altitude = 6.
48. 120°.
49. 7.07.

Exercise 21

1. 12560.57.
2. 5911.7.
3. 172.756.
4. 545.44.
5. 3122.
6. 21519.5.
7. .19936.
8. 202281.818.

9. b = 3.416.
c = 4.2331.
A = 36° 11′ 53″.
10. a = 2.67815.
b = 5.41875.
c = 6.0445.
11. a = 13.1945.
b = 8.4405.
A = 57° 23′ 36″.
12. 42.847.

13. .088996.
14. .0287326.
15. 244.79.
16. 300.61.
17. h = 5.2496.
l = 6.1403.
A = 58° 45′ 17″.
18. l = 1.5086.
c = 2.6811.
h = .69175.

19. l = 7.1773.
c = 12.299.
h = 3.7011.
20. .7723.
21. 9.58675.
22. 1.5458.
23. .8874.
24. R = 3.22046.
c = 2.2029.
r = 3.0263.

25. Perimeter $= 21.265$.
26. $p = 23.187$.
$R = 3.9448$.
28. 938.
29. 47577.
30. 882.
31. .01618.
32. 31.47.
33. 137.33.
34. 6000000.
35. .00003529.
36. $a = 8.283$.
$A = 52.44°$.
$c = 10.45$.
37. $c = 77.22$.
$a = 68.9$.
$b = 34.84$.
38. Impossible.
39. .13833.
40. 149.07.
41. 4813.3.
42. 151.4.
43. 80.8.
44. .2084.
45. $h = 8.828$.
$A = 22.03°$.
$l = 23.54$.
46. $l = 1.2351$.
$h = .7478$.
$c = 1.9656$.
47. $l = 54.51$.
$c = 91.06$.
$h = 30.04$.
48. $c = .8598$.
$h = .2384$.
$A = 29°$.
49. 58.75.
50. .8308.
51. 36950.
52. 15.172.
53. $R = 2.262$.
$c = 1.9624$.
$r = 2.038$.
54. $R = 18.34$.
$c = 10.3332$.
$r = 17.6$.
55. $R = 4.031$.
$c = 2.7575$.
$r = 3.788$.
56. 101.36.
57. $2886.8 = \frac{10000}{6}\sqrt{3}$.
58. $180000\sqrt{3} = 301760$.
59. 298.78.
60. $4050\sqrt{3} = 7014.6$.
61. $3200\sqrt{3} = 5542.4$.
62. 800.
63. $2000000\sqrt{3} = 3464000$.
64. 7200.
65. $2500\sqrt{3} = 4330$.
66. $\frac{10000}{3}\sqrt{3} = 5773.3$.
67. $400\sqrt{3} = 692.8$.
68. 80,000.

Exercise 22

In this exercise, where two answers are given to an example, the first is the result obtained by use of five-place log tables, and the second answer is the result obtained by use of four-place tables.

1. 389.7 = Ht.
2. 474.788.
474.8.
3. 114.1.
4. 10° 33′ 25″.
10.56°.
5. 491.511.
491.44.
6. Base = 76.79.
Base = 76.8.
Alt. = 49.6955.
Alt. = 49.7.
Area = 1908.5.
Area = 1908.08.
7. 37° 58′ 46″.
37.975°.
8. Distance of ladder from house = 12.588.
12.58.
∠ ladder makes with house = 30° 14′ 8″
= 30.22°.
9. 695.414.
695.35.
10. 17° 31′ 7″.
17.52°.
11. 82.056.
82.06.
12. 287.25.
287.47.
13. 231.7.
231.68.
14. 1534.96.
1535.
16. Ht. of hill 1673.038.
Ht. of hill 1673.67.
Dis. of ship 6215.143.
Dis. of ship 6215.7.
17. $KR = 12\sqrt{3} = 20.784$.
$KA = 24$.
$KT = 6\sqrt{3} = 10.392$.
$RT = 18$.
$FT = 18\sqrt{3} = 31.176$.
$RF = 36$.
19. 23.013.
23.012.
20. 5246.25.
5246.6.
21. 43.3 = ht. of tree.
25 = width of river.
22. $KR = 12$.
$RP = 6\sqrt{3} = 10.392$.
$RS = 6\sqrt{6} = 14.694$.
$ST = 12\sqrt{3} = 20.784$.
$SF = 24$.
$TF = 12$.
23. 13.071.
13.053.
24. 71.264.
71.28.
25. 616.771.
616.5.
26. 45° 0′ 37″.
45°.
50.6375.
50.62.

27. $AB = \sin y$.
$OB = \cos y$.
$BC = \sin x \cos y$.
$OC = \cos x \cos y$.

29de $= 26.0.9$
$\measuredangle = 108° 14' 40''$.
$108.26°$.
$71° 45' 20''$.
$71.74°$.

Exercise 23

1. 2. 3. 3. 5. 4. 7. 4. 9. 3. 11. 1. 13. 4. 15. 4.
2. 2. 4. 4. 6. 1. 8. 3. 10. 3. 12. 2. 14. 2.

16. (1) Same as the signs of the functions in the second quadrant.
(3) Same as the signs of the functions in the third quadrant.
(5) Same as the signs of the functions in the fourth quadrant.

17. 385°. 745°. − 335°. − 695°.
18. 330°. 690°. − 390°. − 750°.
19. 460°. 820°. − 260°. − 620°.
20. 260°. 620°. − 460°. − 820°.

21. 65°. 22. 60°. 23. 60°. 24. 155°. 25. 40°. 26. 53°.

27. Second. 29. Second. 31. Fourth.
28. Third. 30. Third. 32. Second.

33. 8.052 (by use of five-place tables). 8.06 (by use of four-place tables).
34. 55.73.

Exercise 24

1. 2. 3. 0. 5. 4. 7. 0.
2. ∞. 4. $c^2 - a^2 + 4ac$. 6. $-2a$. 8. $3m$.

Exercise 25

1. $\sin 390° = \frac{1}{2}$.
$\cos 390° = \frac{1}{2}\sqrt{3}$.
$\tan 390° = \frac{1}{3}\sqrt{3}$.
$\sec 390° = \frac{2}{3}\sqrt{3}$.

2. $\cos 780° = \frac{1}{2}$.
$\tan 780° = \sqrt{3}$.
$\sin 780° = \frac{1}{2}\sqrt{3}$.
$\cot 780° = \frac{1}{3}\sqrt{3}$.

4. $\sin = \frac{1}{2}\sqrt{3}$.
$\cos = \frac{1}{2}$.
$\tan = \sqrt{3}$.
$\cot = \frac{1}{3}\sqrt{3}$.

5. $\sin = \frac{1}{2}$.
$\cos = \frac{1}{2}\sqrt{3}$
$\tan = \frac{1}{3}\sqrt{3}$.
$\cot = \sqrt{3}$.

6. $\sin = \frac{1}{2}\sqrt{2}$.
$\cos = \frac{1}{2}\sqrt{2}$.
$\tan = 1$.
$\cot = 1$.

7. $\sin = \frac{1}{2}$.
$\cos = \frac{1}{2}\sqrt{3}$.
$\tan = \frac{1}{3}\sqrt{3}$.
$\cot = \sqrt{3}$.

8. $\sin = \frac{1}{2}\sqrt{3}$.
$\cos = \frac{1}{2}$.
$\tan = \sqrt{3}$.
$\cot = \frac{1}{3}\sqrt{3}$.

9. $\sin = \frac{1}{2}\sqrt{2}$.
$\cos = \frac{1}{2}\sqrt{2}$.
$\tan = 1$.
$\cot = 1$.

10. $\sin x = \pm \frac{4}{5}$.
$\tan x = \mp \frac{4}{3}$.
$\cot x = \mp \frac{3}{4}$.
$\sec x = -\frac{5}{3}$.
$\csc x = \pm \frac{5}{4}$.

11. $\sin x = \pm \frac{15}{17}$.
$\cos x = \mp \frac{8}{17}$.
$\cot x = -\frac{8}{15}$.
$\sec x = \mp \frac{17}{8}$.
$\csc x = \pm \frac{17}{15}$.

12. $\cos x = \mp \frac{12}{13}$.
$\tan x = \pm \frac{5}{12}$.
$\sec x = \mp \frac{13}{12}$.
$\cot x = \pm \frac{12}{5}$.
$\csc x = -\frac{13}{5}$.

13. $\sin x = -\dfrac{\sqrt{5}}{5}$.
$\cos x = -\dfrac{2\sqrt{5}}{5}$.
$\tan x = \frac{1}{2}$.
$\cot x = 2$.
$\sec x = -\dfrac{\sqrt{5}}{2}$.
$\csc x = -\sqrt{5}$.

14. $\sin x = \dfrac{\sqrt{m^2-1}}{m}$
$\cos x = -\dfrac{1}{m}$.

$\tan x = -\sqrt{m^2 - 1}$.

$\cot x = -\dfrac{\sqrt{m^2-1}}{m^2-1}$.

$\sec x = -m$.

$\csc x = \dfrac{m\sqrt{m^2-1}}{\sqrt{m^2-1}}$.

15. $\sin x = -3\dfrac{\sqrt{10}}{10}$

$\cos x = \dfrac{\sqrt{10}}{10}$.

$\tan x = -3$.

$\cot x = - \frac{1}{3}$.

$\sec x = \sqrt{10}$.

$\csc x = -\dfrac{\sqrt{10}}{3}$.

16. $\sin x = -\dfrac{\sqrt{35}}{6}$.

$\cos x = -\frac{1}{6}$.

$\tan x = \sqrt{35}$.

$\sec x = -6$.

$\csc x = -\dfrac{6\sqrt{35}}{35}$.

$\cot x = \dfrac{\sqrt{35}}{35}$.

18. $\sin y = -\frac{1}{3}\sqrt{5}$.

$\csc y = -\frac{3}{5}\sqrt{5}$.

19. $\sin x = -\frac{1}{2}$.

$\cos x = \dfrac{\sqrt{3}}{2}$.

$\cot x = -\sqrt{3}$.

$\tan x = -\dfrac{\sqrt{3}}{3}$.

$\sec x = 2\dfrac{\sqrt{3}}{3}$.

$\csc x = -2$.

20. $\frac{13}{5}$.

21. $-\frac{25}{1183}$.

Exercise 26

1. $-\frac{1}{2}$. **2.** $\frac{1}{2}$. **3.** $-\sqrt{3}$. **4.** $-\sqrt{3}$. **5.** $-\sqrt{2}$. **6.** -1. **7.** $\frac{1}{3}\sqrt{3}$. **8.** $-\frac{1}{2}$. **9.** $-\frac{1}{2}$.

10. $-\dfrac{\sqrt{2}+5}{2}$.

11. $-\frac{10}{3}\sqrt{3} - 4$.

12. sin 38.

13. $-\tan 17°$.

14. $\sin 40°$.

15. $-\sec 5°$.

16. $\tan 5°$.

17. $-\tan 45°$.

18. $-\sin 20°$.

19. $-\sin 27°$.

20. $-\cot 25°$.

21. $\sec 30°$.

22. $-\sin 27°$.

23. $\cot 22°$.

24. $-\cos 10° 16'$.

25. $-\cot 30° 17'$.

26. $-\sec 25°$.

27. $\sin 8°$.

28. $-\tan 20°$.

29. $-\cot 30°$.

32. $9\frac{1}{2}$.

33. $11 \cos x$.

34. $a \cos x + b \sin x - c \tan x$.

35. $p \sin x \cos x$.

36 $-(a+b)\cos x - (a-b)\sin x$.

Exercise 27

1. $\frac{1}{2}\sqrt{2}$. **2.** $\sqrt{3}$. **3.** $-\frac{1}{2}$. **4.** $-\sqrt{3}$. **5.** $-\sqrt{3}$. **6.** 0. **7.** -2. **8.** $\frac{1}{3}\sqrt{3}$. **9.** $-\frac{1}{2}\sqrt{2}$.

10. $\sin = -\cos 29°$.
$\cos = -\sin 29°$.
$\tan = \cot 29°$.
$\cot = \tan 29°$.
$\sec = -\csc 29°$.
$\csc = -\sec 29°$.

11. $\sin = -\cos 9°$.
$\cos = \sin 9°$.
$\tan = -\cot 9°$.
$\cot = -\tan 9°$.
$\csc = -\sec 9°$.
$\sec = \csc 9°$.

12. $\sin = \sin 15°$.
$\cos = -\cos 15°$.
$\tan = -\tan 15°$.
$\cot = -\cot 15°$.
$\sec = -\sec 15°$.
$\csc = \csc 15°$.

13. $\sin = -\sin 15°$.
$\cos = \cos 15°$.
$\tan = -\tan 15°$.
$\cot = -\cot 15°$.
$\sec = \sec 15°$.
$\csc = -\csc 15°$.

14. $\sin = \cos 17°$.
$\cos = -\sin 17°$.
$\tan = -\cot 17°$.
$\cot = -\tan 17°$.
$\sec = -\csc 17°$.
$\csc = \sec 17°$.

15. $\sin = \cos 10°$.
$\cos = \sin 10°$.
$\tan = \cot 10°$.
$\cot = \tan 10°$.
$\sec = \csc 10°$.
$\csc = \sec 10°$.

16. $\sin = \sin 0°$.
$\cos = -\cos 0°$.
$\tan = \tan 0°$.
$\cot = \cot 0°$.
$\sec = -\sec 0°$.
$\csc = \csc 0°$.

17. $\sin = \sin 36° 43'$.
$\cos = -\cos 36° 43'$.
$\tan = -\tan 36° 43'$.
$\cot = -\cot 36° 43'$.
$\sec = -\sec 36° 43'$.
$\csc = \csc 36° 43'$.

18. $\sin = \cos 37.24°$.
$\cos = \sin 37.24°$.
$\tan = \cot 37.24°$.
$\cot = \tan 37.24°$.
$\sec = \csc 37.24°$.
$\csc = \sec 37.24°$.

21. $-\cos x$. 23. $-\sin x$. 25. $-\sec x$. 27. $-3\cos x$.
22. $-\cos x$. 24. $\tan x$. 26. $-\sec x$.
28. $-a\cos x+b\sin x-c\tan x$. 30. $\sin^2 x\cos x$.
29. $-m\cos A-p\cot A-q\cot A$. 31. $-\cos x$.

Exercise 28

1. 30°, 150°.
2. 30°, 150°, 210°, 330°.
3. 45°, 135°, 225°, 315°.
4. 30°, 150°, 210°, 330°.
5. 30°, 150°.
6. 60°, 300°, 180°.
7. 30°, 150°.
8. 45°, 225°.
9. 45°, 225°.
10. 60°, 240°.
11. 45°, 225°.
12. 45°, 135°, 225°, 315°.
13. 30°, 150°, 45°, 225°, 315°.
14. 60°, 120°, 240°, 300°, 45°, 135°, 225°, 315°.
15. 30°, 150°, 210°, 330°, 60°, 120°, 240°, 300°.
16. 30°, 150°, 210°, 330°.
17. 30°, 150°.
18. 30°, 150°.

Where two answers are given, the first answer is found by the five-place tables, the second answer is found by the four-place tables.

19. 66.35 mi. east. 66.34 mi. east. 27.14 mi. north.
20. 39° 10′ 25″. 39.18°. 21. 760.316. 760.33.
22. Distance of the spring from the house = 217.39. 217.4.
Distance of the spring from the barn = 229.12. 229.16.

Exercise 29

1. $\sin(x+y)=\frac{63}{65}$.
2. $\sin(x-y)=\frac{33}{65}$.
$\cos(x+y)=\frac{16}{65}$.
$\cos(x-y)=\frac{56}{65}$
3. $\sin(x+45°)=\frac{\sqrt{2}}{2}(\sin x+\cos x)$.
$\cos(30°-x)=\frac{\sqrt{3}\cos x+\sin x}{2}$.
$\sin(x-60°)=\frac{\sin x-\cos x\sqrt{3}}{2}$.
4. $(x+y)=\infty$.
5. $\cot(x-y)=0$.
6. $\frac{\sqrt{6}-\sqrt{2}}{4}$.
7. $2+\sqrt{3}$.
8. $\frac{\sqrt{6}+\sqrt{2}}{4}$. 9. $\sqrt{3}-2$. 10. $\frac{\sqrt{6}-\sqrt{2}}{4}$. 11. $\frac{\sqrt{2}-\sqrt{6}}{4}$. 12. $\sin 90°=1$. $\cos 90°=0$.
14. $\tan(45°+y)=\frac{1+\tan y}{1-\tan y}$.
$\tan(45°-y)=\frac{1-\tan y}{1+\tan y}$.
15. $\cot(60°+y)=\frac{\sqrt{3}\cot^2 y-4\cot y+\sqrt{3}}{3\cot^2 y-1}$.
$\cot(30°+y)=\frac{\sqrt{3}\cot^2 y+2\cot y}{\cot^2 y-3}$.

Exercise 30

1. $\sin 60°=\frac{1}{2}\sqrt{3}$.
$\cos 60°=\frac{1}{2}$.
2. $\tan 60°=\sqrt{3}$.
3. $\sin 120°=\frac{1}{2}\sqrt{3}$.
$\tan 120°=-\sqrt{3}$.
9. $3\sin x-4\sin^3 x$.
10. $4\cos^3 x-3\cos x$.
11. $\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$.
13. $-\frac{15}{8}$.
14. $-\frac{7}{24}$.
21. $\frac{1}{8}\cos 4x+\frac{1}{2}\cos 2x+\frac{3}{8}$.

Exercise 31

2. $\sin 15^\circ = \frac{1}{2}\sqrt{2-\sqrt{3}} = .2588.$
$\tan 15^\circ = 2-\sqrt{3} = .2679.$
$\cos 15^\circ = \frac{1}{2}\sqrt{2+\sqrt{3}} = .9659.$

3. $\cot 22\frac{1}{2}^\circ = \sqrt{2}+1 = 2.4142.$
$\cos 22\frac{1}{2} = \frac{1}{2}\sqrt{2+\sqrt{2}} = .9239.$
$\sin 22\frac{1}{2} = \frac{1}{2}\sqrt{2-\sqrt{2}} = .3827.$

4. $\sin 45^\circ = \cos 45^\circ = \frac{1}{2}\sqrt{2} = .7071.$
$\tan 45^\circ = \cot 45^\circ = 1.$
$\sec 45^\circ = \csc 45^\circ = \sqrt{2} = 1.4142.$

5. $\cos \frac{1}{2} A = \frac{1}{6}\sqrt{18+6\sqrt{5}}.$
$\cot \frac{1}{2} A = \frac{3+\sqrt{5}}{2}.$
$\tan \frac{1}{2} A = \frac{3-\sqrt{5}}{2}.$

6. $\cos \frac{\theta}{2} = \frac{1}{2}\sqrt{2+2a}.$
$\cot \frac{\theta}{2} = \frac{1}{1-a}\sqrt{1-a^2}.$
$\tan \frac{\theta}{2} = \frac{1}{1+a}\sqrt{1-a^2}.$

12. $\cos A = \sqrt{\frac{1+\cos 2A}{2}}.$
$\sin A = \sqrt{\frac{1-\cos 2A}{2}}.$
$\cot A = \sqrt{\frac{1+\cos 2A}{1-\cos 2A}}.$

13. $\frac{-15}{4}.$

14. $-\frac{3\sqrt{5}+25}{21}.$

16. $A = 79^\circ\, 36'\, 40''.$
$A = 79.726^\circ.$
$b = 22.$

17. $2^\circ\, 44'\, 40''.$
$2.744^\circ.$

Exercise 32

13. $\sin(A+B) = \frac{\sqrt{15}+\sqrt{3}}{8}.$
$\sin(A-B) = \frac{\sqrt{15}-\sqrt{3}}{8}.$
$\cos(A+B) = \frac{3\sqrt{5}-1}{8}.$
$\cos(A-B) = \frac{3\sqrt{5}+1}{8}.$
$\sin 2A = \frac{1}{2}\sqrt{3}.$
$\sin 2B = \frac{1}{8}\sqrt{15}.$
$\cos 2A = \frac{1}{2}.$
$\cos 2B = \frac{7}{8}.$

14. $\sin(60^\circ + 30^\circ) = 1.$
$\sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}+1}{2}.$

15. $-\frac{\sin 29.5^\circ \cos 7.5^\circ}{\sin 27^\circ \sin 11^\circ}.$

16. $\frac{2\cos 3A \sin A}{\cos 6A}.$

17. $\sin(A+B)\sin(A-B).$

18. 3.44.
.2136.

Exercise 34

1. $\csc\theta = -\frac{5}{4}.$
$\cot\theta = \frac{3}{4}.$
$\sin\frac{1}{2}\theta = \frac{2}{5}\sqrt{5}.$
$\tan(180^\circ-\theta) = -\frac{4}{3}.$
$\sin(-\theta) = \frac{4}{5}.$

2. $\frac{4}{5}.$

3. $2+\sqrt{3}.$

4. $\sin 2x = \pm\frac{3}{32}\sqrt{7},$
the sign depending on whether $\frac{1}{2}x$ is taken in the first or fourth quadrants.
In like manner:
$\tan 2x = \mp\frac{3}{31}\sqrt{7}.$

5. $\cos 15^\circ = \frac{1}{2}\sqrt{2+\sqrt{3}}.$
$\csc 15^\circ = 2\sqrt{2+\sqrt{3}}.$
$\tan 15^\circ = 2-\sqrt{3}.$

6. (a) $= \frac{3-4\sqrt{3}}{10}.$
(b) $= \frac{4-3\sqrt{3}}{10}.$
(c) $= \frac{4+3\sqrt{3}}{10}.$

(d) $= \frac{1}{2}\sqrt{3}$.
(e) $= -\frac{1}{2}$.
(f) $= -\sqrt{3}$.
(g) $= \frac{7}{24}$.
(h) $= \frac{25\sqrt{3}-48}{11}$.
(i) $= \frac{25\sqrt{3}-48}{39}$.
(j) $= \frac{1}{2}\sqrt{3}$.

7. (a) $= -\frac{\sqrt{5}}{2}$.
(b) $= \frac{1}{2}$.
(c) $= -2$.
(d) $-\frac{2}{5}\sqrt{5}$.

8. (a) $\begin{cases} \sin\left(x-\frac{\pi}{2}\right) = -\cos x. \\ \cos\left(x-\frac{\pi}{2}\right) = \sin x. \\ \tan\left(x-\frac{\pi}{2}\right) = -\cot x. \\ \cot\left(x-\frac{\pi}{2}\right) = -\tan x. \end{cases}$

(b) $\begin{cases} \sin(\pi-\theta) = \sin\theta. \\ \cos(\pi-\theta) = -\cos\theta. \\ \tan(\pi-\theta) = -\tan\theta. \\ \cot(\pi-\theta) = -\cot\theta. \end{cases}$

(c) $\begin{cases} \sin\left(x-\frac{3\pi}{2}\right) = \cos x. \\ \cos\left(x-\frac{3\pi}{2}\right) = -\sin x. \\ \tan\left(x-\frac{3\pi}{2}\right) = -\cot x. \\ \cot\left(x-\frac{3\pi}{2}\right) = -\tan x. \end{cases}$

(d) $\begin{cases} \sin(\pi+x) = -\sin x. \\ \cos(\pi+x) = -\cos x. \\ \tan(\pi+x) = \tan x. \\ \cot(\pi+x) = \cot x. \end{cases}$

34. $-\frac{1}{2}$. **35.** $-\frac{4}{5}$. **36.** $\frac{3}{2}$. **37.** $-\frac{2}{3}\sqrt{3}$. **38.** $-2b$.

39. $\tan\theta = \frac{3}{4}$.
$\sin\theta = -\frac{3}{5}$.

41. $-\frac{17}{25}$.

53. $\frac{3 - 4\cos 4x + \cos 8x}{128}$.

54. $\frac{1}{128}(35 - 64\cos 2x + 32\sin^2 2x\cos 2x + 28\cos 4x + \cos 8x)$.

Exercise 35

3. $a = c\cos B$.

7. (I) $\frac{a-b}{a+b} = \tan(A - 45°)$ and a right triangle.

(II) $a + b = (a-b)(2+\sqrt{3})$ an isosceles triangle with the angles 30°, 30°, 120°.

9. $\sin B = \frac{b}{a}$.

$\sin A = \frac{a}{b}$.

Exercise 36

1. $c = 9.1226$.
$C = 41° 7'$.
$b = 13.288$.

2. $A = 134° 20'$.
$b = 74.9916$
$c = 242.755$

3. $A = 57° 52'$.
$a = 1116.98$.
$c = 1260.26$.

4. $A = 109° 19'$.
$a = 4899.56$.
$b = 4106$.

5. $C = 69° 57' 36''$.
$a = .85442$.
$b = .81196$.

6. $A = 29° 1' 2'$.
$a = 56.541$.
$b = 90.164$.

7. $A = 99° 29' 12$.
$b = 1.0943$.
$c = .488667$.

8. $B = 43° 18' 36''$.
$b = 1.3487$.
$c = 1.8286$.

9. $C = 68° 15' 30'$.
$a = .182095$.
$b = .188745$.

10. $b = 5.267\sqrt{2}$.
$= 7.4486$.
$c = 2.6335(\sqrt{6}+\sqrt{2})$.
$= 11.175$.
$C = 105°$.

11. $C = 75°$.
$a = 500(3\sqrt{2} - \sqrt{6})$.
$= 896.55$.
$b = 500(2\sqrt{3} - 2)$.
$= 732.1$.

12. 4.0954. 11.697.

13. $b = 17.08$.
$c = 15.097$.
$C = 56.73°$.

14. $a = 634.3$.
$b = 632.89$.
$A = 81.32°$.

15. $c = 1.022$.
$a = 1.4815$.
$B = 25.57°$.

16. $c = 38.52$.
$b = 57.412$.
$A = 79.9°$.

17. $a = 13283.34$.
$c = 13346.67$.
$A = 80° 46'$.

18. $a = 600.4$.
$b = 602$.
$C = .75°$.

19. $c = 7.295$.
$b = 14.83$.
$A = 117.67°$.

20. $b = .2592$.
$a = .2181$.
$C = 55.87°$.

21. $a = 186.25$.
$c = 32.5$.
$A = 101.96°$.

22. $c = 4377$.
$b = 5641.43$.
$A = 55.69°$.

23. $a = 20.343$.
$c = 28.66$.
$B = 27.77°$.

24. $a = 838.83$.
$b = 595.1$.
$C = 56.6°$.

25. $b = c = a = 100$.
$B = C = A = 60°$.

26. $C = 30°$.
$a = 200\sqrt{3} = 346.42$.
$b = c = 200$.

27. $C = 45°$.
$b = 250(3\sqrt{2} - \sqrt{6}) = 448.3$.
$c = 250(2\sqrt{3} - 2) = 365.7$.

28. $B = 30°$.
$c = 200\sqrt{2} = 282.8$.
$a = 100(\sqrt{6} + \sqrt{2}) = 386.4$.

29. 925.8.

30. Distance of balloon from first point = 2033 yd.
Distance of balloon from second point = 2363 yd.
Height of balloon = 1739 yd.

Exercise 37

1. $c = 26.8675$.
$B = 39° 45' 17''$.
$A = 72° 14' 43''$.

2. $a = 385.43$.
$B = 74° 38' 19''$.
$C = 37° 3' 41''$.

3. $C = 110° 22' 10''$.
$B = 39° 25' 30''$.
$a = .1912$.

4. $A = 48° 42' 12''$.
$C = 67° 42' 18''$.
$b = .0748566$.

5. $C = 34° 6' 36''$.
$B = 22° 36' 54''$.
$a = 4.70177$.

6. $a = 336.446$.
$B = 99° 55' 36''$.
$C = 27° 58' 24''$.

7. $8.185 = c$.

8. $C = 109° 36' 5''$.
$B = 38° 5' 25''$.
$a = 14.962$.

9. $C = 6° 49' 41''$.
$b = 317.8$.
$A = 4° 51' 41''$.

10. $A = 49.06°$.
$c = 208.1$.
$B = 79.117°$.

11. $a = .9418$.
$B = 117.99°$.
$C = 33.85°$.

12. $A = 32.24°$.
$C = 35.58°$.
$b = .6566$.

13. $B = 141.99°$.
$A = 25.89°$.
$c = 3.972$.

14. $A = 79.82°$.
$C = 21.56°$.
$b = 1712.3$.

15. $a = 7.93$.

16. $B = 6.23°$.
$C = 4.97°$.
$a = 5.906$.

17. $c = 102.425$.
$A = 65.83°$.
$B = 45.93°$.

18. $A = 33.84°$.
$B = 102.98°$.
$c = 1474.67$.

19. $b = 10.7$.

Where two answers are given, the first answer is obtained by using the five-place tables, and the second answer is obtained by the use of the four-place tables.

20. Distance = 234.34 ft.
Distance = 234.32 ft.

21. 4.36 mi.

22. Resultant = 14.989.
Resultant = 15.08.
$\angle$ with $OA = 77° 11' 20''$.
$\angle$ with $OA = 77.23°$.
$\angle$ with $OB = 43° 31' 40''$.
$\angle$ with $OB = 43.49°$.

23. 3.59.

24. 152.268.
152.22.
238.31.
238.22.

Exercise 38

1. $A = 78° 5' 36''$. 78.1°.
$B = 58° 23' 28''$. 58.38°.
$C = 43° 30' 58''$. 43.52°.

2. $A = 44° 32' 4''$. 44.53°.
$B = 86° 25'$. 86.41°.
$C = 49° 2' 58''$. 49.05°.

3. $A = 26° 19' 54''$. 26.33°.
$B = 98° 18' 54''$. 98.32°.
$C = 55° 21' 14''$. 55.36°.

4. $A = 45° 11' 50''$. 45.19°.
$B = 101° 22' 18''$. 101.38°.
$C = 33° 25' 58''$. 33.43°.

5. $A = 43° 53' 14''$. 43.88°.
$B = 60° 3' 36''$. 60.06°.
$C = 76° 3' 18''$. 76.06°.

6. $A = 61° 53' 38''$. 61.88°.
$B = 72° 46' 4''$. 72.78°.
$C = 45° 20' 20''$. 45.34°.

7. $A = 91° 48'$. 91.80°.
$B = 47° 36' 56''$. 47.61°.
$C = 40° 35' 10''$. 40.59°.

8. $A = 37° 50' 40''$. 37.84°.
$B = 127° 3'$. 127.05°.
$C = 15° 6' 22''$. 15.11°.

9. $A = 40° 38' 22''$. 40.64°.
$B = 49° 21' 56''$. 49.36°.
$C = 89° 59' 46''$. 90°.

10. $A = 52° 20' 30''$. 52.34°.
$B = 107° 19' 12''$. 107.32°.
$C = 20° 20' 26''$. 20.34°.

11. $A = 13° 12' 8''$. 13.2°.
$B = 30° 2' 46''$. 30.04°.
$C = 136° 45' 6''$. 136.76°.

12. $A = 46° 19' 52''$. 46.33°.
$B = 31° 17' 50''$. 31.3°.
$C = 102° 22' 18''$. 102.37°.

13. $A = 107° 55' 12$. 107.92°.
$B = 35° 15' 34''$. 35.26°.
$C = 36° 49' 18''$. 36.82°.

14. 104° 28′ 42″. 104.48°.

15. 16° 44′ 6″. 16.736°.

16. .53224. .5323.

17. .1188.

18. 14.8586. 14.86.

20. Q is 53° 7′ 48″ (53.14°) north of west from P.
Q is 38° 52′ 48″ (38.88°) north of west from R.
P is due west of R.
P is 36° 52′ 12″ (36.86°) east of south from Q.
R is due east of P.
R is 38° 52′ 48″ (38.88°) south of east from Q.
When R is northeast from P :
Q is 8° 7′ 48″ (8.14°) north of west from P.
Q is 6° 7′ 12″ (6.12°) south of west from R.
R is 6° 7′ 12″ (6.12°) north of east from Q.
P is southwest from R. P is 8° 7′ 48″ (8.14°) south of east from Q.

21. 28° 57′ 17″. 28.96°.

Exercise 39

1. One solution.

2. Two solutions.

3. One solution.

4. No solution.

5. No solution.

6. One solution.

7. One solution, a right △.

8. No solution.

9. Two solutions.

10. $B = 32° 36' 33''$.
$C = 109° 5' 27''$.
$c = 211.48$.

11. $B = 40° 40'$.
$B' = 16° 44'$.
$C = 78° 2'$.
$C' = 101° 58'$.
$b = 15.787$.
$b' = 6.9753$

12. $B = 42° 44' 23''$. .
$A = 33° 1' 49''$.
$a = 92.942$.

13. $A = 18° 19' 43''$.
$C = 139° 17' 59''$.
$c = 1.3952$.

14. $B = 70° 47'$.
$B' = 14° 35'$.
$C = 61° 54'$.
$C' = 118° 6'$.
$b = 128.455$.
$b' = 34.2515$.

15. $A = 32° 55' 57''$.
$A' = 147° 4' 3''$.
$C = 131° 33' 51''$.
$C' = 17° 25' 45''$.
$c = 1643.96$.
$c' = 661.15$.

16. $A = 43° 38'$.
$B = 58° 3' 42''$.
$b = .32868$.

17. $A = 90°$.
$c = 25.64$.

18. $B = 28° 16' 25''$.
$C = 20° 25' 11''$.
$b = .56045$.

19. $A = 103° 50' 22''$.
$A' = 13° 7' 8'' = A$.
$a = 15.354$.
$a' = 3.589$.
$B = 44° 38' 23''$.
$B' = 135° 21' 37''$.

20. $A = 35.91°$.
$A' = 144.09°$.
$C = 111.72°$.
$C' = 3.54°$.
$c = 219.7$.
$c' = 14.6$.

21. $B = 55°$.
$B' = 10.26°$.
$C = 67.63°$.
$C' = 112.37°$.
$b = 20.118$.
$b' = 4.372°$.

22. $A = 25.22°$.
$C = 49.51°$.
$a = 135.46$.

23. $A = 20.79°$.
$B = 132.99°$.
$b = 136.733$.

24. $A = 16.25°$.
$A' = 163.75°$.
$C = 149.45°$.
$C' = 1.95°$.
$c = 36.63$.
$c' = 2.4518$.

25. $B = 122.81°$.
$B' = 12.45°$.
$C = 34.81°$.
$C' = 145.19°$.
$b = 441.7$.
$b' = 113.2$.

26. $A = 70.78°$.
$C = 45.91°$.
$a = 10.08$.

27. $A = 72.16°$.
$A' = 9.22°$.
$B = 58.53°$.
$B' = 121.47°$.
$a = .19685$.
$a' = .03313$.

28. Other side $= \begin{cases} 129.1. \\ 129.125. \end{cases}$

Other diagonal $= \begin{cases} 41.66. \\ 41.62. \end{cases}$

Larger angle of parallelogram $= \begin{cases} 173° 15' 8''. \\ 173.26°. \end{cases}$

Smaller angle of parallelogram $= \begin{cases} 6° 44' 52''. \\ 6.74°. \end{cases}$

29. 1010.58.
1010.2.

Exercise 40

1. 106.79.
 106.8.
2. .30733.
 .30726.
3. 125.229.
 125.225.
4. 14290.6.
 14290.
5. 38983.64.
 38983.33.
6. 113.55.
7. .054776.
 .0547875.
8. 1056.66.
 1056.25.
9. 1283.5.
10. 42150.
 42130.77.
11. Area of parallelogram $= cd \sin A$.
13. $600\sqrt{3} = 1039.2$.
14. 106.798.
 106.8.

Exercise 41

In this exercise when two answers are given to an example, the first answer is found by the use of five-place tables, and the second answer is found by four-place tables.

4. 69.372.
 69.37.
5. 72.268.
 72.27.
6. 8968.5 ft. above the Colorado plain.
 8958 ft. above the Colorado plain.
 14144.5 ft. above sea level.
 14134 ft. above sea level.
7. 373.3.
8. 69.98.
9. 136.9.
10. 1016.6.
 1016.8.
11. Height = 97.083.
 Height = 97.08.
 Distance = 71.787.
 Distance = 71.78.
12. 10.274.
 6.61.
13. 16.83.
14. Other side $= 43.43$.
 Other diagonal $= \begin{cases} 58.342. \\ 58.346. \end{cases}$
 $\begin{cases} 146° 52' 47''. \\ 146.88°. \end{cases}$
 $\begin{cases} 33° 7' 13''. \\ 33. \quad 12°. \end{cases}$
15. Height = 42.93.
 Height = 42.92 ft.
 Distance = 104.63.
 Distance = 104.675 ft.
16. 11.36.
 5.573.
17. .1189.
18. 4.2818.
 4.283.
19. 1496.517.
 1496.66.
20. First answer = 4.4867 mi., 4.488 mi.
 Second answer = 9.16 mi.
21. 996.94.
 997.6.
22. 401.52.
 401.54.
23. 443.54.
 443.5.
24. 974.145.
 973.9.
25. 220.7.
26. 16.58.
27. 6739 m.
 6740 m.
28. 9° 6′.
29. Difference of latitude = difference of departure = 247.5 mi.
 New latitude = 34° 23′ North.
 New longitude = 48° 9′ W.
30. 152.69 ft.
 152.7 ft.
31. 114.5 ft.
32. $\left.\begin{matrix} 85.854 \text{ ft.} \\ 85.89 \text{ ft.} \end{matrix}\right\} =$ distance between observers.
 38.566 ft. = distance of first observer from the rock.
33. 2008 = resultant.
 $\left.\begin{matrix} 72° 16' \\ 72.27° \end{matrix}\right\} =$ angle the resultant makes with OX.

35. 298.

36. 1813 ft.

37. 161.8 ft.

38. $97° 2' 32$
$97.06°$ and
$14° 57' 28''$
$14.94°$ respectively.

39. 367.89 ft., 367.9 ft. } = side opposite tower.
90.032 ft., 90.04 ft. and 379.125 ft., 379.1 ft. } = the other two sides respectively.

40. 48 ft. and 108 ft. respectively.

41. $40° 0' 16''$, $40°$ } = angle the slope makes with the embankment.
29.45 ft. = width of base.

42. 161.3.

43. $22° 49' 46''$.
$22.83°$.

44. 85.27 mi.

Exercise 42

1. $30° = \frac{\pi}{6}$.
$135° = \frac{3\pi}{4}$.
$60° = \frac{\pi}{3}$.
$90° = \frac{\pi}{2}$.
$210° = \frac{7\pi}{6}$.
$270° = \frac{3\pi}{2}$.
$225° = \frac{5\pi}{4}$.
$72° = \frac{2\pi}{5}$.
$315° = \frac{7\pi}{4}$.

2. $\frac{\pi}{6} = 30°$.
$\frac{\pi}{4} = 45°$.
$\frac{\pi}{3} = 60°$.
$\frac{2\pi}{3} = 120°$.
$\frac{4\pi}{5} = 144°$.
$\frac{3\pi}{5} = 108°$.
$\frac{7\pi}{5} = 252°$.
$\frac{8\pi}{15} = 96°$.

3. $1° = .01745$ radian.
$16'' = .00007757$ radian.
$2' 15'' = .0006545$ radian.
$5° 14' = .0913374$ radian.

4. 2 radians $= 114° 35' 30''$.
3.2 radians $= 183° 20' 48''$.
.003 radian $= 0° 10' 18.8''$.

5. Arc 21 in. long $= \frac{3}{2}$ radian.
Arc 7 in. long $= \frac{1}{2}$ radian.

6. $R = 28$.

7. Radians = 1.118.
Angle $= 64° 3' 22.5''$.

8. Angles $= 85°;\ 25°$.
= 1,47325 radians; .43625 radian.

9. Complement of $\frac{\pi}{6} = \frac{\pi}{3}$, 60°; supplement $= \frac{5\pi}{6}$, 150°.

Complement of $\frac{\pi}{3} = \frac{\pi}{6}$, 30°; supplement $= \frac{2\pi}{3}$, 120°.

Complement of $\frac{\pi}{4}$, $45° = \frac{\pi}{4}$, 45°; supplement $= \frac{3\pi}{4}$, 135°.

Complement of $\frac{\pi}{9} = \frac{7\pi}{18}$, 70°; supplement $= \frac{8\pi}{9}$, 160°.

Complement of $\frac{5\pi}{18} = \frac{2\pi}{9}$, 40°; supplement $= \frac{13\pi}{18}$, 130°.

10. $\sin\frac{\pi}{6}=\frac{1}{2}$. $\cos=\frac{1}{2}\sqrt{3}$.
$\tan=\frac{1}{3}\sqrt{3}$. $\cot=\sqrt{3}$.
$\sec=\frac{2}{3}\sqrt{3}$. $\csc=2$.

$\sin\frac{\pi}{3}=\frac{1}{2}\sqrt{3}$. $\cos=\frac{1}{2}$.
$\tan=\sqrt{3}$. $\cot=\frac{1}{3}\sqrt{3}$.
$\sec=2$. $\csc=\frac{2}{3}\sqrt{3}$.

$\sin\frac{\pi}{4}=\cos\frac{\pi}{4}=\frac{1}{2}\sqrt{2}$.

$\tan\frac{\pi}{4}=\cot\frac{\pi}{4}=1$.

$\sec\frac{\pi}{4}=\csc\frac{\pi}{4}=\sqrt{2}$.

$\sin\frac{\pi}{2}=1$. $\cot\frac{\pi}{2}=0$.

$\cos\frac{\pi}{2}=0$. $\sec\frac{\pi}{2}=\infty$.

$\tan\frac{\pi}{2}=\infty$. $\csc\frac{\pi}{2}=1$.

$\sin\frac{3\pi}{4}=\frac{1}{2}\sqrt{2}$. $\cos=-\frac{1}{2}\sqrt{2}$.
$\tan=\cot=-1$.
$\sec=-\sqrt{2}$. $\csc=\sqrt{2}$.

$\sin\frac{7\pi}{6}=-\frac{1}{2}$. $\cos=-\frac{1}{2}\sqrt{3}$.
$\tan=\frac{1}{3}\sqrt{3}$. $\cot=\sqrt{3}$.
$\sec=-\frac{2}{3}\sqrt{3}$. $\csc=-2$.

$\sin\frac{7\pi}{4}=-\frac{1}{2}\sqrt{2}$. $\cos=\frac{1}{2}\sqrt{2}$.
$\tan=\cot=-1$.
$\sec=\sqrt{2}$. $\csc=-\sqrt{2}$.

11. $1\frac{1}{5}$ radians $=68°\ 45'\ 18''$.

13. $R=4$.
$A=143°\ 14'\ 22.5''$.

14. $a=12.5$.
$A=14°\ 19'\ 26\frac{1}{4}''$.

15. $\rho=8$.
$A=458°\ 22'$.

16. $\rho=.26175$.
$a=10.9935$.

17. $\rho=.64565$.
$R=154.89$.

18. $4'\ 35''$.

19. 69.102 ft.

20. $4'\ 20''$.

21. 1117 mi.

22. 437320 mi.

23. 35374500 mi.

24. $\frac{3}{2}\sqrt{2}-6$.

Exercise 44

2. $\frac{\pi}{3}, \frac{2\pi}{2}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

3. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

4. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

5. $\frac{\pi}{6}, \frac{5\pi}{6}$.

6. $\frac{\pi}{3}, \frac{5\pi}{3}$.

7. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

8. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

9. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$.

10. $\frac{\pi}{6}, \frac{5\pi}{6}$.

11. $0°, \pi$.

12. $\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}$.

13. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

14. $\frac{\pi}{3}, \frac{2\pi}{3}, \pi$.

15. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

16. $\frac{\pi}{6}, \frac{7\pi}{6}$.

17. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, 0, \frac{\pi}{3}$.

18. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{3}$.

19. $0, \frac{\pi}{3}, \frac{\pi}{6}, \frac{5\pi}{6}$.

20. $0, \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{3\pi}{2}$.

21. $0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

22. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

Exercise 45

1. $\theta = 30°, 210°.$ $x = 100, -100.$
2. $\theta = 36.5°, 216.5°.$ $x = 200, -200.$
3. $\theta = 58.51°, 301.49°.$ $x = 500, -500.$
4. $x = 50°.$ $y = 40°.$
5. $x = 1000.$ $y = 2000.$
6. $x = 60°.$ $y = 45°.$
7. $x = 36.87°.$ $y = 22.62°.$
8. $x = 1000.$ $\theta = 72.5°.$
9. $x = a\cos A + b\sin A.$ $y = b\cos A - a\sin A.$

Exercise 46

1. $\cos^{-1}\frac{1}{2}\sqrt{2} = 45°, \frac{\pi}{4}.$

   $\tan^{-1}\sqrt{3} = 60°, \frac{\pi}{3}.$

   $\sin^{-1}\frac{1}{2} = 30°, \frac{\pi}{6}.$

   $\sec^{-1}\sqrt{2} = 45°, \frac{\pi}{4}.$

   $\csc^{-1}\frac{2}{3}\sqrt{3} = 60°, \frac{\pi}{3}.$

   $\cot^{-1}\sqrt{3} = 30°, \frac{\pi}{6}.$

   $\cos^{-1}\frac{1}{2} = 60°, \frac{\pi}{3}.$

   $\sec^{-1}2 = 60°, \frac{\pi}{3}.$

   $\sin^{-1}\frac{1}{2}\sqrt{3} = 60°, \frac{\pi}{3}.$

   $\cot^{-1}\frac{1}{3}\sqrt{3} = 60°, \frac{\pi}{3}.$

   $\tan^{-1}\frac{1}{3}\sqrt{3} = 30°, \frac{\pi}{6}.$

2. $\cos(\cot^{-1}\frac{3}{4}) = \frac{3}{5}.$
3. $\tan(\sin^{-1}\frac{5}{13}) = \frac{5}{12}.$
4. $\sec(\tan^{-1}\frac{8}{15}) = \frac{17}{15}.$
5. $\sin(\cot^{-1}a) = \dfrac{\sqrt{1+a^2}}{1+a^2}.$
6. $\cot\left(\cos^{-1}\dfrac{a}{b}\right) = \dfrac{a\sqrt{b^2-a^2}}{b^2-a^2}.$
7. $\tan(2\sin^{-1}\frac{1}{2}) = \sqrt{3}.$
8. $\sin(2\tan^{-1}\frac{5}{12}) = \frac{120}{169}.$
9. $\cos(2\sec^{-1}\frac{17}{8}) = -\frac{161}{289}.$
10. $\sin(\frac{1}{2}\cos^{-1}\frac{1}{3}) = \frac{1}{3}\sqrt{3}.$
11. $\cot(\frac{1}{2}\tan^{-1}\frac{15}{8}) = \pm\frac{5}{3}.$
12. $\sin(3\sin^{-1}\frac{1}{2}) = 1.$
13. $\sin(\sin^{-1}\frac{1}{2} - \cos^{-1}\frac{2}{3}) = \dfrac{2-\sqrt{15}}{6}.$
14. $\tan(\tan^{-1}2 + \cot^{-1}3) = 7.$

30. $\frac{\pi}{6} \pm 2n\pi,$ $\frac{5\pi}{6} \pm 2n\pi.$
31. $\frac{\pi}{6} \pm 2n\pi,$ $\frac{7\pi}{6} \pm 2n\pi.$
32. $\frac{\pi}{4} \pm 2n\pi,$ $\frac{7\pi}{4} \pm 2n\pi.$
33. $\frac{\pi}{3} \pm 2n\pi,$ $\frac{4\pi}{3} \pm 2n\pi.$
34. $\frac{\pi}{3} \pm 2n\pi,$ $\frac{2\pi}{3} \pm 2n\pi.$
35. $\frac{\pi}{6} \pm 2n\pi,$ $\frac{11\pi}{6} \pm 2n\pi.$
36. $\frac{\pi}{2} \pm 2n\pi,$ $\frac{3\pi}{2} \pm 2n\pi.$
37. $\frac{\pi}{6} \pm 2n\pi,$ $\frac{7\pi}{6} \pm 2n\pi.$
38. $\frac{\pi}{4} \pm 2n\pi,$ $\frac{7\pi}{4} \pm 2n\pi.$
39. $\frac{7\pi}{6} \pm 2n\pi,$ $\frac{11\pi}{6} \pm 2n\pi.$
42. $x = \frac{4\pi}{3}.$

43. $30^\circ = \sin^{-1}\frac{1}{2} = \cos^{-1}\frac{1}{2}\sqrt{3} = \tan^{-1}\frac{1}{3}\sqrt{3} = \cot^{-1}\sqrt{3}.$

$60^\circ = \sin^{-1}\frac{1}{2}\sqrt{3} = \cos^{-1}\frac{1}{2} = \tan^{-1}\sqrt{3} = \cot^{-1}\frac{1}{3}\sqrt{3}.$

$90^\circ = \sin^{-1}1 = \cos^{-1}0 = \tan^{-1}\infty = \cot^{-1}0.$

$45^\circ = \sin^{-1}\frac{1}{2}\sqrt{2} = \cos^{-1}\frac{1}{2}\sqrt{2} = \tan^{-1}1 = \cot^{-1}1.$

$0^\circ = \sin^{-1}0 = \cos^{-1}1 = \tan^{-1}0 = \cot^{-1}\infty.$

$n\,180^\circ = \sin^{-1}0 = \tan^{-1}0.$

$n\,90^\circ = \cos^{-1}0 = \cot^{-1}0.$